

- The capacitance of a two-conductor body and resistance of the medium between them can be computed from knowledge of the electric field in that medium.
- The electrostatic energy density stored in a dielectric medium is  $w_e = \frac{1}{2}\epsilon E^2$  (J/m<sup>3</sup>).
- When a charge configuration exists above an infinite, perfectly conducting plane, the induced field  $\mathbf{E}$  is the same as that due to the configuration itself and its image with the conducting plane removed.

### GLOSSARY OF IMPORTANT TERMS

Provide definitions or explain the meaning of the following terms:

static condition  
 electrostatics  
 volume, surface, and line charge densities  
 current density  $\mathbf{J}$   
 conduction current  
 convection current  
 electric field intensity  $\mathbf{E}$   
 electric flux density  $\mathbf{D}$   
 Coulomb's law  
 Gauss's law  
 Gaussian surface  
 conservative field  
 electric potential  
 electric dipole  
 dipole moment  $\mathbf{p}$   
 constitutive parameters  
 conductivity  $\sigma$   
 conductor  
 dielectric material  
 semiconductor  
 superconductor  
 electron drift velocity  $\mathbf{u}_e$   
 hole drift velocity  $\mathbf{u}_h$   
 perfect conductor

perfect dielectric  
 electron mobility  $\mu_e$   
 hole mobility  $\mu_h$   
 Ohm's law  
 conductance  $G$   
 Joule's law  
 boundary conditions  
 polarization vector  $\mathbf{P}$   
 linear material  
 isotropic material  
 homogeneous material  
 electric susceptibility  $\chi_e$   
 permittivity  $\epsilon$   
 relative permittivity  $\epsilon_r$   
 dielectric strength  
 dielectric breakdown  
 capacitance  $C$   
 electrostatic potential energy  $W_e$   
 electrostatic energy density  $w_e$   
 image method

### PROBLEMS

#### Sections 4-2: Charge and Current Distributions

- 4.1 A cube 2 m on a side is located in the first octant in a Cartesian coordinate system, with one of its corners at the origin. Find the total charge contained in the cube if the charge density is given by  $\rho_v = xy^2e^{-2z}$  (mC/m<sup>3</sup>).
- 4.2 Find the total charge contained in a cylindrical volume defined by  $r \leq 2$  m and  $0 \leq z \leq 3$  m if  $\rho_v = 20rz$  (mC/m<sup>3</sup>).
- 4.3 Find the total charge contained in a cone defined by  $R \leq 2$  m and  $0 \leq \theta \leq \pi/4$ , given that  $\rho_v = 10R^2 \cos^2 \theta$  (mC/m<sup>3</sup>).
- 4.4 If the line charge density is given by  $\rho_l = 24y^2$  (mC/m), find the total charge distributed on the  $y$ -axis from  $y = -5$  to  $y = 5$ .

\* Answer(s) available in Appendix D.

\* Solution available in CD-ROM.

4.5 Find the total charge on a circular disk defined by  $r \leq a$  and  $z = 0$  if:

(a)  $\rho_s = \rho_{s0} \cos \phi$  (C/m<sup>2</sup>)

(b)  $\rho_s = \rho_{s0} \sin^2 \phi$  (C/m<sup>2</sup>)

\* (c)  $\rho_s = \rho_{s0} e^{-r}$  (C/m<sup>2</sup>)

(d)  $\rho_s = \rho_{s0} e^{-r} \sin^2 \phi$  (C/m<sup>2</sup>)

where  $\rho_{s0}$  is a constant.

4.6 If  $\mathbf{J} = \hat{y}4xz$  (A/m<sup>2</sup>), find the current  $I$  flowing through a square with corners at (0, 0, 0), (2, 0, 0), (2, 0, 2), and (0, 0, 2).

4.7 If  $\mathbf{J} = \hat{R}5/R$  (A/m<sup>2</sup>), find  $I$  through the surface  $R = 5$  m.

4.8 An electron beam shaped like a circular cylinder of radius  $r_0$  carries a charge density given by

$$\rho_v = \left( \frac{-\rho_0}{1+r^2} \right) \quad (\text{C/m}^3)$$

where  $\rho_0$  is a positive constant and the beam's axis is coincident with the  $z$ -axis.

(a) Determine the total charge contained in length  $L$  of the beam.

(b) If the electrons are moving in the  $+z$ -direction with uniform speed  $u$ , determine the magnitude and direction of the current crossing the  $z$ -plane.

#### Section 4-3: Coulomb's Law

4.9 A square with sides of 2 m has a charge of  $40 \mu\text{C}$  at each of its four corners. Determine the electric field at a point 5 m above the center of the square.

\* 4.10 Three point charges, each with  $q = 3 \text{ nC}$ , are located at the corners of a triangle in the  $x$ - $y$  plane, with one corner at the origin, another at (2 cm, 0, 0), and the

third at (0, 2 cm, 0). Find the force acting on the charge located at the origin.

