Chapter 1

Electrostatics

1.1 Maxwell's Equations

Electromagnetic behavior can be described using a set of four fundamental relations known as *Maxwell's Equations*. Note that these equations are observed, not derived. In general, these equations are given by

$$\nabla \cdot \overline{D} = \rho_v \tag{1.1}$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \tag{1.2}$$

$$\nabla \cdot \overline{B} = 0 \tag{1.3}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \tag{1.4}$$

where

$\overline{E}$:	Electric field intensity	V/m
$\overline{H}$:	Magnetic field intensity	A/m
$\overline{D}$:	Electric flux density	C/m^2
$\overline{B}$:	Magnetic flux density	Wb/m^2
ρ_v:	Electric charge density	C/m^3
$\overline{J}$:	Electric current density	A/m^2

There is also an integral form of these equations that can be represented as

$$\oint_S \overline{D} \cdot d\overline{s} = \int_V \rho_v \, dV = Q \tag{1.5}$$

$$\oint_C \overline{E} \cdot d\overline{\ell} = -\frac{d}{dt} \int_A \overline{B} \cdot d\overline{s} \tag{1.6}$$

$$\oint_S \overline{B} \cdot d\overline{s} = 0 \tag{1.7}$$

$$\oint_C \overline{H} \cdot d\overline{\ell} = \int_A \overline{J} \cdot d\overline{s} + \frac{d}{dt} \int_A \overline{D} \cdot d\overline{s} \tag{1.8}$$

where S is the closed surface bounding the volume V and C is the closed path bounding the area A. We will actually show how the integral and differential forms of these equations are related a little later. For now, just take them on faith.

Suppose that the fields do not change in time (*static fields*). All of the time derivatives go to zero, and so Maxwell's equations become

$$\nabla \cdot \overline{D} = \rho_v \tag{1.9}$$

$$\nabla \times \overline{E} = 0 \tag{1.10}$$

$$\nabla \cdot \overline{B} = 0 \tag{1.11}$$

$$\nabla \times \overline{H} = \overline{J} \tag{1.12}$$

Note that for the case of static fields, the electric and magnetic fields are no longer coupled. Therefore, we can treat them separately. For electric fields, we are dealing with *electrostatics*. For magnetic fields, we are dealing with *magnetostatics*.

1.2 Charge and Current Distributions

We need to remind ourselves about charges. The volume charge density is the amount of charge per unit volume. It can vary with position in the volume. The total charge is the integral of the charge density over the volume V, or

$$Q = \int_V \rho_v \, dV \tag{1.13}$$

where Q is the charge in coulombs (C). When dealing with things like conductors, the charge may be distributed on the *surface* of a material. We therefore are interested in the *surface charge density* ρ_s with units of C/m^2. This is the amount of charge per unit area on the surface. The total charge Q would be the integral of ρ_s over the surface. Finally, we can have a *line charge density* ρ_ℓ with units of C/m which is the amount of charge per unit distance along a line. An example of this might be a very thin wire. The total charge Q would be the integral of ρ_ℓ over the length of the line segment.

By a similar token, we can discuss current density. $\overline{J}$ is the volume current density, measured in units of A/m^2. It represents the amount of current flowing through a unit surface area. The total current flowing through a surface A is therefore

$$I = \int_A \overline{J} \cdot d\overline{s} \tag{1.14}$$

We can also define a *surface current density* $\overline{J}_s$ with units of A/m. This means the current is confined to the surface, and so $\overline{J}_s$ is the amount of current per unit length, where the length represents the "cross-section" of the surface.

If we have a volume charge density ρ_v moving at a velocity $\overline{u}$ (note that we are including the vector direction in the velocity), then $\overline{J} = \rho_v \overline{u}$.

1.3 Electric Fields and Flux

Let's also quickly review electric fields and flux. Coulomb's law states that

1. an isolated charge q induces an electric field $\overline{E}$ at every point in space. At an observation point a distance R from this charge, the electric field is

$$\overline{E} = \hat{R}\frac{q}{4\pi\epsilon R^2} \tag{1.15}$$

   where $\hat{R}$ is the unit vector pointing from the charge to the observation point. ϵ is called the *permittivity* of the medium.

