## Schedule...

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Equivalence - Equality

Mosiah 29: 38

38 Therefore they relinquished their desires for a king, and became exceedingly anxious that every man should have an equal chance throughout all the land; yea, and every man expressed a willingness to answer for his own sins.
Current Sources

◆ All current sources can be modeled as voltage sources (and vice-versa)
  ▲ Many sources are best modeled as voltage sources (batteries, electric outlets etc.)
  ▲ There are some things that are best modeled as current sources:
    • Van de Graaff generator

Behaves as a current source because of its very high output voltage coupled with its very high output resistance and so it supplies the same few microamps at any output voltage up to hundreds of thousands of volts
Lecture 9 – Equivalent Circuits

Thévenin Equivalent
Norton Equivalent
Network Analysis

Network Analysis Methods:

✓ Node voltage method
✓ Mesh current method
✓ Superposition

ṟ Equivalent circuits
  ✓ Source transformation
  ũ Thévenin equivalent
  ũ Norton equivalent
Equivalent Circuits

It is always possible to view a complicated circuit in terms of a much simpler equivalent source and equivalent load circuit.
It is always possible to view a complicated circuit in terms of a much simpler equivalent source and equivalent load circuit.
Equivalent Circuits

Equivalent circuits fall into one of two classes:

- **Thévenin**: voltage source \( v_T \) and series resistor \( R_T \)
- **Norton**: current source \( i_N \) and parallel resistor \( R_N \)

\[
\begin{align*}
\text{Thévenin Equivalent} & \quad \text{Norton Equivalent} \\
\begin{array}{c}
\text{Load} \\
R_T \\
i \\
v \\
\end{array} & \quad \begin{array}{c}
\text{Load} \\
i_N \\
R_N \\
v \\
\end{array}
\]

NB: \( R_T = R_N \)
**Equivalent Circuits**

**Thévenin Theorem**: when viewed from the load, any network comprised of independent sources and linear elements (resistors), may be represented by an equivalent circuit.

- Equivalent circuit consists of an ideal voltage source $v_T$ in series with an **equivalent resistance** $R_T$

A fancy way of saying: “The circuit that includes everything except for the load”
Norton Theorem: when viewed from the load, any network comprised of independent sources and linear elements (resistors), may be represented by an equivalent circuit.

- Equivalent circuit consists of an ideal current source $i_N$ in parallel with an equivalent resistance $R_N$

A fancy way of saying: “The circuit that includes everything except for the load”
Thévenin and Norton Resistances

Computation of Thévenin and Norton Resistances:

1. Remove the load (open circuit at load terminal)
2. Zero all independent sources
   - Voltage sources $\rightarrow$ short circuit ($v = 0$)
   - Current sources $\rightarrow$ open circuit ($i = 0$)
3. Compute equivalent resistance (with load removed)
Thévenin and Norton Resistances

**Example 1:** find the equivalent resistance as seen by the load $R_L$

$i_s = 0.5\, \text{A}, \quad v_s = 10\, \text{V}, \quad R_1 = 4\, \Omega, \quad R_2 = 6\, \Omega, \quad R_3 = 10\, \Omega, \quad R_4 = 2\, \Omega, \quad R_5 = 2\, \Omega, \quad R_6 = 3\, \Omega, \quad R_7 = 5\, \Omega$
Thévenin and Norton Resistances

**Example 1**: find the equivalent resistance as seen by the load $R_L$

$\begin{align*}
I_s &= 0.5\,\text{A}, \quad V_s = 10\,\text{V}, \quad R_1 = 4\,\Omega, \quad R_2 = 6\,\Omega, \quad R_3 = 10\,\Omega, \quad R_4 = 2\,\Omega, \quad R_5 = 2\,\Omega, \quad R_6 = 3\,\Omega, \quad R_7 = 5\,\Omega
\end{align*}$

1. Remove the load
Thévenin and Norton Resistances

**Example 1**: find the equivalent resistance as seen by the load $R_L$

- $i_s = 0.5\,\text{A}$, $v_s = 10\,\text{V}$, $R_1 = 4\,\Omega$, $R_2 = 6\,\Omega$, $R_3 = 10\,\Omega$, $R_4 = 2\,\Omega$, $R_5 = 2\,\Omega$, $R_6 = 3\,\Omega$, $R_7 = 5\,\Omega$

