1. Consider a random process $X(t)$ defined by $X(t) = U \cos \omega t + V \sin \omega t$ where $-\infty < t < \infty$ and $\omega$ is a constant and $U$ and $V$ are random variables.
   a) Show that the condition $E(U) = E(V) = 0$ is necessary for $X(t)$ to be stationary.
   
   $E[X(t)] = E[U] \cos \omega t + E[V] \sin \omega t$
   
   This must be independent of $t$ for stationarity.
   
   $E[X(t)] = 0$ iff $E[U] = 0 \Rightarrow E[V] = 0$.

   b) Show that $X(t)$ is WSS if $U$ and $V$ are uncorrelated with equal variance, $E(UV) = 0$ and $E(U^2) = E(V^2) = \sigma^2$
   
   $R_X(t, t+\tau) = E[X(t)X(t+\tau)]$
   
   $= E[(U \cos \omega t + V \sin \omega t)(U \cos (\omega(t+\tau)) + V \sin (\omega(t+\tau)))]$
   
   $= E[U^2 \cos \omega t \cos (\omega(t+\tau)) + U^2 \sin \omega t \sin (\omega(t+\tau))]$
   
   $= \sigma^2 \cos \omega \tau$ which is only a function of the time difference.

   Thus, $X(t)$ is WSS.

2. Suppose I have a random process $\{X_t; t \in \mathbb{R}\}$ that has the following property for a k-dimensional random vector selected from this process:
   
   $P_{X_{t_0}, X_{t_1}, \ldots, X_{t_{k-1}}} = P_{X_{t_0+\tau}, X_{t_1+\tau}, \ldots, X_{t_{k-1}+\tau}}$ for all $k, t_0, t_1, \ldots, t_{k-1}, \tau$

   Will this process be Wide-Sense-Stationary (WSS)? Why or why not?

   This is the definition for strict stationarity. It will definitely be WSS because the $E[X]$ and $R_X(t)$ will not be affected by time shifts.

3. Suppose I have the following joint probability distribution for $X$ and $Y$:
   
   $p_{X,Y}(x, y) = \begin{cases} 1/4, & \text{for } (x, y) = (-2, -1), (-1, 1), (1, 1), (2, -1) \\ 0, & \text{otherwise} \end{cases}$

   Find the affine MMSE, the linear MMSE, and the general MMSE? Note, you do not need to do much calculation.