Acoustic Based Angle-Of-Arrival Estimation in the Presence of Interference

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***Abstract*—Before radar systems gained widespread use, passive sound-detection based systems were employed in Great Britain to detect incoming aircraft. This paper examines a proposed system that utilizes two microphone systems, with the goal of measuring the angle of arrival of an aircraft. Noise from nearby automobile traffic will be modeled, and the effects on system performance will be estimated and simulated.**

I. Introduction

Between World Wars I and II, many resources were devoted to aircraft detection and tracking. Before radar systems became widely available, much of this research focused on audio-based detection systems. In the late 1920s to early 1930s, England worked on many large dome and wall-shaped detectors (such as those shown in Figure 1) faced out over the English Channel in the hope of detecting aircraft raids. They were used primarily experimentally, as the advent of radar resulted in exponentially better performance; at their peak, most audio-based systems would only give a few minutes of warning before enemy aircraft came into visual range. As aircraft began to get faster, the benefits of the audio-based systems decreased.

However, as radar was for the most part classified during World War II, nations on both sides attempted to spread misinformation that acoustic systems were the primary means of aircraft detection until the very late years of the war.

In this paper, an angle of arrival system based equipped with two microphones will be proposed. The basic equation for finding the angle of arrival will be derived, and conditions under which the equation is valid will be discussed. The basic acoustic model for the detection system will be outlined, and basic system performance based on this model will be shown. Interference in the form of automobile noise will be added into the system, and the resulting effects on the maximum detectable range and angle of the aircraft will be simulated and shown. Finally, the overall system performance and feasibility will be discussed.

Fig. 1. Two audio mirrors and the audio wall at Denge, in Kent, England. Image provided courtesy of the Geograph Britain and Ireland Project.

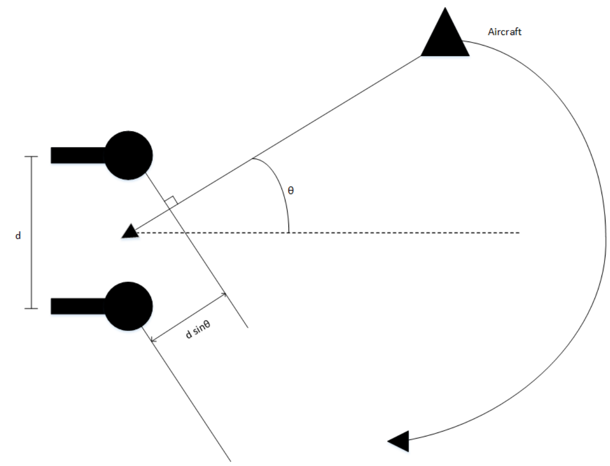

Fig. 2. Diagram of angle of arrival equation derivation

II. Proposed System

A. Basic System Setup

The system proposed will consist of two listening stations in the model of sound wall in figure 1, each with a separate microphone, each having one square meter of aperture area. The microphones in the system will be assumed to have ideal response to sound powers as low as 10^{-12} W, relative to a 20 uPa sound pressure level. This level is often defined as the minimum pressure level detectable by the human ear. The microphones will sample at 51 KHz. For simplicity, we will assume that the frequency response is flat across the entire range that the aircraft will produce. In this paper, all sound power levels given in dB units will be relative to the 10^{-12} limit.

B. Angle of Arrival Equation

Given the two microphones, the equation for the angle of arrival is derived as below. See Figure 2 for reference. For two microphones separated by distance d, we define the angle from the aircraft to the center of the microphones as θ. Assuming that the sound wave propagates as a plane wave outwards, the signal will reach one microphone before the other if θ is not zero. The extra distance the sound wave will have to travel is defined as d*sin(θ). Assuming that the speed of the sound wave is constant over this interval (which we will assume is the speed of sound, c), the difference in arrival times at the two microphones Δt is:

$$\Delta t = \frac{d * sin(\theta)}{c} \tag{1}$$

Given that we know the distance, and assuming that the speed of sound is always constant for our experiment, when the receivers hear a sound with time delta t, we can solve for theta as below.

$$\theta = arcsin(\frac{\Delta t * c}{d}) \tag{2}$$

Our signal processing system must be able to determine Δt for us to calculate theta.