4.11\* Charge  $q_1 = 6 \mu\text{C}$  is located at (1 cm, 1 cm, 0) and charge  $q_2$  is located at (0, 0, 4 cm). What should  $q_2$  be so that  $\mathbf{E}$  at (0, 2 cm, 0) has no  $y$ -component?

4.12 A line of charge with uniform density  $\rho_l = 8 (\mu\text{C/m})$  exists in air along the  $z$ -axis between  $z = 0$  and  $z = 5$  cm. Find  $\mathbf{E}$  at (0, 10 cm, 0).

4.13\* Electric charge is distributed along an arc located in the  $x$ - $y$  plane and defined by  $r = 2$  cm and  $0 \leq \phi \leq \pi/4$ . If  $\rho_l = 5 (\mu\text{C/m})$ , find  $\mathbf{E}$  at (0, 0,  $z$ ) and then evaluate it at:

(a) The origin.

(b)  $z = 5$  cm

(c)  $z = -5$  cm

4.14 A line of charge with uniform density  $\rho_l$  extends between  $z = -L/2$  and  $z = L/2$  along the  $z$ -axis. Apply Coulomb's law to obtain an expression for the electric field at any point  $P(r, \phi, 0)$  on the  $x$ - $y$  plane. Show that your result reduces to the expression given by (4.33) as the length  $L$  is extended to infinity.

4.15\* Repeat Example 4-5 for the circular disk of charge of radius  $a$ , but in the present case, assume the surface charge density to vary with  $r$  as

$$\rho_s = \rho_{s0} r^2 \quad (\text{C/m}^2)$$

where  $\rho_{s0}$  is a constant.

4.16 Multiple charges at different locations are said to be in equilibrium if the force acting on any one of them is identical in magnitude and direction to the force acting on any of the others. Suppose we have two negative charges, one located at the origin and carrying charge  $-9e$ , and the other located on the positive  $x$ -axis at a distance  $d$  from the first one and carrying charge  $-36e$ .

Determine the location, polarity and magnitude of a third charge whose placement would bring the entire system into equilibrium.

#### Section 4-4: Gauss's Law

- 4.17<sup>\*</sup> Three infinite lines of charge, all parallel to the  $z$ -axis, are located at the three corners of the kite-shaped arrangement shown in Fig. 4-29. If the two right triangles are symmetrical and of equal corresponding sides, show that the electric field is zero at the origin.

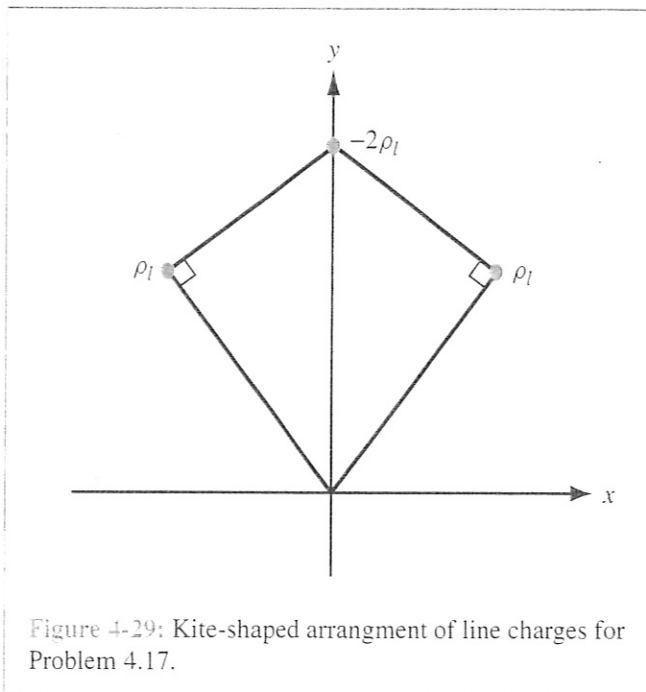


Figure 4-29: Kite-shaped arrangement of line charges for Problem 4.17.

- 4.18 Three infinite lines of charge,  $\rho_{l_1} = 3$  (nC/m),  $\rho_{l_2} = -3$  (nC/m), and  $\rho_{l_3} = 3$  (nC/m), are all parallel to the  $z$ -axis. If they pass through the respective points  $(0, -b)$ ,  $(0, 0)$ , and  $(0, b)$  in the  $x$ - $y$  plane, find the electric field at  $(a, 0, 0)$ . Evaluate your result for  $a = 2$  cm and  $b = 1$  cm.

- 4.19 A horizontal strip lying in the  $x$ - $y$  plane is of width  $d$  in the  $y$ -direction and infinitely long in the  $x$ -direction. If the strip is in air and has a uniform charge distribution  $\rho_s$ , use Coulomb's law to obtain an explicit expression for the electric field at a point  $P$  located at a distance  $h$  above the centerline of the strip. Extend your result to the special case where  $d$  is infinite and compare it with Eq. (4.25).

