Analyzing Video Game Weather
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Abstract— The Mind Weave weather model is designed to loosely model real world weather systems while allowing magic to affect parameters of the model. The Mind Weave magic system can effect changes in atmospheric moisture, temperature, and winds. The purpose of this paper is to analyze the behavior of this weather model and produce correlation matrices that help us to predict how a spell cast by a mage will affect weather locally within the game. The weather model is highly complex and we will be making certain assumptions to simplify analysis.

Keywords: weather, magic, probability, ARMA, Fourier Transform

I. INTRODUCTION

Mind Weave is an online role-playing platform under development. One of its defining characteristics is a magic system that is based on elemental energies used to invent spells dynamically. In order for this magic system to support weather affecting magic, Mind Weave includes a weather model that is centered on key parameters such as wind, atmospheric moisture, and temperature that can be affected by the elemental magic in order to cause effects like rain or lack thereof.

In this paper we analyze the behavior of this weather model as a stochastic process. Our intention is to determine what correlation there is between certain parameters in the weather model. With this information we will be able to predict how magical energy can most efficiently be used to produce a desired effect on the weather. In our case, we will focus on the proposition of producing a rain storm by magically changing the atmospheric moisture and/or temperature in the wizard’s cell.

We will also examine the temporal and spatial correlation of weather parameters in the model. We expect to find that the weather processes are asymptotically uncorrelated in both time and space. This result will serve to demonstrate the potential longevity of a magically created spell. It will also help to verify or discount the accuracy of the weather model, since real world weather is asymptotically uncorrelated in time and space.

II. THEORY

Weather is a highly complicated process. The Mind Weave weather model reflects this complexity with a number of interconnecting parameters including wind speed and direction, cloud cover, rainfall rate, atmospheric moisture content, sun intensity, temperature, pressure, and humidity [1]. The behavior of these parameters can be dependent on climate, season, and time of day.

These parameters interconnect in complicated ways that make analysis difficult. We will therefore make certain assumptions that simplify the analysis without significantly deviating from the behavior of the full model. Our primary assumptions include

1) The region is highly moist and the pdfs for change in cloud cover and rainfall rate have a very low probability of being truncated.
2) Geography in the region is such that wind blows primarily from the North-East to the South-West, at least during the experiment period.
3) The region has uniform climate characteristics, so climate related parameters including season are constant throughout.
4) Sun intensity varies only with cloud cover in the experiment period. This can be achieved by
   - Performing the experiment only at night
   - Or using an experiment period that is very small compared to the day cycle of the region.

These assumptions allow us to evaluate the change in cloud cover and rainfall rate as Laplacian and Gaussian random variables respectively, avoid the complexity of the dynamic wind model, and ignore the cyclical behavior of the system over multiple day cycles.

The assumption of a single wind direction, for example North-East to South-West, allows us to treat the system as causal spatially (because all dependence is one directional in both spatial dimensions) as well as temporally. This means that the cloud cover and rainfall rate can be modeled as a three dimensional (3D) ARMA process as show in Figure 1 where n is the temporal dimension and i and j are spatial dimensions. In this diagram $\Delta(n)$, $\Delta(i)$, and $\Delta(j)$ represent “delays” or negative offsets in their respective dimensions. The values $\tau_c$, $d_c$, and $Z$ come from the weather model where $\tau_c$ is the update period, $d_c$ is the side length of a cell, and $Z$ is the wind vector, which in our case is uniform for all cells. Other factors in the ARMA coefficients have
been derived from the weather model assuming a $-135^\circ$ wind direction.

Having shown that the spatial and temporal rainfall can be modeled as a finite-order ARMA process with coefficients of magnitude less than 1, we can surmise that the process is asymptotically uncorrelated in both time and space. For simplicity, we will demonstrate this with a first-order, one-dimensional, auto-regressive process where

$$Y_k = a^k Y_0 + \sum_{i=1}^{k} (a^{k-i} X_i). \quad (1)$$

By inspection, we can see that $K_Y(k) \to 0$ as $k \to \infty$ because $a < 1 \Rightarrow a^k \to 0$ as $k \to \infty$.