2. The force on a charge q' due to an electric field is

$$\overline{F} = q'\overline{E} \tag{1.16}$$

Now, the permittivity is:

$\epsilon = \epsilon_0\epsilon_r$	
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	permittivity of vacuum (free space)
ϵ_r	relative permittivity or dielectric constant of material

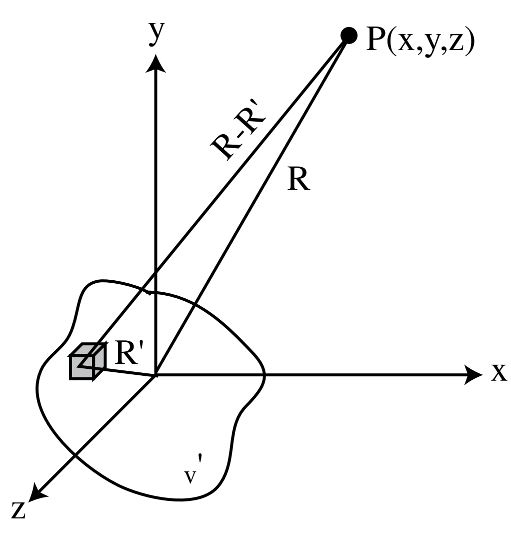
The electric flux density due to an electric field is

$$\overline{D} = \epsilon\overline{E} \tag{1.17}$$

The electric field for a charge distribution is given by

$$\overline{E} = \frac{1}{4\pi\epsilon}\int_{v'} \rho'_v \frac{\overline{R} - \overline{R}'}{|\overline{R} - \overline{R}'|^3} dv' \tag{1.18}$$

In cartesian coordinates this becomes



$$R' = x'\hat{x} + y'\hat{y} + z'\hat{z} \tag{1.19}$$

$$R = x\hat{x} + y\hat{y} + z\hat{z} \tag{1.20}$$

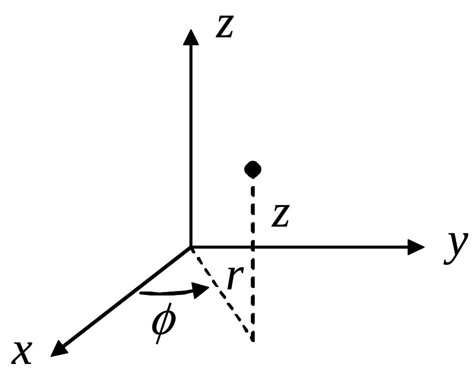
$$dv' = dx'dy'dz' \tag{1.21}$$

$$\overline{E}(x,y,z) = \frac{1}{4\pi\epsilon}\int_{v'} \rho_v'(x',y',z')\frac{(x-x')\hat{x} + (y-y')\hat{y} + (z-z')}{[(x-x')^2+(y-y')^2+(z-z')^2]^{3/2}}dx'\ dy'\ dz' \tag{1.22}$$

Even for a fairly simple charge distribution ρ_v' this equation is very complicated.

1.4 Review of Cylindrical and Spherical Coordinates

1.4.1 Cylindrical Coordinates



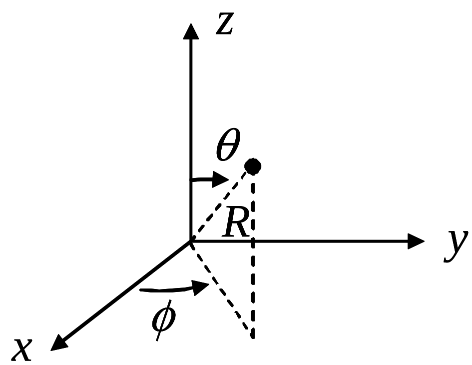
(r, ϕ, z)
$r = \sqrt{x^2 + y^2}$
$\phi = \tan^{-1}(y/x) =$ angle from $+x$ axis

Swept Variable	Differential Length
r	dr
ϕ	$rd\phi$
z	dz

1.4.2 Spherical Coordinates

(R, θ, ϕ)
$R = \sqrt{x^2 + y^2 + z^2}$
$\phi = \tan^{-1}(y/x) =$ angle from $+x$ axis
$\theta = \cos^{-1}(z/R) =$ angle from $+z$ axis