- 4.20 Given the electric flux density

$$\mathbf{D} = \hat{\mathbf{x}}2(x + y) + \hat{\mathbf{y}}(3x - 2y) \quad (\text{C/m}^2)$$

determine

- (a)  $\rho_v$  by applying Eq. (4.26).  
 (b) The total charge  $Q$  enclosed in a cube 2 m on a side, located in the first octant with three of its sides coincident with the  $x$ -,  $y$ -, and  $z$ -axes and one of its corners at the origin.  
 (c) The total charge  $Q$  in the cube, obtained by applying Eq. (4.29).

- 4.21<sup>\*</sup> Repeat Problem 4.20 for  $\mathbf{D} = \hat{\mathbf{x}}xy^3z^3$  (C/m<sup>2</sup>).

- 4.22 Charge  $Q_1$  is uniformly distributed over a thin spherical shell of radius  $a$ , and charge  $Q_2$  is uniformly distributed over a second spherical shell of radius  $b$ , with  $b > a$ . Apply Gauss's law to find  $\mathbf{E}$  in the regions  $R < a$ ,  $a < R < b$ , and  $R > b$ .

- 4.23<sup>\*</sup> The electric flux density inside a dielectric sphere of radius  $a$  centered at the origin is given by

$$\mathbf{D} = \hat{\mathbf{R}}\rho_0 R \quad (\text{C/m}^2)$$

where  $\rho_0$  is a constant. Find the total charge inside the sphere.

4.29 In a certain region of space, the charge density is given in cylindrical coordinates by the function:

$$\rho_v = 5re^{-r} \quad (\text{C/m}^3)$$

Apply Gauss's law to find  $\mathbf{D}$ .

4.29 An infinitely long cylindrical shell extending between  $r = 1$  m and  $r = 3$  m contains a uniform charge density  $\rho_{v0}$ . Apply Gauss's law to find  $\mathbf{D}$  in all regions.

4.29 If the charge density increases linearly with distance from the origin such that  $\rho_v = 0$  at the origin and  $\rho_v = 4 \text{ C/m}^3$  at  $R = 2$  m, find the corresponding variation of  $\mathbf{D}$ .

#### Section 4-5: Electric Potential

4.29 A square in the  $x$ - $y$  plane in free space has a point charge of  $+Q$  at corner  $(a/2, a/2)$ , the same at corner  $(a/2, -a/2)$ , and a point charge of  $-Q$  at each of the other two corners.

- Find the electric potential at any point  $P$  along the  $x$ -axis.
- Evaluate  $V$  at  $x = a/2$ .

4.29 The circular disk of radius  $a$  shown in Fig. 4-7 has uniform charge density  $\rho_s$  across its surface.

- Obtain an expression for the electric potential  $V$  at a point  $P(0, 0, z)$  on the  $z$ -axis.
- Use your result to find  $\mathbf{E}$  and then evaluate it for  $z = h$ . Compare your final expression with (4.24), which was obtained on the basis of Coulomb's law.

4.29 A circular ring of charge of radius  $a$  lies in the  $x$ - $y$  plane and is centered at the origin. Assume also that the ring is in air and carries a uniform density  $\rho_l$ .

- Show that the electrical potential at  $(0, 0, z)$  is given by  $V = \rho_l a / [2\epsilon_0(a^2 + z^2)^{1/2}]$ .
- Find the corresponding electric field  $\mathbf{E}$ .

4.30 Show that the electric potential difference  $V_{12}$  between two points in air at radial distances  $r_1$  and  $r_2$  from an infinite line of charge with density  $\rho_l$  along the  $z$ -axis is  $V_{12} = (\rho_l / 2\pi\epsilon_0) \ln(r_2/r_1)$ .

4.31 Find the electric potential  $V$  at a location a distance  $b$  from the origin in the  $x$ - $y$  plane due to a line charge with charge density  $\rho_l$  and of length  $l$ . The line charge is coincident with the  $z$ -axis and extends from  $z = -l/2$  to  $z = l/2$ .

4.32 For the electric dipole shown in Fig. 4-13,  $d = 1$  cm and  $|\mathbf{E}| = 4$  (mV/m) at  $R = 1$  m and  $\theta = 0^\circ$ . Find  $\mathbf{E}$  at  $R = 2$  m and  $\theta = 90^\circ$ .

4.33 For each of the distributions of the electric potential  $V$  shown in Fig. 4-30, sketch the corresponding distribution of  $\mathbf{E}$  (in all cases, the vertical axis is in volts and the horizontal axis is in meters).

