An Improved Unwrap for Data Corrupted by Overflow

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Abstract—Overflow occurs when a value is represented in binary using an insufficient number of bits. Values that lie outside the bit range are misinterpreted as values within the bit range. Continuous signals that suffer from integer overflow can be correcting using unwrap functions, but this correction fails when significant noise is present. A new method of correcting signals is proposed which uses moving average filters to correct for integer overflow. The moving average is used to determine areas of active overflow. In the corrected signal, the areas of active overflow are removed, and jumps caused by overflow are corrected for. The unwrap method and the new moving average method are compared using various examples.

I. INTRODUCTION

When designing a digital signal processing system, hardware limitations often require that signals be represented using a minimal number of bits. This limits the range of values that the system can represent. If values outside of these limits are passed through a bit-limited system they will be misinterpreted as values that exists within the limited range. This results in a type of data corruption called overflow. An example of a signal which has been affected by overflow is shown in Fig. 1.

![Fig. 1. An example of how overflow affects a sine wave.](image1)

For truly random processes, it is impossible to tell if individual values have been affected by overflow. It can, however, be determined if the signal as a whole has been affected by overflow as the distribution will be limited to values within the representable range. See Fig. 2 and 3. Overflow corruption in continuous noise-free signals can easily

![Fig. 2. The pdf of gaussian noise, with the 8-bit limits shown.](image2)

![Fig. 3. Pdf of the same gaussian noise as Fig. 2 but limited to 8-bits.](image3)
be identified as it results in discontinuities when the signal reaches the bit limits. These signals can be corrected, or unwrapped, by identifying these discontinuities and removing them by adding or subtracting the bit width. This paper investigates overflow of signals with both continuous and random properties. When such signals suffers from overflow there is not a single discontinuous point, rather a range of discontinuous points. As a result, the traditional unwrapping function used with continuous signals is no longer reliable.

We present an alternative unwrapping approach, which utilizes a moving average (MA) filter to identify the areas affected directly by overflow. This new correction method removes areas of active overflow, and corrects the remaining signal. This new method is presented, and the limitations are discussed.

II. THEORY

A. Unwrapping

The traditional unwrap function is used primarily in the context of phase functions, where a function is limited between $-\pi$ and $\pi$. Instead of increasing past the value of $\pi$, a phase function will jump down to values above $-\pi$. The unwrap function finds these sudden large jumps and corrects the signal by adding or subtracting $2\pi$ to the function after these jumps. This transforms phase functions with discontinuities into continuous functions that are not limited between $-\pi$ and $\pi$. Though designed for phase functions, the unwrap function can also be used to correct continuous signals corrupted by overflow. Instead of the function being limited between $-\pi$ and $\pi$, the function would be limited between $-2^{b-1}$ and $2^{b-1} - 1$, where $b$ is the number of bits used to represent the signal values. Instead of $2\pi$, a correction factor of $2^b$ will be used. When a noisy signal is presented, however, the basic unwrap function cannot correctly correct for overflow issues. For this reason, a new method is investigated.

B. Signal Model

In this paper, we investigate signals that are sinusoidal and have added Gaussian noise. The sinusoid can be described by

$$X(n) = \sum_{j} v_j \sin(2\pi f_j n + \phi_j)$$  \hspace{1cm} (1)

where $v_j$, $f_j$, $\phi_j$ are the amplitude, frequency and phase of the $j^{th}$ sinusoidal term and $n$ is a discrete time index. The noise, $W(n)$ is a simple zero-mean Gaussian random variable with some variance, $\sigma^2$. Thus, the signals we are concerned with can be described by the following equation:

$$Y(n) = X(n) + W(n).$$ \hspace{1cm} (2)

When a signal of this form gets corrupted by overflow, the resulting signal is given by the following equation:

$$Y_{o_f}(n) = X(n) + W(n) + o_f(n)$$  \hspace{1cm} (3)

where $o_f(n) = m2^b$, $b$ is the new smaller bit-width, and $m$ is some integer that describes the direction and amount of the overflow. For example, if a signal went just above the maximum positive value of new bit-width, $m$ would be $-1$. If the signal continued to increase and go above the maximum positive value again, $m$ would now be $-2$.

It is easy to see that a simple correction factor of $-o_f(n)$ can be added to $Y_{o_f}(n)$ to retrieve the original signal $Y(n)$. To use this correction factor, both $n$ and $m$ must be determined. This paper proposes a method to choose the values by using a moving average estimator, described below.

C. Moving Average Estimator

To help with the unwrapping, a moving average estimator is used to find where the overflow discontinuities are. As the noisy signal progresses, the MA estimator smooths out the noisiness and gives a good estimate of the underlying signal.