2. Zero all independent sources
   - short circuit voltage sources
   - Open circuit current sources
**Example 1**: find the equivalent resistance as seen by the load $R_L$

- $i_s = 0.5\, \text{A}$, $v_s = 10\, \text{V}$, $R_1 = 4\, \Omega$, $R_2 = 6\, \Omega$, $R_3 = 10\, \Omega$, $R_4 = 2\, \Omega$, $R_5 = 2\, \Omega$, $R_6 = 3\, \Omega$, $R_7 = 5\, \Omega$

3. Compute equivalent resistance

$$R_{EQ1} = R_1 + R_2 = 4 + 6 = 10\, \Omega$$

$$R_{EQ2} = \frac{R_4 R_5}{R_4 + R_5} = \frac{2 \times 2}{(2) + (2)} = \frac{4}{4} = 1\, \Omega$$
Thévenin and Norton Resistances

Example 1: find the equivalent resistance as seen by the load \( R_L \)

\[ i_s = 0.5A, \ v_s = 10V, \ R_1 = 4\Omega, \ R_2 = 6\Omega, \ R_3 = 10\Omega, \ R_4 = 2\Omega, \ R_5 = 2\Omega, \ R_6 = 3\Omega, \ R_7 = 5\Omega \]

3. Compute equivalent resistance

\[
R_{EQ3} = \frac{R_{EQ1}R_3}{R_{EQ1} + R_3}
\]

\[
= \frac{10 \times 10}{10 + 10}
\]

\[
= \frac{100}{20}
\]

\[
= 5\Omega
\]
Thévenin and Norton Resistances

**Example 1**: find the equivalent resistance as seen by the load $R_L$

- $i_s = 0.5$ A, $v_s = 10$ V, $R_1 = 4$ Ω, $R_2 = 6$ Ω, $R_3 = 10$ Ω, $R_4 = 2$ Ω, $R_5 = 2$ Ω, $R_6 = 3$ Ω, $R_7 = 5$ Ω

3. Compute equivalent resistance

$$R_{EQ4} = R_{EQ3} + R_{EQ2}$$

$$= 5 + 1$$

$$= 6$$ Ω

\[ R_{EQ2} = 1 \] Ω

\[ R_{EQ3} = 5 \] Ω
Thévenin and Norton Resistances

**Example 1**: find the equivalent resistance as seen by the load $R_L$

Given: $i_s = 0.5\, \text{A}$, $v_s = 10\, \text{V}$, $R_1 = 4\, \Omega$, $R_2 = 6\, \Omega$, $R_3 = 10\, \Omega$, $R_4 = 2\, \Omega$, $R_5 = 2\, \Omega$, $R_6 = 3\, \Omega$, $R_7 = 5\, \Omega$

3. Compute equivalent resistance

$$R_{EQ5} = \frac{R_{EQ4} \cdot R_6}{R_{EQ4} + R_6} = \frac{(6)(3)}{(6) + (3)} = \frac{18}{9} = 2\, \Omega$$
Thévenin and Norton Resistances

**Example 1**: find the equivalent resistance as seen by the load $R_L$

- $i_s = 0.5\,\text{A}$, $v_s = 10\,\text{V}$, $R_1 = 4\,\Omega$, $R_2 = 6\,\Omega$, $R_3 = 10\,\Omega$, $R_4 = 2\,\Omega$, $R_5 = 2\,\Omega$, $R_6 = 3\,\Omega$, $R_7 = 5\,\Omega$

![Diagram of circuit with Thévenin resistance](image)

3. Compute equivalent resistance

$$R_{EQ} = R_{EQ5} + R_7 = 2 + 5 = 7\,\Omega$$

$$R_{EQ5} = 2\,\Omega$$
Thévenin and Norton Resistances

**Example 1**: find the equivalent resistance as seen by the load $R_L$

$\begin{align*}
i_s &= 0.5\, \text{A}, \quad v_s = 10\, \text{V}, \\
R_1 &= 4\, \Omega, \quad R_2 = 6\, \Omega, \quad R_3 = 10\, \Omega, \quad R_4 = 2\, \Omega, \quad R_5 = 2\, \Omega, \quad R_6 = 3\, \Omega, \quad R_7 = 5\, \Omega
\end{align*}$