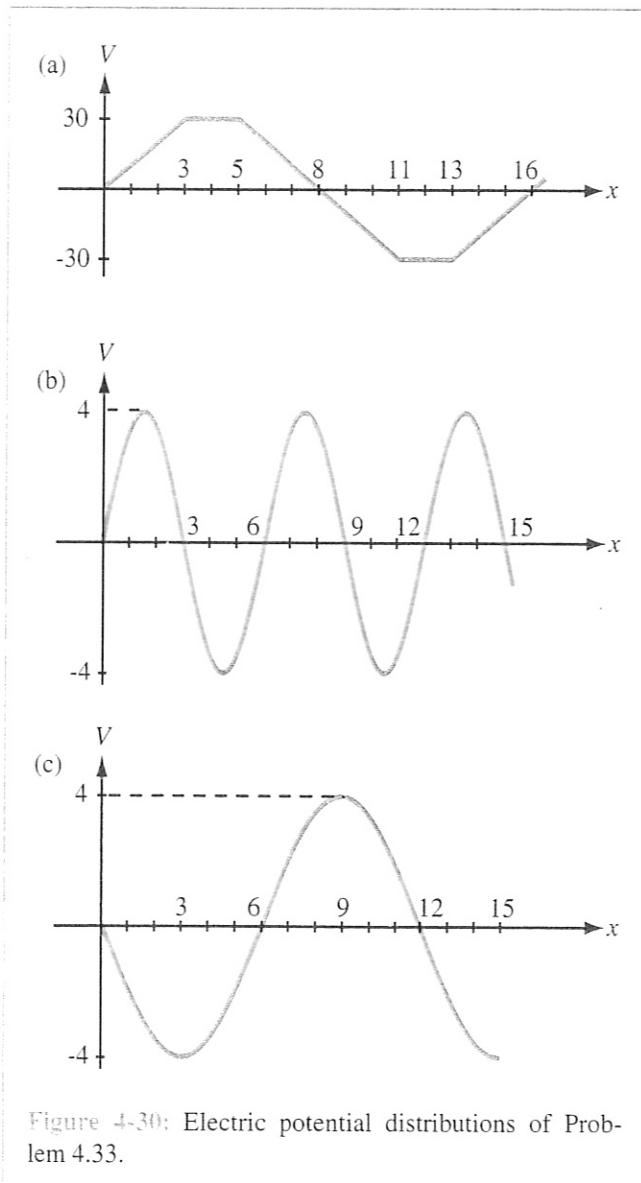
4.34 Given the electric field

$$\mathbf{E} = \hat{\mathbf{R}} \frac{18}{R^2} \quad (\text{V/m})$$

find the electric potential of point  $A$  with respect to point  $B$  where  $A$  is at  $+2$  m and  $B$  at  $-4$  m, both on the  $z$ -axis.

4.35 An infinitely long line of charge with uniform density  $\rho_l = 9$  (nC/m) lies in the  $x$ - $y$  plane parallel to the  $y$ -axis at  $x = 2$  m. Find the potential  $V_{AB}$  at point  $A(3 \text{ m}, 0, 4 \text{ m})$  in Cartesian coordinates with respect to point  $B(0, 0, 0)$  by applying the result of Problem 4.30.

4.36 The  $x$ - $y$  plane contains a uniform sheet of charge with  $\rho_{s1} = 0.2$  (nC/m<sup>2</sup>). A second sheet with  $\rho_{s2} = -0.2$  (nC/m<sup>2</sup>) occupies the plane  $z = 6$  m. Find  $V_{AB}$ ,  $V_{BC}$ , and  $V_{AC}$  for  $A(0, 0, 6 \text{ m})$ ,  $B(0, 0, 0)$ , and  $C(0, -2 \text{ m}, 2 \text{ m})$ .



## Section 4-7: Conductors

4.37 A cylindrical bar of silicon has a radius of 4 mm and a length of 8 cm. If a voltage of 5 V is applied between the ends of the bar and  $\mu_e = 0.13 \text{ (m}^2/\text{V}\cdot\text{s)}$ ,  $\mu_h = 0.05$

$\text{(m}^2/\text{V}\cdot\text{s)}$ ,  $N_e = 1.5 \times 10^{16} \text{ electrons/m}^3$ , and  $N_h = N_e$ , find the following:

- The conductivity of silicon.
- The current  $I$  flowing in the bar.
- The drift velocities  $\mathbf{u}_e$  and  $\mathbf{u}_h$ .
- The resistance of the bar.
- The power dissipated in the bar.

4.38 Repeat Problem 4.37 for a bar of germanium with  $\mu_e = 0.4 \text{ (m}^2/\text{V}\cdot\text{s)}$ ,  $\mu_h = 0.2 \text{ (m}^2/\text{V}\cdot\text{s)}$ , and  $N_e = N_h = 2.4 \times 10^{19} \text{ electrons or holes/m}^3$ .

4.39 A 100-m-long conductor of uniform cross-section has a voltage drop of 4 V between its ends. If the density of the current flowing through it is  $1.4 \times 10^6 \text{ (A/m}^2)$ , identify the material of the conductor.

4.40 A coaxial resistor of length  $l$  consists of two concentric cylinders. The inner cylinder has radius  $a$  and is made of a material with conductivity  $\sigma_1$ , and the outer cylinder, extending between  $r = a$  and  $r = b$ , is made of a material with conductivity  $\sigma_2$ . If the two ends of the resistor are capped with conducting plates, show that the resistance between the two ends is  $R = l/[\pi(\sigma_1 a^2 + \sigma_2(b^2 - a^2))]$ .