Our process is a first order ARMA process in 3 dimensions. This principle can be extended to show that the process is asymptotically uncorrelated in both time and space. The same is true of cloud cover, temperature, and atmospheric moisture.

A multidimensional Fourier transform of the ARMA process produces

$$\mathcal{F}(f_1, f_2, f_3) = \sum_{m} \sum_{k} h_{0kl} e^{-j2\pi (kf_2 + kf_3)} \sum_{i} \sum_{l} A_{mkl} e^{-j2\pi (mf_1 + kf_2 + kf_3)}$$

where $\mathcal{H}(\vec{f})$ is the transfer function due to wind migration including an input expressed in Figure 1 in the Fourier domain, $\mathcal{Y}(\vec{f})$ is the Fourier transform of the rainfall rate, $\mathcal{X}(\vec{f})$ is the Fourier transform of the change in rainfall rate, $\mathcal{Z}(\vec{f})$ is the Fourier transform of the cloud cover change, and $\mathcal{C}(\vec{f})$ is the Fourier transform of the cloud cover change.

From these Fourier domain results we can derive the correlation relationship between rainfall and cloud cover

$$R_{Y,\lambda} \leftarrow |\mathcal{F}|^2 \mathcal{X}(\vec{f}) \mathcal{C}^*(\vec{f}) \quad (5)$$

$$\mathcal{X}(\vec{f}) = e^{j|\vec{v}|\cdot \vec{f}} e^{-\frac{1}{2}(\vec{a} \cdot \vec{f})^2} \quad (6)$$

$$\mathcal{C}(\vec{f}) = \frac{e^{-j\vec{f} \cdot \vec{r}}}{1 + \lambda^2 \vec{f}^2} \quad (7)$$

Due to the symmetry in the ARMA coefficients, this simplifies to a convolution of the change random variables of the form

$$R_{Y,\lambda} = \frac{1}{2\pi} (X * C) \delta(n). \quad (8)$$

The convolution shows that the correlation between rainfall and cloud cover is highly dependent on the moisture and humidity of the system. When humidity is high, they are highly correlated, but when the humidity is low, the variance of both is reduced and they approach of covariance of 0. Thus, clouds are a strong indicator for rain when it is humid, but when it is dry, clouds are rare and rain is nonexistent and they are uncorrelated.

A key question for a player in Mind Weave is how magic affects the weather. Magic can change temperature and moisture directly, so how these relate to rainfall becomes an important question. These relationships are given by a conditional expectation estimator of the form

$$E[Y_{ni,j} | M_{ni,j} = m, L_{ni,j} = e^{1056/t}] \propto m^9 e^{-8(20.386-5132/t)} \quad (9)$$

where for the sake of this analysis all values except for $m$ and $t$ are held constant. This then simplifies to

$$E[Y_{ni,j} | M_{ni,j} = m, L_{ni,j} = e^{1056/t}] \propto m^9 e^{-8(20.386-5132/t)} \quad (10)$$

In Section V we will discuss the implications of this relationship for spell casting.

### III. Experiment

To obtain experimental results true to the Mind Weave model without simplifying assumptions, we produced a working matlab simulation of the model. Certain aspects of the model, like dynamic wind, did not work correctly and needed to be simplified in order to avoid wild variations in temperature.

Using this matlab simulation of the model, we were able to check the model behavior against our theoretical analysis of of the model and confirm the validity of our assumptions.
IV. RESULTS

A. Asymptotic Decorrelation

To prove our original assumption of asymptotic decorrelation we analyzed the correlation coefficient of cloud cover in a single cell over 100 time steps. As time increased between the first and the last time sample the correlation decreased and this is shown in Figure 2. We also took a cell on the edge and evaluated the correlation coefficient to other cells increasing in distance away. The correlation coefficient $\rho$ is

$$
\rho(t, s) = \frac{E[(X_t - \mu_t)(X_s - \mu_s)]}{\sigma_t \sigma_s}
$$

(12)

where $t$ and $s$ were different time points in the first case and different location points in last case as shown in Figure 2.