3. Compute equivalent resistance

$R_{EQ} = 7\, \Omega$
Thévenin Voltage

**Thévenin equivalent voltage**: equal to the open-circuit voltage ($v_{oc}$) present at the load terminals (load removed)

\[ v_T + R_T i = v_{oc} \]

Remove load
Thévenin Voltage

**Computing Thévenin voltage:**

1. Remove the load (open circuit at load terminals)
2. Define the open-circuit voltage ($v_{oc}$) across the load terminals
3. Choose a network analysis method to find $v_{oc}$
   - node, mesh, superposition, etc.
4. Thévenin voltage $v_T = v_{oc}$
Thévenin Voltage

**Example 2**: find the Thévenin voltage

\[ v_s = 10V, \quad R_1 = 4\Omega, \quad R_2 = 6\Omega, \quad R_3 = 10\Omega \]
Thévenin Voltage

**Example 2**: find the Thévenin voltage

\[ V_s = 10\text{V}, \quad R_1 = 4\Omega, \quad R_2 = 6\Omega, \quad R_3 = 10\Omega \]

1. Remove the load
2. Define \( v_{oc} \)
Thévenin Voltage

Example 2: find the Thévenin voltage

\[ v_s = 10 \text{V}, \quad R_1 = 4 \Omega, \quad R_2 = 6 \Omega, \quad R_3 = 10 \Omega \]

\[ i_3 = 0 \]

3. Choose a network analysis method
   - Voltage divider

\[ v_{oc} = \frac{R_2}{R_1 + R_2} v_s \]
**Example 2**: find the Thévenin voltage

\[ v_s = 10\text{V}, \quad R_1 = 4\Omega, \quad R_2 = 6\Omega, \quad R_3 = 10\Omega \]

\[ v_T = \frac{R_2}{R_1 + R_2} v_s \]
Thévenin Equivalent Circuit

Computing Thévenin Equivalent Circuit:

1. Compute the Thévenin resistance $R_T$
2. Compute the Thévenin voltage $v_T$
Example 3: find $i_L$ by finding the Thévenin equivalent circuit

$V_s = 10V$, $R_1 = 4\Omega$, $R_2 = 6\Omega$, $R_3 = 10\Omega$, $R_L = 10\Omega$
Thévenin Equivalent Circuit

◆ **Example 3**: find $i_L$ by finding the Thévenin equivalent circuit

$V_s = 10V$, $R_1 = 4\Omega$, $R_2 = 6\Omega$, $R_3 = 10\Omega$, $R_L = 10\Omega$

1. Compute $R_T$
   - Remove $R_L$

![Image of Thévenin Equivalent Circuit]
Thévenin Equivalent Circuit

Example 3: find \( i_L \) by finding the Thévenin equivalent circuit
\[ v_s = 10 \text{V}, \ R_1 = 4 \Omega, \ R_2 = 6 \Omega, \ R_3 = 10 \Omega, \ R_L = 10 \Omega \]

1. Compute \( R_T \)
   - Remove \( R_L \)
   - Zero sources
Thévenin Equivalent Circuit

Example 3: find $i_L$ by finding the Thévenin equivalent circuit

$V_s = 10V, R_1 = 4\Omega, R_2 = 6\Omega, R_3 = 10\Omega, R_L = 10\Omega$

1. Compute $R_T$
   - Remove $R_L$
   - Zero sources
   - Compute $R_T = R_{EQ}$

\[ R_T = R_3 + R_1 \parallel R_2 \]
Example 3: find $i_L$ by finding the Thévenin equivalent circuit

$v_s = 10\text{V}$, $R_1 = 4\Omega$, $R_2 = 6\Omega$, $R_3 = 10\Omega$, $R_L = 10\Omega$