C. Acoustic Properties

In order to simply our calculations, it will be assumed that the acoustic properties of the air are ideal; thus, no acoustic absorption or attenuation will take place. The model will assume a very basic propagation model for the sound, considering only the intensity, and the aperture size of the receiver walls.

Our equation for received power will be simple. Sound intensity Q_s, radiating isotropically, can be calculated as

$$Q_s = \frac{P_s}{4\pi R^2} \tag{3}$$

where P_s is the sound power at the signal source (in this case, the sound power of the aircraft engines at zero distance), and R is the one-way distance from the source to the microphone. This equation has no loss terms in the denominator because of the ideal propagation medium assumptions. Given an aperture size A, the power received P_r at our listening stations will be modeled by the following equation:

$$P_r = \frac{P_s A}{4\pi R^2} \tag{4}$$

D. Signal Processing

In order to detect the aircraft, it is assumed that the waveform that the aircraft engine generates has previously been recorded, and is known. The aircraft signal is expected to loop or repeat every second.

In order to calculate the Δt value necessary for angle of arrival estimation, the system will utilize a basic correlation system on the raw samples taken by the microphones. The samples will be correlated with the recorded aircraft signal, and a threshold will be examined. The sample numbers of the correlation peaks from both listening stations t_1 and t_2 will be compared, Δt will be found using the following formula:

$$\Delta t = \frac{t_2 - t_1}{SamplingRate} = \frac{t_2 - t_1}{51000\frac{1}{sec}} \tag{5}$$

If either station fails to have a peak cross the threshold, then it is assumed that the aircraft signal was not found, and instead of calculating a Δt, no calculation will be done. This means that no angle will be calculated for that particular second – and error code will be produced instead. In order for the system to practically useful, it is desired that the system will have a probability of detection greater than 90%.

III. Initial System Performance

Assuming that there is no noise present in the system, the maximum range at which an aircraft can be from the listening stations and still be tracked is calculated. It is assumed that the system is listening for a jet liner aircraft with turbofan engines, which will have a sound power of 160 dB. We first calculate this power in watts:

$$P_s = (10^{-12}W) * 10^{\frac{160}{10}} = 10,000W \tag{6}$$

If the power received at both listening stations must be at least at the threshold of human hearing (10^{-12} W), the range R can be solved for in equation 4:

$$R = \sqrt{\frac{10000W * 1m^2}{4\pi * 10^{-12}W}} \approx 28,209km \tag{7}$$

It is obvious that this is an unrealistically large distance. For reference, most GPS satellites orbit at an altitude of about 20,000 km, far beyond the limit of the atmosphere, and certainly far beyond the altitude any jet liner can fly at. Note that this value is caused by the assumption that air has ideal acoustic propagation properties; the model as chosen cannot account for real-world acoustic losses.

IV. System Performance With Automobile Noise

The system model will now be expanded to include a nearby highway, where cars will be moving at high speeds and generating high amounts of noise. The noise received at the listening stations will be estimated and simulated.

A. Highway Setup

For simplicity, the highway will be placed 100 meters behind the listening stations in the direction opposite to the direction the domes are facing. The highway will run parallel to the line made by the receiving stations, and will stretch from -1 km to 1 km in these directions. Outside of this range, we will assume that no noise from the cars can reach the stations (perhaps because both sides of the highway then become a tunnel, for example). See figure 3 for the basic layout.