Swept Variable	Differential Length
R	dR
θ	$R\ d\theta$
ϕ	$R\sin\theta d\phi$



1.5 Gauss's Law

Starting with the point form of one of Maxwell's equations

$$\nabla \cdot \overline{D} \quad = \quad \rho_v \tag{1.23}$$

we want to convert it into the integral for. We do this by integrating both sides over a particular volume to get

$$\int_v \nabla \cdot \overline{D} dv \quad = \quad \int_v \rho_v dv. \tag{1.24}$$

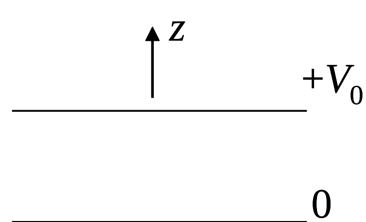
The divergence theorem is then used to convert the volume integral on teh left side to a surface integral as given by

$$\oint_S \overline{D} \cdot d\overline{s} = \int_v \rho_v dV = Q. \tag{1.25}$$

This equation is Gauss's Law.

Let's look at a parallel plate capacitor. We know that we have charge (equal and opposite) on the two conductors. Gauss's Law says that the flux equals the charge:

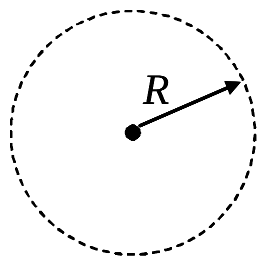
$$\psi_e = \oint_S \overline{D} \cdot d\overline{s} = \int_V \rho_v dV = Q \tag{1.26}$$



1.5.1 Applying Gauss's Law

1. Use symmetry to guess direction of $\overline{D}$.
2. Express flux as a vector (use unit vectors) with an unknown constant.
3. Pick surface to use for applying Gauss's Law.
4. Apply Gauss's Law.
5. Solve for unknown constant.

1.5.2 Point Charge



1. Flux must extend radially from point charge.
2. Flux density: $\overline{D} = D_o\hat{R}$.
3. Surface = sphere of radius R.
4. Gauss's Law:

$$\oint_S \overline{D} \cdot d\overline{s} = \int_V \rho_v dV = Q \tag{1.27}$$

$$\oint_S \overline{D} \cdot d\overline{s} = \int_0^{2\pi} \int_0^{\pi} D_o\hat{R} \cdot \hat{R}R^2 \sin\theta d\theta d\phi \tag{1.28}$$

$$= 2\pi D_o R^2 \int_0^{\pi} \sin\theta d\theta = 4\pi D_o R^2 = Q \tag{1.29}$$

5.

$$D_o = \frac{Q}{4\pi R^2} \tag{1.30}$$

$$\overline{D} = \frac{Q}{4\pi R^2}\hat{R} \tag{1.31}$$

So, the flux density falls off as $1/4\pi R^2 = 1/$surface area of sphere of radius R.

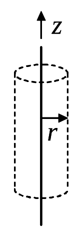
Note that we often use $\sin\theta d\theta d\phi$, which is a differential *solid angle*. A full sphere has 4π steradians of solid angle.

1.5.3 Line Charge

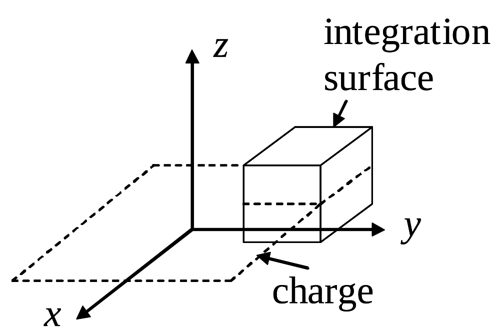
$$\begin{aligned}
\overline{D} &= D_o \hat{r} &(1.32)\\
\int_0^L \int_0^{2\pi} D_o \hat{r}\cdot\hat{r} r d\phi dz &= 2\pi r L D_o = Q &(1.33)\\
D_o &= \frac{Q}{2\pi r L} &(1.34)\\
\overline{D} &= \frac{Q}{2\pi r L}\hat{r} = \frac{Q}{L}\frac{1}{2\pi r}\hat{r} &(1.35)
\end{aligned}$$

Note that $Q/L = \rho_\ell$ which is the line charge density. So

$$\overline{D} = \frac{\rho_\ell}{2\pi r}\hat{r} \qquad (1.36)$$



1.5.4 Plane Charge



Suppose we have charge uniformly distributed over a plane. We choose the x-y plane for simplicity. Since the flux will be uniform in x and y, just look over a small area.

