

ECEn 487 - Introduction to Digital Signal Processing

Winter 2013

Quiz 1

1. The following is the z-transform for an ^{LTI system} ~~sequence~~:

$$H(z) = \frac{8 + 15z^{-1} + 3z^{-2}}{1 + \frac{7}{2}z^{-1} + \frac{3}{2}z^{-2}} = \frac{8 + 15z^{-1} + 3z^{-2}}{(1 + \frac{1}{2}z^{-1})(1 + 3z^{-1})}$$

(2 pts) What are the possible regions of convergence?

$$\text{Poles at } z = -\frac{1}{2}, z = -3$$

$$\text{ROCs: } |z| < \frac{1}{2}$$

$$\frac{1}{2} < |z| < 3$$

$$|z| > 3$$

2. (2 pts) Which of the above regions of convergence has a Fourier transform?

$$\frac{1}{2} < |z| < 3 \quad \text{because it contains the unit circle}$$

3. (6 pts) What ^{is} ~~are~~ the ^{impulse response} ~~possible~~ discrete-time ^{system if it is causal?} ~~sequences~~ for this z-transform?

$$\begin{array}{r} 2 \\ \frac{3}{2}z^{-2} + \frac{7}{2}z^{-1} + 1 \overline{) 3z^{-2} + 15z^{-1} + 8} \\ \underline{3z^{-2} + 7z^{-1} + 2} \\ 8z^{-1} + 6 \end{array}$$

$$H(z) = 2 + \frac{8z^{-1} + 6}{(1 + \frac{1}{2}z^{-1})(1 + 3z^{-1})}$$

$$\frac{A_1}{(1 + \frac{1}{2}z^{-1})} + \frac{A_2}{(1 + 3z^{-1})}$$

$$A_1 = (1 + \frac{1}{2}z^{-1}) H_1(z) \Big|_{z = -\frac{1}{2}} = \frac{8(-\frac{1}{2}) + 6}{1 - 6} = \frac{-4 + 6}{-5} = \frac{2}{-5} = -\frac{2}{5}$$

$$A_2 = (1 + 3z^{-1}) H_1(z) \Big|_{z = -3} = \frac{8(\frac{1}{3}) + 6}{1 - \frac{1}{6}} = \frac{\frac{8}{3} + 6}{\frac{5}{6}} = \frac{-16 + 36}{5} = \frac{20}{5} = 4$$

$$H(z) = 2 + \frac{2}{(1 + \frac{1}{2}z^{-1})} + \frac{4}{(1 + 3z^{-1})}$$

For causal, ROC: $|z| > 3$

$$h[n] = 2\delta[n] + 2(-\frac{1}{2})^n u[n] + 4(-3)^n u[n]$$