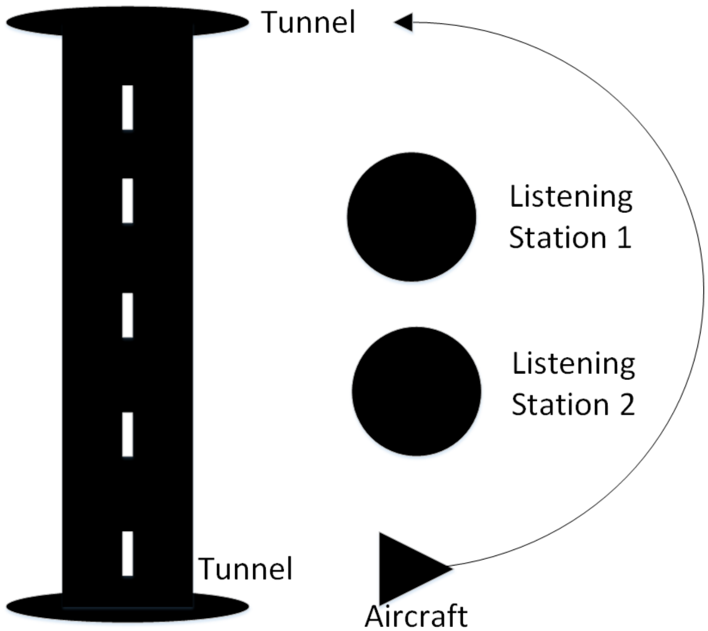

Fig. 3. The basic geometry of the simulation. The highway lies behind the listening stations, while the aircraft flies in a circle on the opposite side.

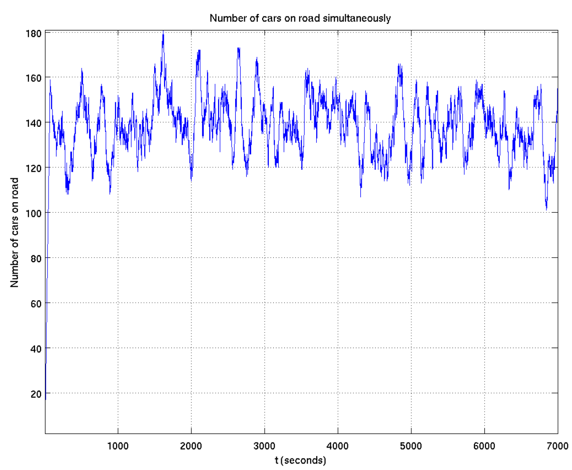

Fig. 4. Number of cars on the stretch of highway, based on the Poisson arrival model

B. Car Arrival Distribution

In order to model the incoming traffic, a Poisson random variable with parameter λ of 2 cars per second will be used. This can represent a busy highway during rush hour. It will further be assumed that the cars will be moving at a speed of 105 km/hr (or 65 miles/hr). At this rate, it will take a car 68.5 seconds to move through the section of the highway where it can contribute to the noise received at the listening stations. See Figure 4 for an example simulation, showing the number of cars that will be on the road at any given second in time. Given how long the cars are on the road, and the λ chosen, the expected number of cars on the road is $68.5 * 2 = 137$. The simulation confirms this result.