$$\overline{D} = \begin{cases} D_o \hat{z} & z > 0 \\ -D_o\ \hat{z} & z < 0 \end{cases} \qquad (1.37)$$

Our surface for applying Gauss's law is the cube with the plane of charge cutting it in half.

$$\int_0^{L_x}\int_0^{L_y} D_o\hat{z}\cdot\hat{z}dxdy - -\int_0^{L_x}\int_0^{L_y} D_o\hat{z}\cdot\hat{z}dxdy = Q \quad (1.38)$$

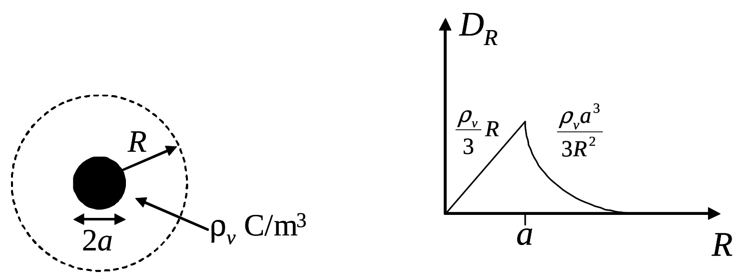
$$2D_oL_xL_y = Q \quad (1.39)$$

$$D_o = \frac{Q}{2L_xL_y} \quad (1.40)$$

$$\overline{D} = \frac{1}{2}\frac{Q}{L_xL_y}\hat{z}, \qquad z>0 \quad (1.41)$$

Note that $Q/L_xL_y = \rho_s$. Note also that the factor of 2 can be explained by the fact that 1/2 of the flux goes in the $+z$ direction and 1/2 goes in the $-z$ direction.

1.5.5 Sphere of Charge



Consider a sphere of radius a filled with charge at a volume charge density of ρ_v. The flux will be $\overline{D} = D_o\hat{R}$.

For $R \leq a$:

$$\int_0^{2\pi}\int_0^{\pi} D_o\hat{R}\cdot\hat{R}R^2\sin\theta d\theta d\phi = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho_v R^2\sin\theta dRd\theta d\phi \quad (1.42)$$

$$4\pi R^2 D_o = \frac{\rho_v}{3}R^3 4\pi \quad (1.43)$$

$$D_o = \frac{\rho_v R}{3} \quad (1.44)$$

$$\overline{D} = \frac{\rho_v}{3}R\hat{R} \quad (1.45)$$

For $R \geq a$:

$$\int_0^{2\pi}\int_0^{\pi} D_o\hat{R}\cdot\hat{R}R^2\sin\theta d\theta d\phi = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_v R^2\sin\theta dRd\theta d\phi \quad (1.46)$$

$$4\pi R^2 D_o = \frac{\rho_v}{3}a^3 4\pi \quad (1.47)$$

$$D_o = \frac{\rho_v a^3}{3R^2} \quad (1.48)$$

$$\overline{D} = \frac{\rho_v a^3}{3R^2}\hat{R} \quad (1.49)$$

1.6 Electric Potential

Suppose that we have a positive charge in an electric field. To move the charge against the electric field requires that we apply a force:

$$\overline{F}_{ext} = -q\overline{E} \tag{1.50}$$

The energy expended is:

$$dW = -q\overline{E} \cdot d\overline{\ell} \tag{1.51}$$

The differential energy per unit charge is defined as the differential electric potential:

$$dV = \frac{dW}{q} = -\overline{E} \cdot d\overline{\ell} \tag{1.52}$$

with units of volts = J/C.

To find the potential difference, we integrate dV:

$$V_{21} = V_2 - V_1 = \int_{P_1}^{P_2} dV = -\int_{P_1}^{P_2} \overline{E} \cdot d\overline{\ell} \tag{1.53}$$

Let's carefully consider the sign. If we have $\overline{E} = E_o\hat{x}$, and we integrate in $d\overline{\ell}$ from x_1 to x_2, where $x_2 > x_1$, then $V_2 < V_1$. Therefore, $V_2 - V_1 < 0$, meaning we have experienced a voltage drop. The equation is:

$$V_{21} = V_2 - V_1 = -\int_{x_1}^{x_2} E_o dx = -E_o(x_2 - x_1) \tag{1.54}$$

For static fields, V_{21} is independent of the path taken. This also implies that

$$\oint_C \overline{E} \cdot d\overline{\ell} = 0 \tag{1.55}$$

Note that this is one of Maxwell's equations for statics. In fact, there is a theorem in vector calculus known as Stokes' Theorem which states that

$$\int_S (\nabla \times \overline{E}) \cdot d\overline{s} = \oint_C \overline{E} \cdot d\overline{\ell} \tag{1.56}$$

So, using (**??**) in Stokes' Theorem results in

$$\nabla \times \overline{E} = 0 \tag{1.57}$$

which is the differential form of Faraday's Law for statics.

