The following Markov chain represents transitions every day for our unmarried BYU engineer upon meeting a new acquaintance:

1. (3 pts) Given the engineer is in the DTR/dating class, what is the probability that he is in the DTR state?

\[ \pi_3 = \pi_3 p_{33} + \pi_4 p_{43} = \pi_3 (0.7) + \pi_4 (0.5) \]

\[ \Rightarrow 0.6 \pi_3 = 0.5 \pi_4 \quad \Rightarrow \quad 1.2 \pi_3 = \pi_4 \]

\[ \pi_3 + \pi_4 = 1 \]

\[ \pi_3 + 1.2 \pi_3 = 1 \]

\[ \pi_3 = \frac{2}{2.2} = \frac{10}{11} \]

2. (3 pts) Given the engineer is in the new acquaintance state, how many days on average will it take to go to the friend or DTR/dating class?

\[ \mu_1 = 0 \]

\[ \mu_3 = 0 \]

\[ \mu_2 = 1 + p_{22} \mu_2 + p_{23} \mu_3 + p_{24} \mu_3 = 1 + 0.2(0) + 0.7 \mu_1 + 0.1 \mu_3 \]

\[ 0.3 \mu_2 = 1 \]

\[ \mu_2 = \frac{1}{0.3} = \frac{10}{3} \text{ days} \]

3. (3 pts) Given the engineer is in the new acquaintance state, what is the probability of ending up in the DTR/dating class?

\[ a_3 = 1 \quad a_1 = 0 \]

\[ a_2 = p_{21} a_1 + p_{22} a_2 + p_{23} a_3 \]

\[ a_2 = (0.2) a_2 + 0.7 a_2 + 0.1 (1) \]

\[ 0.3 a_2 = 0.1 \]

\[ a_2 = \frac{1}{3} \]