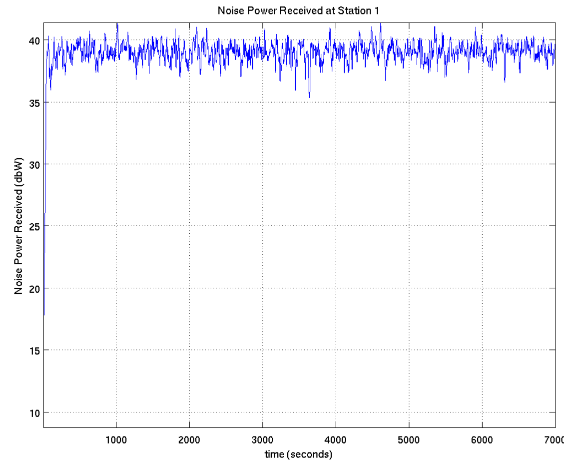

Fig. 5. The noise power received at station 1 as a function of time

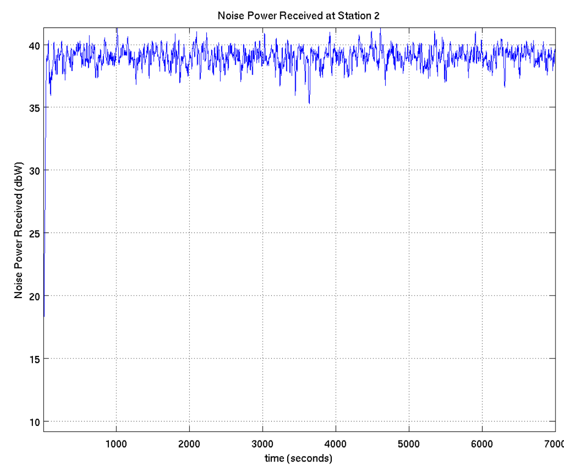

Fig. 6. The noise power received at station 2 as a function of time

C. Car Noise Simulation

Now the noise power from the automobiles generated will be calculated. Each individual car will be modeled as producing 77 dBW of sound power at the source. As the cars move through the highway, the worst-case scenario will be examined: the noise from all of the cars will be modeled as adding in phase at the receiver, resulting in the greatest level of noise power. The actual noise of the cars will be modeled as white noise.

In figures 5 and 6, the simulated noise received is shown. The expected value of noise is found to be about 39 dB.

D. Noise Effects on Signal Processing

In order for the system to be useful, the correlation in the signal processing must be able to find the aircraft signal buried in the noise. Given the noise power level of 39 dB, the SNR necessary to meet the 90% probability of detection was found empirically by simulating the correlation 1000 times at SNR values of 0 dB to -30 dB. It was discovered that at an SNR of

-7dB or less, the correlation failed over 50% of the time. The only value that succeeded in meeting the system requirements was 0 dB, which failed an average of 77 times this computation was run. At -1dB, the probability of detection was about 88%. Thus, the SNR value of 0 dB is chosen for the simulation below.

E. Noise Effects on Maximum Detectable Aircraft Range

Given the necessary SNR of 0 dB, the power received at the listening stations must also be 39 dB. This corresponds to a power value of 8.3*10^{-9} W. Using the acoustic propagation model and equation 4, the maximum range the aircraft can be from the listening stations and still be detected can be calculated:

$$R_{aircraft_{max}} = \sqrt{\frac{10000W * 1m^2}{4 * \pi * (8.3 * 10^{-9}W)}} = 313.6km \quad (8)$$

This value is just over 1% of the maximum range of the noiseless simulation. This is still considered low earth orbit; the International Space Station orbits at about 320 km at its lowest point. Noise considerations have not fixed the model from having issues with real-world modeling.

V. Simulation And Results

To test the performance of the system in noise over a long period of time, and to test the maximum range performance of the system, a large-scale simulation was executed. The aircraft is set to fly in a circle around the center point of the listening stations, with a radius of the maximum value set above. Since the noise in figures 5 and 6 is shown to occasionally rise above the expected value of 39 dB from which the maximum range was calculated, it is expected that the system will occasionally fail and be unable to track the target for short periods of time.

A. Doppler Considerations

In addition to testing the range, flying the aircraft in a circle greatly reduces the doppler considerations on the final simulation. If the aircraft had been chosen to fly in a straight line, simulation would have been much more difficult. While some doppler will still be present in the model (since the plane is flying around the center point of the two listening stations, and not around one of the listening stations themselves), there will be a small amount of doppler shift present; however, it was determined that this shift was only a few cycles, and was negligible. It is thus assumed that there will be no doppler shift in the final simulation model.

B. Aircraft Flight Parameters

The simulation was carried out in MATLAB. The aircraft was set to have a maximum velocity of 180 meters/second (645 km/hr), which is an average speed for a jetliner. The plane was simulated, flying from -90 degrees to 90 degrees around the center point of the listening stations. At the velocity given, this results in a flight time of just over an hour and a half.