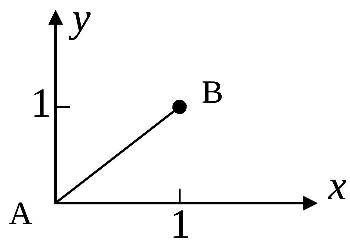
1.6.1 Example 1

Let

$$\overline{E} = x\hat{x} + y\hat{y} = r\cos\phi\hat{x} + r\sin\phi\hat{y} = r(\hat{x}\cos\phi + \hat{y}\sin\phi) = r\hat{r} \tag{1.58}$$

We want to find the potenial for the path defined by $x = y$ from $(0, 0)$ to $(1, 1)$.

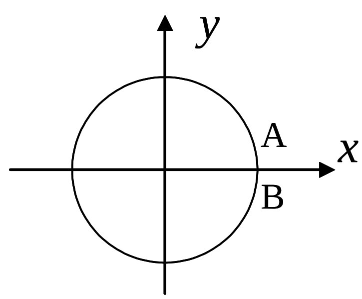
$$V_{BA} = V_B - V_A = -\int_0^{\sqrt{2}} r dr = -\left.\frac{r^2}{2}\right|_0^{\sqrt{2}} = -1 \tag{1.59}$$



1.6.2 Example 2

Let $\overline{E} = x\hat{x} = r\cos\phi\hat{x}$. We want to find the potenial for the circle of radius $r = 1$ about the origin in the x-y plane. Therefore, $d\bar{\ell} = \hat{\phi} r d\phi$ (don't forget to plug in $r = 1$ everywhere).

$$\begin{aligned} V_{BA} &= -\int_0^{2\pi} x\hat{x}\cdot\hat{\phi}d\phi \\ &= -\int_0^{2\pi} \cos\phi(-\sin\phi)d\phi \\ &= \frac{1}{2}\int_0^{2\pi} \sin 2\phi d\phi = 0 \end{aligned}$$



1.6.3 Example 3

Let $\overline{E} = y\hat{x} + x\hat{y} = r\sin\phi\hat{x} + r\cos\phi\hat{y}$. We will use the same unit circle as above.

$$\begin{aligned} V_{BA} &= -\int_0^{2\pi} [\sin\phi(\hat{x}\cdot\hat{\phi}) + \cos\phi(\hat{y}\cdot\hat{\phi})]d\phi \\ &= -\int_0^{2\pi} [-\sin^2\phi + \cos^2\phi]d\phi \\ &= -\int_0^{2\pi} \cos 2\phi d\phi = 0 \end{aligned}$$

1.6.4 Poisson's and Laplace's Equations

We have examined the integral relation between electric field and potential. However, if we know the potential, how do we obtain the electric field? First, we need to recall from vector calculus that the differential of any scalar can be written as

$$\begin{aligned} dT &= \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz && (1.60) \\ &= \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right) \cdot (dx\hat{x} + dy\hat{y} + dz\hat{z}) && (1.61) \\ &= \nabla T \cdot d\bar{\ell} && (1.62) \end{aligned}$$

Therefore, we can rewrite (**??**) as

$$dV = -\overline{E} \cdot d\bar{\ell} = \nabla V \cdot d\bar{\ell} \tag{1.63}$$

and can conclude from this that

$$\overline{E} = -\nabla V \tag{1.64}$$

Now, using Gauss's Law in differential form leads to

$$\begin{aligned} \nabla \cdot \overline{D} &= \nabla \cdot \epsilon \overline{E} = -\epsilon \nabla \cdot \nabla V = \rho_v && (1.65) \\ \nabla \cdot \nabla V &= -\frac{\rho_v}{\epsilon} && (1.66) \\ \nabla^2 V &= -\frac{\rho_v}{\epsilon} && (1.67) \end{aligned}$$

This is known as *Poisson's Equation*. If we have a charge-free region ($\rho_v = 0$) then

$$\nabla^2 V = 0 \tag{1.68}$$

which is known as *Laplace's Equation*. Now we have mechanisms for finding the field if we know the potential Note that in Cartesian coordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \tag{1.69}$$

We will actually solve these equations in the laboratory.