4.41 Apply the result of Problem 4.40 to find the resistance of a 20-cm-long hollow cylinder (Fig. 4-31) made of carbon with  $\sigma = 3 \times 10^4 \text{ (S/m)}$ .

4.42 A  $2 \times 10^{-3}$ -mm-thick square sheet of aluminum has 5 cm  $\times$  5 cm faces. Find the following:

- The resistance between opposite edges on a square face.
- The resistance between the two square faces. (See Appendix B for the electrical constants of materials.)

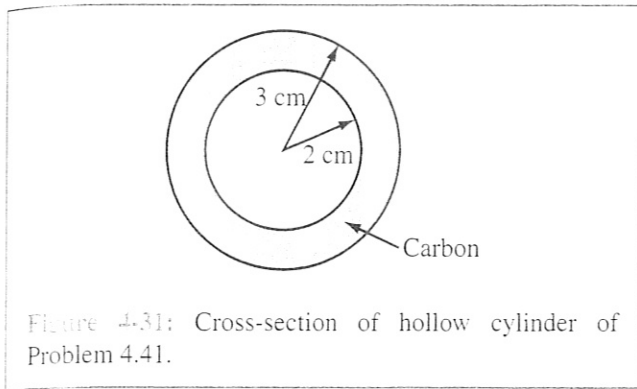


Figure 4-31: Cross-section of hollow cylinder of Problem 4.41.

Section 4-9: Boundary Conditions

4.43 With reference to Fig. 4-19, find  $\mathbf{E}_1$  if  $\mathbf{E}_2 = \hat{x}3 - \hat{y}2 + \hat{z}2$  (V/m),  $\epsilon_1 = 2\epsilon_0$ ,  $\epsilon_2 = 18\epsilon_0$ , and the boundary has a surface charge density  $\rho_s = 3.54 \times 10^{-11}$  (C/m<sup>2</sup>). What angle does  $\mathbf{E}_2$  make with the  $z$ -axis?

4.44 An infinitely long conducting cylinder of radius  $a$  has a surface charge density  $\rho_s$ . The cylinder is surrounded by a dielectric medium with  $\epsilon_r = 4$  and contains no free charges. If the tangential component of the electric field in the region  $r \geq a$  is given by  $\mathbf{E}_t = -\hat{\phi} \cos^2 \phi / r^2$ , find  $\rho_s$ .

4.45 A 2-cm conducting sphere is embedded in a charge-free dielectric medium with  $\epsilon_{2r} = 9$ . If  $\mathbf{E}_2 = \hat{R}3 \cos \theta - \hat{\theta}3 \sin \theta$  (V/m) in the surrounding region, find the charge density on the sphere's surface.

4.46 If  $\mathbf{E} = \hat{R}150$  (V/m) at the surface of a 5-cm conducting sphere centered at the origin, what is the total charge  $Q$  on the sphere's surface?

4.47 Figure 4-32 shows three planar dielectric slabs of equal thickness but with different dielectric constants. If  $\mathbf{E}_0$  in air makes an angle of  $45^\circ$  with respect to the  $z$ -axis, find the angle of  $\mathbf{E}$  in each of the other layers.

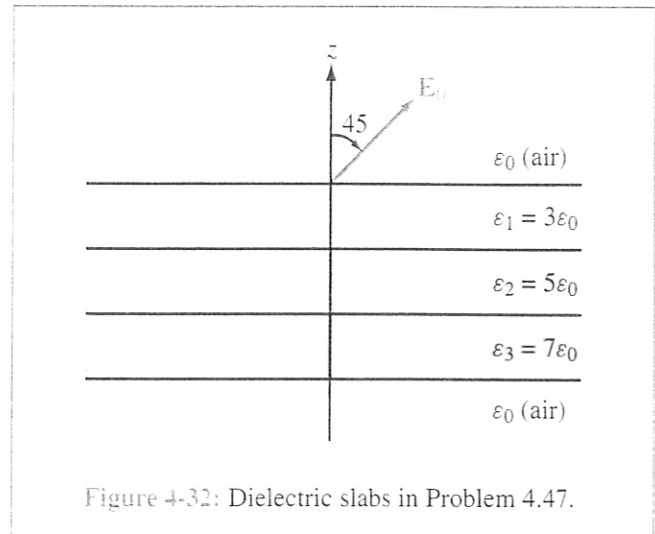


Figure 4-32: Dielectric slabs in Problem 4.47.

Sections 4-10 and 4-11: Capacitance and Electrical Energy

4.48 Determine the force of attraction in a parallel-plate capacitor with  $A = 5 \text{ cm}^2$ ,  $d = 2 \text{ cm}$ , and  $\epsilon_r = 4$  if the voltage across it is 50 V.