![Graph showing that the correlation between cells approaches zero as time or space goes to infinity.](image)

Fig. 2. Graph showing that the correlation between cells approaches zero as time or space goes to infinity.

B. Covariance of Rain and Cloud Cover

The two random variables used in this experiment were cloud cover and rain. We evaluated the covariance of these two variables using the equation

$$
K(c, r) = E[CR] - E[C]E[R]
$$

(13)

where $C$ is Cloud cover and $R$ is Rainfall. Because we were calculating the data over 100 time samples we multiplied $C$ and $R$ as matrices, subtracted out the averages multiplied, and then took the diagonals of the resulting matrix to obtain the values that we wanted. These diagonals are the covariances between the variables at each instance in time. We then plot this covariance and it is shown in Figure 3 along with the rain and cloud cover values. The covariance between the variables reaches a maximum of 40 at the time when conditions are the wettest.

![Plot showing the Covariance between clouds and rain as well as the cloud and rain levels over time.](image)

Fig. 3. Plot showing the Covariance between clouds and rain as well as the cloud and rain levels over time.

C. Rain Estimate Error

Our rain values can be estimated using Equation 9. We then compare these to our actual rain values to obtain our estimator error.

$$
\text{Error} = \frac{\hat{Y} - Y}{Y} \times 100
$$

(14)

A plot of this error is shown in Figure 4 for all 2500 grid cells. This was for one time sample around 5am in the weather model. The error has an average of around 2 percent. We took the average of error over all time and obtained the plot in Figure 5. What is interesting are the sudden changes around time 20 and 90. These turned out to correspond with the rising and setting of the sun.

V. ANALYSIS

Our simulation results in general support the theoretical results we derived. They also say something about the effectiveness of the weather model as a part of a game. For challenging games, it is important that the rules be consistent. To be suitable for a game, a weather model should agree with expectations based on reality and should behave consistently.
As predicted in the theoretical section, the simulation results show that features of the weather model are asymptotically uncorrelated in both time and space. This is an important result for validating the model’s semblance to real world weather. Future weather becomes harder to predict based on current conditions as time goes on. Similarly, weather somewhere else is hard to predict based on local conditions as it gets further away. Having this be the case in the game helps with immersion and prevents players from thinking they can predict the weather any more effectively than real world weathermen.

Our results also show that the correlation between cloud cover and rainfall rate is highly dependent on humidity. This agrees with our theoretical results and again fulfills reality based assumptions. In reality, white, fluffy clouds on a dry day are not expected to imply rain. On the other hand, if it is humid and thick, dark clouds are in the air, rain is expected to come with them. This grounds the model in reality and provides a useful tool for players to know if it is going to rain or not in much the same way they do in real life.

Finally, our results show that the conditional expectation rain estimator given in Equation 9 is accurate at night with a bias toward high estimations. During the day, it is not as accurate, possibly due to the reduction in rainfall during the day lagging to the estimator. This lag would explain the positive error during the night as well when the rainfall rate is increasing in general.

Weighting this estimated impact on rainfall from changes in temperature and atmospheric moisture by the cost of those changes in the Mind Weave magic system provides a rule of thumb for casting efficient rainfall spells: cold magic is nearly 3 times as power efficient as water magic for causing rainfall. Other factors such as casting time and casting fatigue make it advantageous to still include some water in the spell, but this analysis shows that cold should be a dominating part of the spell.

VI. CONCLUSION

A probabilistic analysis of this video game weather model can be used to show its effectiveness and provide insight into best practices of game play within the model. We have shown that the weather is asymptotically uncorrelated in time and space; that rainfall and cloud cover are more highly correlated when humidity is high; and that a conditional expectation estimator for rainfall behavior is accurate enough to help guide spell formulation in the game.

REFERENCES