1. Compute $R_T$
2. Compute $v_T$
   - (previously computed)

$$v_T = \frac{R_2}{R_1 + R_2} v_s$$
Example 3: find $i_L$ by finding the Thévenin equivalent circuit

$V_s = 10V, R_1 = 4\Omega, R_2 = 6\Omega, R_3 = 10\Omega, R_L = 10\Omega$

$R_T = R_3 + R_1 \parallel R_2$

$V_T = \frac{R_2}{R_1 + R_2} V_s$

**NB**: $i_L$ is the same in both circuits
Thévenin Equivalent Circuit

**Example 3**: find $i_L$ by finding the Thévenin equivalent circuit

$V_s = 10\, \text{V}, \quad R_1 = 4\, \Omega, \quad R_2 = 6\, \Omega, \quad R_3 = 10\, \Omega, \quad R_L = 10\, \Omega$

$$R_T = R_3 + R_1 \parallel R_2$$

$$v_T = \frac{R_2}{R_1 + R_2} v_s$$

$$i_L = \frac{v_T}{R_T + R_L} = \left( \frac{R_2}{R_1 + R_2} v_s \right) \cdot \frac{1}{R_3 + R_1 \parallel R_2 + R_L}$$

$$= \left( \frac{(6)(10)}{(4) + (6)} \right) \cdot \frac{1}{(10) + (4) \parallel (6) + (10)}$$

$$= \left( \frac{60}{10} \right) \cdot \frac{1}{20 + 2.4}$$

$$= \frac{6}{22.4}$$

$$= 0.27\, \text{A}$$
**Norton Current**

**Current equivalent current**: equal to the short-circuit current \( i_{sc} \) present at the load terminals (load replaced with short circuit)

\[
i_N = i_{sc}
\]

\[
\begin{align*}
V & = i_N R_N \\
i & = \frac{V}{R_N}
\end{align*}
\]
Norton Current

Computing Norton current:

1. Replace the load with a short circuit
2. Define the short-circuit current ($i_{sc}$) across the load terminals
3. Choose a network analysis method to find $i_{sc}$
   - node, mesh, superposition, etc.
4. Norton current $i_N = i_{sc}$
Norton Current

**Example 4:** find the Norton equivalent current $i_N$

- $v_s = 6V$, $i_s = 2A$, $R_1 = 6\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_L = 10\Omega$
Norton Current

Example 4: find the Norton equivalent current $i_N$

$V_s = 6V$, $i_s = 2A$, $R_1 = 6\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_L = 10\Omega$

1. Short-circuit the load
2. Define $i_{sc}$
Nature Current

**Example 4:** find the Norton equivalent current $i_N$

- $v_s = 6V$, $i_s = 2A$, $R_1 = 6\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_L = 10\Omega$

1. $v_a$ is independent
2. $v_b$ is dependent (actually $v_a$ and $v_b$ are dependent on each other but choose $v_b$) $v_b = v_a + v_s$

3. Choose a network analysis method
   - Node voltage
Norton Current

**Example 4**: find the Norton equivalent current $i_N$

- $v_s = 6\,\text{V}$, $i_s = 2\,\text{A}$, $R_1 = 6\,\Omega$, $R_2 = 3\,\Omega$, $R_3 = 2\,\Omega$, $R_L = 10\,\Omega$

Choose a network analysis method
- Node voltage

KCL at node a:

$$i_s - i_1 - i_v = 0$$

$$\frac{v_a}{R_1} + i_v = i_s$$
Norton Current

**Example 4**: find the Norton equivalent current \( i_N \)

\[ v_s = 6 \text{V}, \quad i_s = 2 \text{A}, \quad R_1 = 6 \Omega, \quad R_2 = 3 \Omega, \quad R_3 = 2 \Omega, \quad R_L = 10 \Omega \]

Choose a network analysis method
- Node voltage

KCL at node b:

\[ i_v - i_2 - i_3 = 0 \]
\[ i_v - \frac{v_b}{R_2} - \frac{v_b}{R_3} = 0 \]
\[ i_v - \frac{(v_a + v_s)}{R_2} - \frac{(v_a + v_s)}{R_3} = 0 \]

\[ i_v - v_a \left( \frac{1}{R_2} + \frac{1}{R_3} \right) = v_s \left( \frac{1}{R_2} + \frac{1}{R_3} \right) \]
**Norton Current**

