From Bertsekas and Tsitsiklis, *Introduction to Probability, 2nd Ed.*

1. (5 pts) Chapter 7 Problem 1.
2. (5 pts) Chapter 7 Problem 3.
3. (5 pts) Chapter 7 Problem 4.
4. (5 pts) Chapter 7 Problem 10.
5. (5 pts) Chapter 7 Problem 11.
7. MATLAB Problem (25 points)

Here we will employ some of your linear algebra skills to be able to find steady-state probabilities.

a) Find the Markov probability matrix $M$ associated with Problem 7.13 above.

b) Perform the following decomposition in MATLAB.

$$ [V, D] = \text{eig}(M) $$

$V$ is your matrix of eigenvectors. $D$ is your matrix of eigenvalues. What you will notice is that you will have eigenvalues of 1 (steady-state solution) and eigenvalues of $< 1$ which correspond to the transient.

c) Find the steady-state values by the following code:

$$ \text{SS\_matrix} = V * [0 \ 0 \ 0; \ 0 \ 0 \ 0; \ 0 \ 0 \ 1] / V $$

You should then see that you have a steady-state solution which corresponds to the steady-state values that you calculated in Problem 13.

d) Notice that you can achieve the same by executing the code:

$$ V * D^{100} / V $$

This is basically just showing how the transients decay.

e) Then perform the following:

$$ M^{1000} $$

Why does this give you the same result as part d and c?

f) What happens if you take $\text{SS\_matrix}$ to the $n$th power? Why do you get the result that you do?

g) Consider the matrix, $M = \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.3 & 0.6 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$. What happens to $M^n$ for large $n$?

h) Consider the chain given by $M = \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$. What happens to $M^n$ for large $n$? Why?