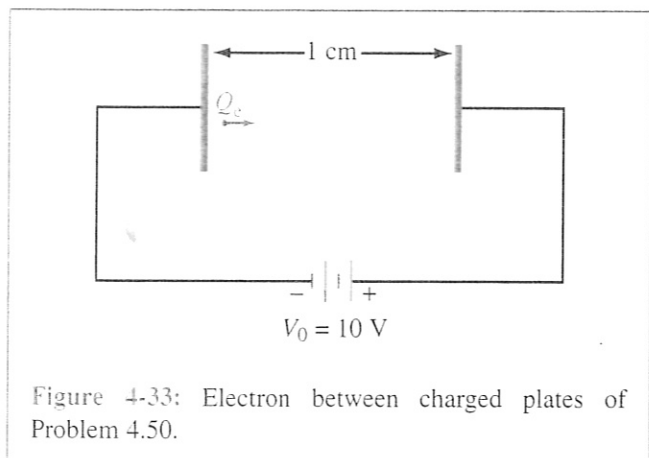
4.49 Dielectric breakdown occurs in a material whenever the magnitude of the field  $\mathbf{E}$  exceeds the dielectric strength anywhere in that material. In the coaxial capacitor of Example 4-12,

- (a) At what value of  $r$  is  $|\mathbf{E}|$  maximum?
- (b) What is the breakdown voltage if  $a = 1 \text{ cm}$ ,  $b = 2 \text{ cm}$ , and the dielectric material is mica with  $\epsilon_r = 6$ ?

4.50 An electron with charge  $Q_e = -1.6 \times 10^{-19} \text{ C}$  and mass  $m_e = 9.1 \times 10^{-31} \text{ kg}$  is injected at a point adjacent to the negatively charged plate in the region between the plates of an air-filled parallel-plate capacitor with separation of 1 cm and rectangular plates each  $10 \text{ cm}^2$  in area (Fig. 4-33). If the voltage across the capacitor is 10 V, find the following:



- (a) The force acting on the electron.  
 (b) The acceleration of the electron.  
 (c) The time it takes the electron to reach the positively charged plate, assuming that it starts from rest.



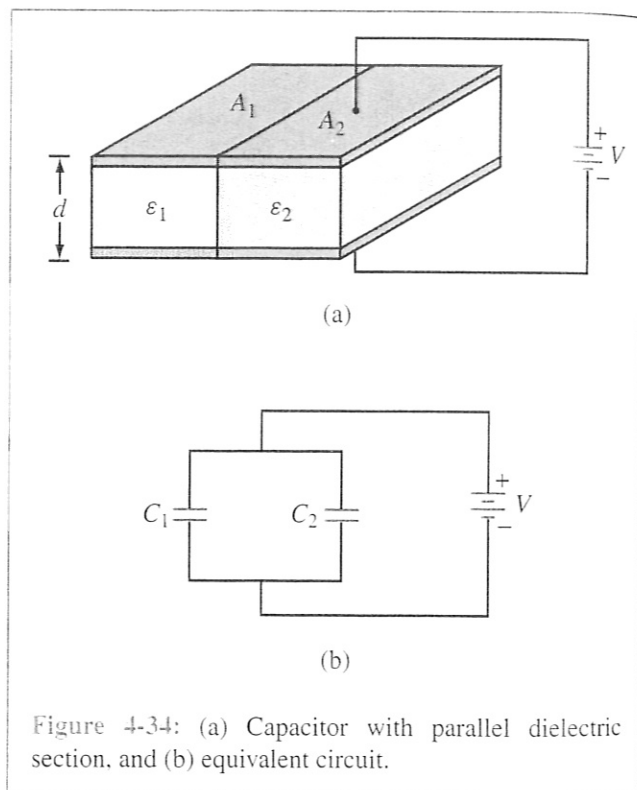
4.51<sup>\*</sup> In a dielectric medium with  $\epsilon_r = 4$ , the electric field is given by

$$\mathbf{E} = \hat{x}(x^2 + 2z) + \hat{y}x^2 - \hat{z}(y + z) \quad (\text{V/m})$$

Calculate the electrostatic energy stored in the region  $-1 \text{ m} \leq x \leq 1 \text{ m}$ ,  $0 \leq y \leq 2 \text{ m}$ , and  $0 \leq z \leq 3 \text{ m}$ .

4.52 Figure 4-34(a) depicts a capacitor consisting of two parallel, conducting plates separated by a distance  $d$ . The space between the plates contains two adjacent dielectrics, one with permittivity  $\epsilon_1$  and surface area  $A_1$  and another with  $\epsilon_2$  and  $A_2$ . The objective of this problem is to show that the capacitance  $C$  of the configuration shown in Fig. 4-34(a) is equivalent to two capacitances in parallel, as illustrated in Fig. 4-34(b), with

$$C = C_1 + C_2 \quad (4.132)$$



where

$$C_1 = \frac{\epsilon_1 A_1}{d} \quad (4.133)$$

$$C_2 = \frac{\epsilon_2 A_2}{d} \quad (4.134)$$

To this end, proceed as follows:

- (a) Find the electric fields  $\mathbf{E}_1$  and  $\mathbf{E}_2$  in the two dielectric layers.  
 (b) Calculate the energy stored in each section and use the result to calculate  $C_1$  and  $C_2$ .  
 (c) Use the total energy stored in the capacitor to obtain an expression for  $C$ . Show that (4.132) is indeed a valid result.