A MA filter uses a combination of previous values of a function to create an average that changes as the function changes:

$$Y_n = \sum_{j=1}^{p} a_j X_{n-i}.$$ \hspace{1cm} (4)

A MA filter can be described in terms of a single vector $a_j$. This vector can be selected to calculate a simple average, or a weighted average. For example, a moving average that uses the average of the past three indices would be calculated as

$$Y_n = \frac{1}{3}(X_{n-2} + X_{n-1} + X_n)$$ \hspace{1cm} (5)

i.e. $a_j = 1/3$ for $j = 1, 2, 3$.

D. New correction method

For the method discussed in this paper, we want an MA filter that gives a smoothed average of a noisy function, but doesn’t react quickly to sudden changes in the function. Determining the $a_j$ vector that achieves this depends on the signal and the amount of noise present. There may be optimal choices for $a_j$, but that has been left to future work. This paper will only use a simple equal-weighted moving average. Once an MA filter is constructed, the output $\hat{Y}(n)$ is used as an estimate for the overflown noisy signal $Y(n)$. This estimate is then compared to overflown signal. Because $\hat{Y}(n)$ does not react instantly to overflow, areas where the signal suffers from overflow will cause large jumps in the difference between the $\hat{Y}(n)$ and $Y(n)$. Subsequently, these peaks can be used to pinpoint areas affected by overflow. To identify these peaks, they must be differentiated from the noise, and as such a threshold parameter, $t_h$, must be chosen for our MA-correction process. An example of this threshold is shown in Fig. 4. To determine when the signal has stopped actively overflown, the difference between the MA filter and the noisy signal must consistently stay under the threshold. Another parameter must be added, which we call the check length $c$, determines how many samples in a row must be under the threshold before the signal is no longer actively overflown. As this system is attempting to differentiate areas of active overflow, it is essential that the signal being corrected does not exhibit active overflow areas at too high a frequency. This makes this
new method only useful for signals with sampling frequencies much higher than the frequencies in the signal.

Once the areas of active overflow are identified, the value of the signal at the edges of the overflow area are determined. Using the two values at the edges of the overflow regions, it can be determined what additive term must be added to correct for the remaining signal. If the edge values differ by more than half the bin-width, it is assumed that the signal has jumped the full bit-width, and the signal is corrected for by adding or subtracting $2^b$. With the overflowed sections corrected, the areas directly affected by overflow can be interpolated.

Overall, this method is dictated by three inputs: $a_j$ of the MA filter, the threshold $t_h$, and the check length $c$. As a rule of thumb, $a_j$ is chosen to implement an equal-weighting average, $t_h$ is chosen to be about $2^b - 1$. A good value of $c$ is very dependent on the amount of noise, but should be bigger than the order of the moving average filter.

III. NUMERICAL SIMULATIONS

To demonstrate the new method proposed in this paper, several examples of signals being corrected will be given.

A simple noise free signal is presented in Fig. 5, along with the same signal corrupted with overflow. Also shown is the signal as corrected by both the unwrapping function and the new MA based correction method. Both methods of correcting clearly result in a signal identical to the original uncorrupted signal.

The signal in Fig. 5 is repeated, but with added gaussian noise limited to 7 bits. This is shown in Fig. 6. It is clear from this figure that, unlike in Fig. 5, the corrupted signal is not discontinuous at a single point, but instead has a range of discontinuity as the continuous function approaches the bit limits. It is also clear from Fig. 6 that the basic unwrap function does not correctly correct for corrupted signals when significant noise is present. The new method, on the other hand, accurately corrects the corrupted signal. One issue with the new method is that it does not correct for the function where the function is actively overflowing, but instead ignores those areas. These gaps can be compensated for with interpolation.

As a final example, Fig. 7 shows the same signal but with added gaussian noise limited to 8 bits. The noise is now able to toggle the sign bit, which indicates that the noise itself can cause overflow. With this much noise, both methods of reconstructing the original signal fail.

It should be noted that all three examples presented here use the same parameters to create the MA filter. These parameters...
are:

\[ b_n = 0.1, \text{ for } 0 < n < 10, \]

\[ c = 35, \]

\[ t_h = 100. \]

It should also be noted that all the signals are sampled at a rate of \( f_s = 10 \text{kHz} \).

Fig. 7. Same as Fig 5, but with noise limited to 8 bits. Both the unwrapping function and the new MA based correction fail to reconstruct the original signal.

Overall, these tests show that this new method can provide accurate reconstructions of signals given overflowed data, given the signal is sampled high enough, the noise is not too great, and the correct parameters are chosen.

**IV. CONCLUSIONS**

Overflow in digital signal processing results when an insufficient number of bits are used to represent a value. A new method is proposed that uses moving averages to locate areas where the signal is being directly corrupted by overflow. These areas are removed, and the remaining signal is corrected for. It has been shown that this new method can accurately correct continuous signals with noise.