Binomial: \( p_S(k) = \binom{n}{k} p^k (1 - p)^{n-k}, \) \( k = 0, 1, \ldots, n; \) \( \mathbb{E}[S] = np, \) \( \text{var}(S) = np(1 - p) \)

Geometric: \( p_T(t) = (1 - p)^{t-1} p, \) \( t = 1, 2, \ldots; \) \( \mathbb{E}[T] = \frac{1}{p}, \) \( \text{var}(T) = \frac{1 - p}{p^2} \)

Pascal: \( p_Y(k) = \binom{t-1}{k-1} p^k (1 - p)^{t-k}, \) \( t = k, k + 1, \ldots, \)

Poisson with \( \lambda: \) \( p_Z(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \) \( k = 0, 1, 2, \ldots; \) \( \mathbb{E}[Z] = \lambda, \) \( \text{var}(Z) = \lambda \)

Poisson with \( \lambda \tau: \) \( p_{N\tau}(k) = \frac{P(k, \tau)}{\tau} = e^{-\lambda \tau} \frac{(\lambda \tau)^k}{k!}, \) \( k = 0, 1, \ldots; \) \( \mathbb{E}[N\tau] = \lambda \tau, \) \( \text{var}(N\tau) = \lambda \tau \)

Exponential: \( f_T(t) = \frac{1}{\lambda} e^{-\lambda t}, \) \( t \geq 0; \) \( \mathbb{E}[T] = \frac{1}{\lambda}, \) \( \text{var}(T) = \frac{1}{\lambda^2} \)

Erlang: \( f_Y(y) = \frac{\lambda^k y^{k-1} e^{-\lambda y}}{(k-1)!}, \) \( y \geq 0 \)

1. Imagine that your missionary friend receives mail every day with a probability of 1/6 (no delivery on Sunday) according to a Bernoulli process.

   a) (1 pt) How many days can he expect to receive mail in any 30-day month?
   \( \mathbb{E}[S] = np = 30 \cdot \frac{1}{6} = 5 \)

   b) (1 pt) How many days is it expected to take until he receives his third mail day?
   \( \mathbb{E}[Y_3] = \mathbb{E}[T_1 + T_2 + T_3] = \mathbb{E}[T_1] + \mathbb{E}[T_2] + \mathbb{E}[T_3] = 3 \cdot 6 = 18 \)

   c) (1 pt) Given that the missionary has not received mail for two weeks, how many days is it expected until he receives a mail day?
   \( \mathbb{E}[T] = \frac{1}{p} = 6 \)

   d) (1 pt) What is the probability of receiving the third mail day on day 6?
   \( p_{Y_3}(5) = \binom{6-1}{5} p^5 (1 - p)^1 = \frac{5}{6} \left( \frac{1}{6} \right)^3 \left( \frac{5}{6} \right) = 10 \cdot \frac{25}{6^5} \approx 0.0268 \)

   e) (2 pt) Given that the missionary “knows” that he will receive at least one mail day in the next two days, what is the probability that he will receive mail on the second day?
   Let \( B \) be the event that the missionary receives at least one mail day in the next three days.
   \( P(B) = 1 - P(\text{no mail days}) = 1 - \binom{3}{0} p^0 (1 - p)^3 = 1 - \frac{1}{6^3} = \frac{11}{36} \)
   \( P(X_2 | B) = \frac{P(X_2 \cap B)}{P(B)} = \frac{\frac{5}{36}}{\frac{11}{36}} = \frac{5}{11} \)

2. (3 pt) Students depart from the BYU Testing Center according to a Poisson process with rate 100 students per hour. Each leaving student has passed his or her exam with probability 80%. During a particular hour, what is the probability that no leaving students have failed?

   Effectively this means that you have a rate of 80 students per hour that pass exams and 20 students per hour that fail exams. The probability of no failures with rate \( \lambda = 20 \) is thus \( P(0, 1) = e^{-20}. \)