4.53 Use the result of Problem 4.52 to determine the capacitance for each of the following configurations:

- (a) Conducting plates are on top and bottom faces of the rectangular structure in Fig. 4-35(a).
- (b) Conducting plates are on front and back faces of the structure in Fig. 4-35(a).
- (c) Conducting plates are on top and bottom faces of the cylindrical structure in Fig. 4-35(b).

4.54 The capacitor shown in Fig. 4-36 consists of two parallel dielectric layers. Use energy considerations to show that the equivalent capacitance of the overall capacitor,  $C$ , is equal to the series combination of the capacitances of the individual layers,  $C_1$  and  $C_2$ , namely

$$C = \frac{C_1 C_2}{C_1 + C_2} \quad (4.135)$$

where

$$C_1 = \epsilon_1 \frac{A}{d_1}, \quad C_2 = \epsilon_2 \frac{A}{d_2}$$

- (a) Let  $V_1$  and  $V_2$  be the electric potentials across the upper and lower dielectrics, respectively. What are the corresponding electric fields  $E_1$  and  $E_2$ ? By applying the appropriate boundary condition at the interface between the two dielectrics, obtain explicit expressions for  $E_1$  and  $E_2$  in terms of  $\epsilon_1$ ,  $\epsilon_2$ ,  $V$ , and the indicated dimensions of the capacitor.
- (b) Calculate the energy stored in each of the dielectric layers and then use the sum to obtain an expression for  $C$ .
- (c) Show that  $C$  is given by Eq. (4.135).

4.55 Use the expressions given in Problem 4.54 to determine the capacitance for the configurations in Fig. 4-35(a) when the conducting plates are placed on the right and left faces of the structure.

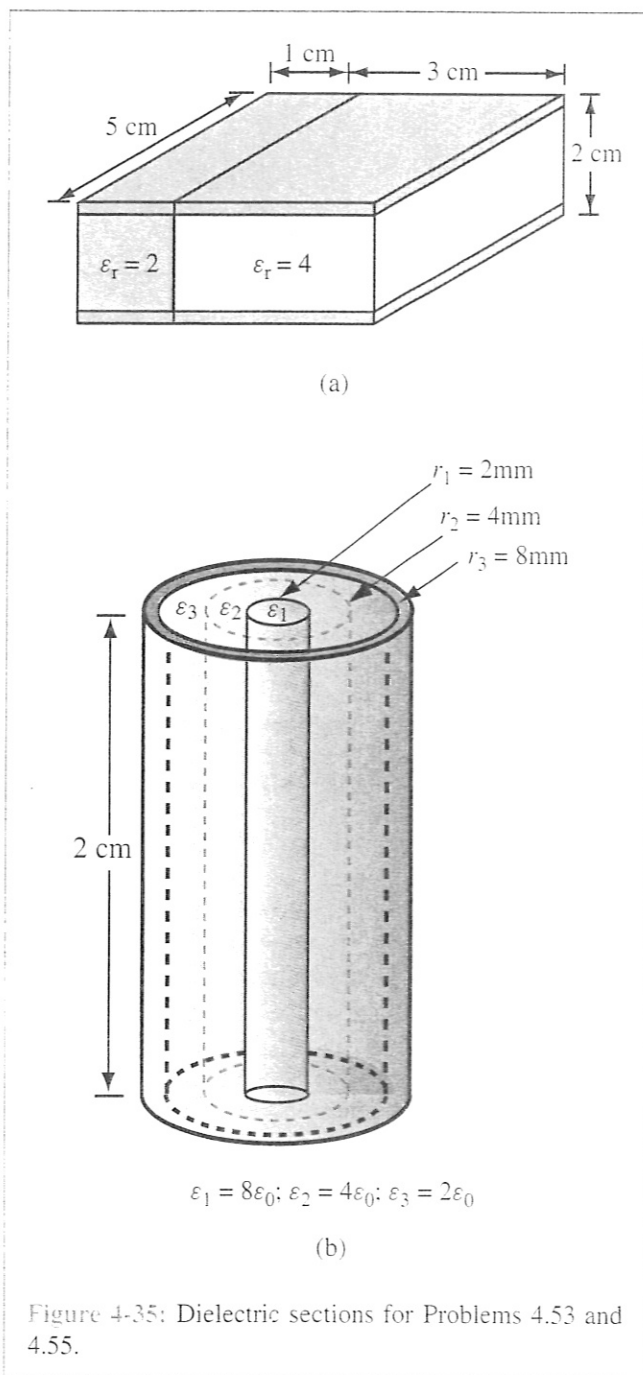


Figure 4-35: Dielectric sections for Problems 4.53 and 4.55.