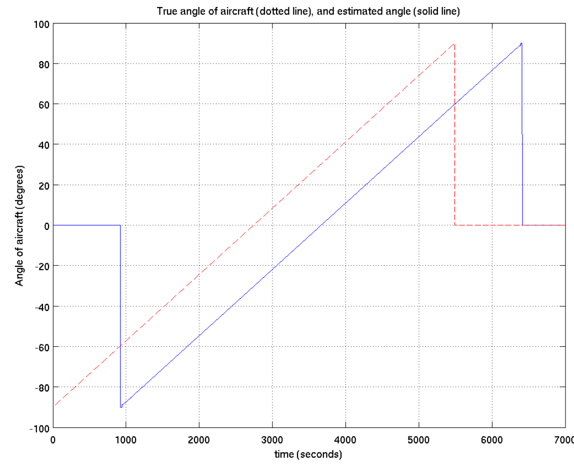

Fig. 7. The estimated angle and true angle of the aircraft in the simulation

C. Aircraft Signal Simulation

In order to simulate the sound from the aircraft reaching the listening stations, the speed of sound c was assumed to be a constant 340.29 m/s throughout the entire simulation. As the aircraft generated each second of its signal, the time delay for the signal to reach each of the listening stations was calculated.

To simplify the generation of the total received signal at each station, it was assumed that the noise power and aircraft signal power was constant over each second. The whole waveform was built, and run through the correlation system as described above.

The results can be seen in figure 7. Note the long delay between the true angle of the plane, and the estimated value. This is because of the long range of the aircraft; the delay for the first detection should occur when the aircraft signal has reached both stations 1 and 2, at a time of $\frac{313{,}600m+25m}{324.29\frac{m}{s}} \approx 921s$, which is exactly what is observed.

It should be noted that even though the probability of detection when the noise value was greater than 39 dB was less than 90%, the simulation was able to find the correct angle (although at a large time delay) for the entirety of the simulation.

D. Simulation at Greater Range

To confirm the 0 dB SNR estimation rule, the simulation was also run with the aircraft set at a longer range, resulting in -3dB SNR. The range is found to be 443.5 km. Because of the larger radius, the simulation time had to be increased from 7000 seconds to 8000 seconds.

The results are shown in figure 8. It is observed that at larger angles, the system is no longer able to cope with the noise; many false angle readings are detected. The angular performance of the system is degraded to about ± 30 degrees before performance falls in acceptable limits. Thus, the 0 dB limit set above appears to be appropriate for the system as currently designed.

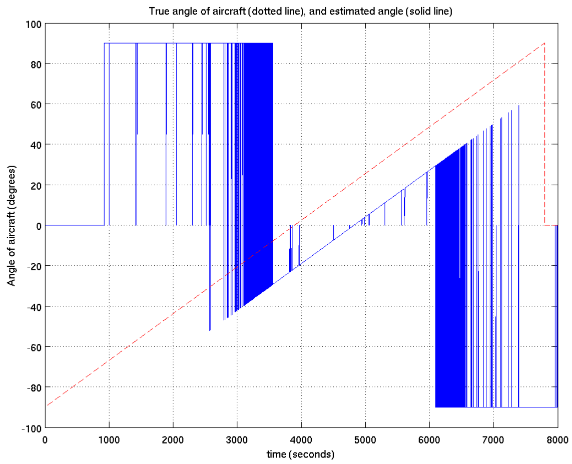

Fig. 8. The estimated angle and true angle of the aircraft in the longer range simulation

VI. Conclusion

The proposed system is shown to have fairly good performance if the SNR value is high. Obviously, a better model would take into account more of real-world acoustics, but the comparison between no noise and having the noise is stark. The basic signal processing means the system cannot handle low SNR values, but this could be improved by adding in some integration or filtering before applying the correlation detection scheme.

The simulation has illuminated some of the deficiencies of an acoustic detection system. The speed of sound is quite low compared to the velocities of aircraft. If an aircraft were coming at speeds greater than Mach 1 the system would be rendered completely useless; it might be possible to track the angle of the plane long after it passed, but in a military environment this voids the purpose of early detection. Even at sub-mach aircraft velocities, the long time delays at long ranges mean that by the time the station hears the signal, the plane is potentially several minutes closer.

Acoustic detection, then, might be more useful in situations where passive low-power systems are required, and where the targets are guaranteed to have velocities much lower than the speed of sound.