**Example 4:** find the Norton equivalent current $i_N$

$V_s = 6V$, $i_s = 2A$, $R_1 = 6\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_L = 10\Omega$

3. Choose a network analysis method
   - Node voltage

$$6i_v - 5v_a = 30$$
$$6i_v + v_a = 12$$

$$i_v = 2.5A$$
$$v_a = -3V$$
**Example 4:** find the Norton equivalent current \( i_N \)

\[ v_s = 6 \text{V}, \ i_s = 2 \text{A}, \ R_1 = 6 \Omega, \ R_2 = 3 \Omega, \ R_3 = 2 \Omega, \ R_L = 10 \Omega \]

3. Choose a network analysis method
   - Node voltage

\[
 i_{sc} = \frac{v_b}{R_3} = \frac{(v_s - v_b)}{2} = \frac{3}{2} \text{A}
\]
**Norton Current**

**Example 4:** find the Norton equivalent current $i_N$

$V_s = 6\, \text{V}, \ i_s = 2\, \text{A}, \ R_1 = 6\, \Omega, \ R_2 = 3\, \Omega, \ R_3 = 2\, \Omega, \ R_L = 10\, \Omega$

\[ i_{N} = i_{sc} \]

\[ i_{N} = 1.5\, \text{A} \]
Norton Equivalent Circuit

Computing Norton Equivalent Circuit:

1. Compute the Norton resistance $R_N$
2. Compute the Norton current $i_N$
Example 5: find the Norton equivalent circuit

\[ v_s = 6\,\text{V}, \quad i_s = 2\,\text{A}, \quad R_1 = 6\,\Omega, \quad R_2 = 3\,\Omega, \quad R_3 = 2\,\Omega, \quad R_L = 10\,\Omega \]
Example 5: find the Norton equivalent circuit

\[ V_s = 6V, \quad i_s = 2A, \quad R_1 = 6\Omega, \quad R_2 = 3\Omega, \quad R_3 = 2\Omega, \quad R_L = 10\Omega \]

1. Compute \( R_N \)
   - Remove \( R_L \)
**Example 5**: find the Norton equivalent circuit

\[ v_s = 6V, \ i_s = 2A, \ R_1 = 6\Omega, \ R_2 = 3\Omega, \ R_3 = 2\Omega, \ R_L = 10\Omega \]

1. Compute \( R_N \)
   - Remove \( R_L \)
   - Zero sources
**Norton Equivalent Circuit**

◆ **Example 5**: find the Norton equivalent circuit

\[ v_s = 6V, \ i_s = 2A, \ R_1 = 6\Omega, \ R_2 = 3\Omega, \ R_3 = 2\Omega, \ R_L = 10\Omega \]

1. Compute \( R_N \)
   * Remove \( R_L \)
   * Zero sources
   * Compute \( R_N = R_{EQ} \)

\[ R_N = R_3 + R_1 \parallel R_2 \]
**Norton Equivalent Circuit**

**Example 5**: find the Norton equivalent circuit

\[ v_s = 6V, \ i_s = 2A, \ R_1 = 6\Omega, \ R_2 = 3\Omega, \ R_3 = 2\Omega, \ R_L = 10\Omega \]

![Diagram of Norton equivalent circuit]

1. Compute \( R_N \)
2. Compute \( i_N \)
   - (previously computed)

\[ i_N = i_{sc} = 1.5A \]

\[ R_N = R_3 + R_1 \parallel R_2 = 4\Omega \]
Norton Equivalent Circuit

**Example 5**: find the Norton equivalent circuit

\[ V_s = 6\, \text{V}, \quad i_s = 2\, \text{A}, \quad R_1 = 6\, \Omega, \quad R_2 = 3\, \Omega, \quad R_3 = 2\, \Omega, \quad R_L = 10\, \Omega \]

\[ i_N = 1.5\, \text{A}, \quad R_N = 4\, \Omega \]