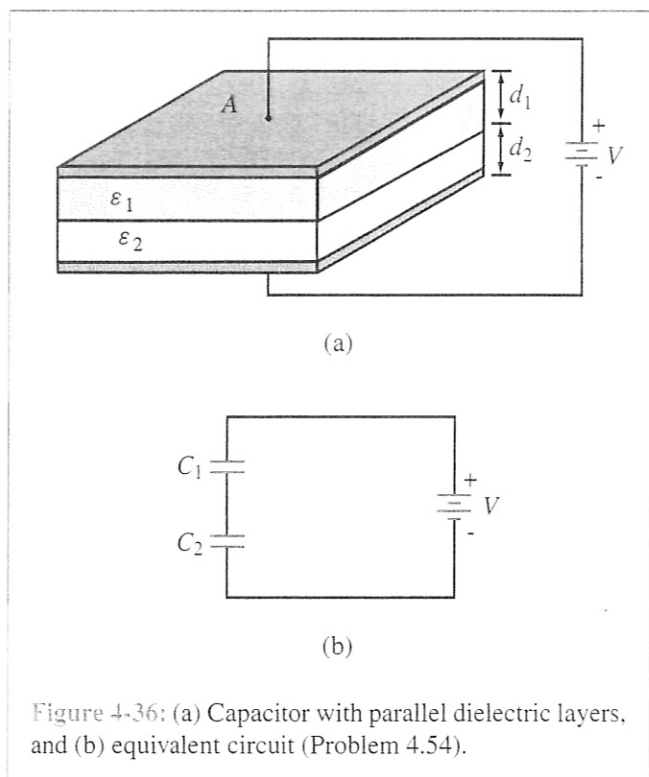


Figure 4-36: (a) Capacitor with parallel dielectric layers, and (b) equivalent circuit (Problem 4.54).

#### Section 4-12: Image Method

4.56 With reference to Fig. 4-37, charge  $Q$  is located at a distance  $d$  above a grounded half-plane located in the  $x$ - $y$  plane and at a distance  $d$  from another grounded half-plane in the  $x$ - $z$  plane. Use the image method to

- Establish the magnitudes, polarities, and locations of the images of charge  $Q$  with respect to each of the two ground planes (as if each is infinite in extent).
- Find the electric potential and electric field at an arbitrary point  $P(0, y, z)$ .

4.57 Conducting wires above a conducting plane carry currents  $I_1$  and  $I_2$  in the directions shown in Fig. 4-38.

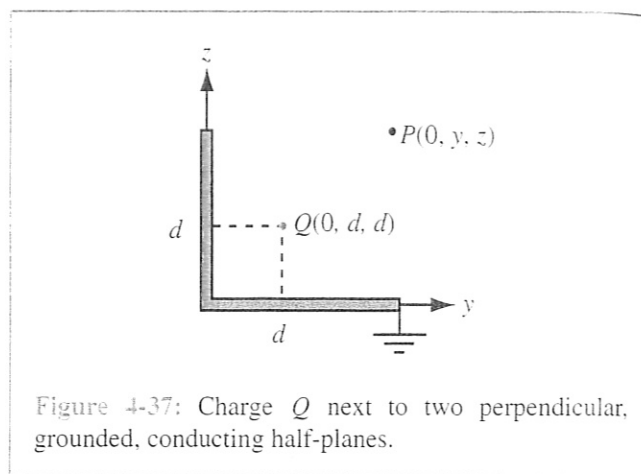


Figure 4-37: Charge  $Q$  next to two perpendicular, grounded, conducting half-planes.

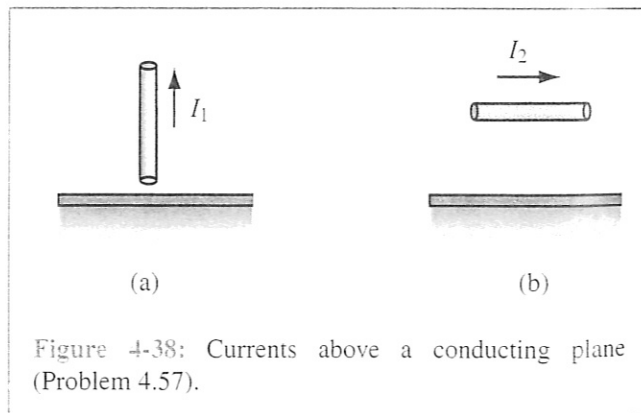


Figure 4-38: Currents above a conducting plane (Problem 4.57).

Keeping in mind that the direction of a current is defined in terms of the movement of positive charges, what are the directions of the image currents corresponding to  $I_1$  and  $I_2$ ?

4.58 Use the image method to find the capacitance per unit length of an infinitely long conducting cylinder of radius  $a$  situated at a distance  $d$  from a parallel conducting plane, as shown in Fig. 4-39.