ECEn 487 - Introduction to Digital Signal Processing

Winter 2013

Quiz 8

1. (2 pts) A discrete-time lowpass filter is to be designed by applying the impulse invariance method to a continuous-tim Butterworth filter having magnitude-squared function

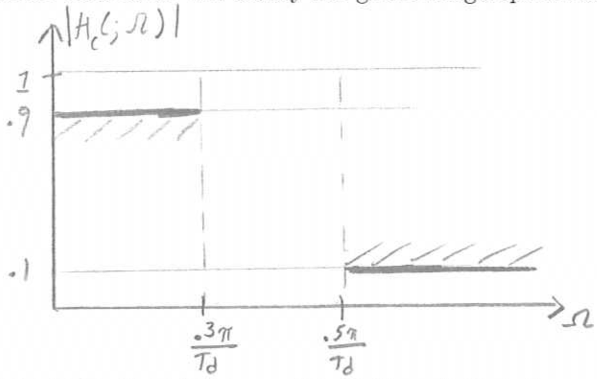
$$|H_C(j\Omega)^2| = \frac{1}{1 + (\Omega/\Omega_c)^{2N}}$$

The specifications for the discrete-time system are as follows:

$$\begin{array}{ll} 0.9 \le |H(e^{j\omega})| \le 1, & 0 \le |\omega| \le 0.3\pi \\ |H(e^{j\omega})| \le 0.1, & 0.5\pi \le |\omega| \le \pi \end{array}$$

Assume that aliasing will not be a problem.

a) (3 pts) Sketch the tolerance bounds of on the magnitude of the frequency response, $|H_c(j\Omega)|$, of the continuous-time Butterworth filter such that after the applicationof the impulse invariance method ($h[n] = T_d h_c(nT_d)$), the resulting discrete-time filter will satisfy the given design specifications.



b) (7 pts) Determine the integer order N and the quantity $T_d\Omega_c$ such that the continuous-time Butterworth filter exactly meets the specifications determined in part (a) at the passband edge.

$$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

Passband

$$(0.9)^2 = \frac{1}{1+(\frac{.3\pi}{T_d}/\Omega_c)^{2N}}$$

Stopband

$$(0.1)^2 = \frac{1}{1+(\frac{.5\pi}{T_d}/\Omega_c)^{2N}}$$

$$(0.9)^2\left[1+\left(\frac{.3\pi}{T_d\Omega_c}\right)^{2N}\right] = 1$$

$$\left(\frac{.3\pi}{T_d\Omega_c}\right)^{2N} = \frac{1}{(0.9)^2} - 1 = .2346$$

$$\left(\frac{.5\pi}{T_d\Omega_c}\right)^{2N} = \frac{1}{(0.1)^2} - 1 = 99$$

$$\left(\frac{3}{5}\right)^{2N} = \frac{.2346}{99}$$

$$2N = \log_{3/5}\left(\frac{.2346}{99}\right)$$

$$N = \frac{1}{2}\log_{3/5}\left(\frac{.2346}{99}\right) = 5.9$$

$N = 6$

$$\left(\frac{.3\pi}{T_d\Omega_c}\right)^{12} = .2346$$

$$\frac{.3\pi}{T_d\Omega_c} = (.2346)^{1/12}$$

$$T_d\Omega_c = \frac{.3\pi}{(.2346)^{1/12}} = 1.0635$$

$$\boxed{\begin{array}{l} N = 6 \\ T_d\Omega_c = 1.0635 \end{array}}$$