1. Suppose that I have \( \Omega = \mathbb{R} \) and a field (not a sigma-field), \( F \), of all the sets that are generated from open intervals, \((a, b)\), on \( \mathbb{R} \). Are the elements of singleton points, \( \{a\} \), members of the field, \( F \)? Why or why not?

2. Consider the following:
I have the following spaces: \( \Omega = \mathbb{Z}^+ \) and \( A = \{-2, 0, 2\} \).
I have a function \( f : \Omega \to A \) such that \( f(\omega) = |\omega| - |\omega - 2| \) with domain \( \Omega \) and range \( A \).
I have sets \( F = \{4, 5\} \subset \Omega \), \( G = \{0\} \subset A \), and \( H = \{1, 3\} \subset \Omega \).

a) Find \( f(F) \).

b) Find \( f^{-1}(G) \).

c) Find \( f(F \cup H) \).

3. An abstract space, \( \Omega \), is defined by \( \{0, 1\}^4 \). A product pmf is defined on \( \Omega \):
\[
p(x) = \prod_{i=0}^{3} p_i(x_i)
\]
where the marginal pmf’s, \( p_i(x_i) \), are given by Bernoulli pmf’s:
\[
p(k) = \begin{cases} 
3/4, & \text{if } k = 1 \\
1/4, & \text{if } k = 0 \\
0, & \text{otherwise}
\end{cases}
\]
A sigma-field, \( \mathcal{F} \), is defined as the power set of \( \Omega \). What is the probability measure of the set \( F = \{(0, 1, 1, 1), (0, 0, 0, 1), (1, 1, 0, 1)\} \)?