1.7 Electric Properties of Materials

We classify materials based upon their *constitutive parameters*

ϵ	electrical permittivity	F/m
μ	magnetic permeability	H/m
σ	conductivity	S/m

We will focus on μ later in our discussion of magnetostatics. For now, let's focus on the other two.

Conductors and dielectrics are classified by how well they conduct current. Dielectrics are essentially insulators, meaning that while they may allow small amounts of current flow through them, this current is quite small. Conductors on the other hand allow free flow of current. Materials that are in between are called semiconductors. Semiconductors have the interesting property that we can dope them properly such that we can control current flow and restrict it to given region.

1.7.1 Conductors

We are not going into great detail here. But we need to simply recognize that the conductivity relates current flow to the electric field which supplies the force to move the charges. This relationship is

$$\overline{J} = \sigma \overline{E} \tag{1.70}$$

which looks a lot like Ohm's law (in fact, Ohm's law is a simplification of this). Given this definition, we see that a perfect dielectric has $\sigma = 0$ so that $\overline{J} = 0$. A perfect conductor, on the other hand, has $\sigma \to \infty$, which implies that to have finite current density we must have $\overline{E} = 0$ since $\overline{E} = \overline{J}/\sigma$.

For metals $\sigma \sim 10^{-7}$, so it is common practice to set $\overline{E} = 0$. A perfect conductor is an equipotential medium, meaning that the electric potential is the same everywhere.

1.7.2 Resistance

$$R = \frac{V}{I} = \frac{-\int_l \overline{E} \cdot dl}{\int_S \overline{J} \cdot ds} = \frac{-\int_l \overline{E} \cdot dl}{\int_S \sigma \overline{E} \cdot ds} \tag{1.71}$$

Example: Linear resistor

1. Assume a particular voltage.
2. Calculate the resulting electric field.
3. Calculate the current from teh electric field.
4. Divide voltage by curretn to get resistance.

$$\begin{aligned} V &= -\int_{x_1}^{x_2} \overline{E} \cdot dl && (1.72) \\ &= -\int_{x_1}^{x_2} \hat{x} E_x \cdot \hat{x} dl && (1.73) \\ &= E_x l && (1.74) \end{aligned}$$

$$I = \int_A \overline{J} \cdot ds = \int_A \sigma \overline{E} \cdot ds = \sigma E_x A \tag{1.75}$$

$$R = \frac{V}{I} = \frac{E_x l}{\sigma E_x A} = \frac{l}{\sigma A} \tag{1.76}$$

How does the resistance apply to integrated circuits? A typical process technology has interconnect lines that are $0.4\mu m$ wide and $0.6\mu m$ high. With copper as the metal the resistance is given by

$$R = \frac{l}{(5.810^7)(.24\mu m^2)} = 72\ \Omega/mm \tag{1.77}$$

Example: Conductance of a Coaxial Cable

What is the conduction between the conductors of a coaxial cable?

This time assume a current flowing between the two conductors I and then calculate the voltage from this assumed current.

$$Jv = \hat{r}\frac{I}{A} = \hat{r}\frac{I}{2\pi rl} \tag{1.78}$$

Since $Jv = \sigma\overline{E}$

$$Ev = \hat{r}\frac{I}{2\pi\sigma rl} \tag{1.79}$$

Calculate the voltage from the electric field

$$V_{ab} = -\int_b^a \overline{E} \cdot dl = -\int_b^a \frac{I}{2\pi\sigma l}\frac{\hat{r}\cdot\hat{r}dr}{r} \tag{1.80}$$

$$V_{ab} = \frac{I}{2\pi\sigma l} ln\left(\frac{b}{a}\right) \tag{1.81}$$

$$G' = \frac{G}{l} = \frac{1}{Rl} = \frac{I}{V_{ab}l} = \frac{2\pi\sigma}{ln(b/a)} \tag{1.82}$$

1.7.3 Dielectrics

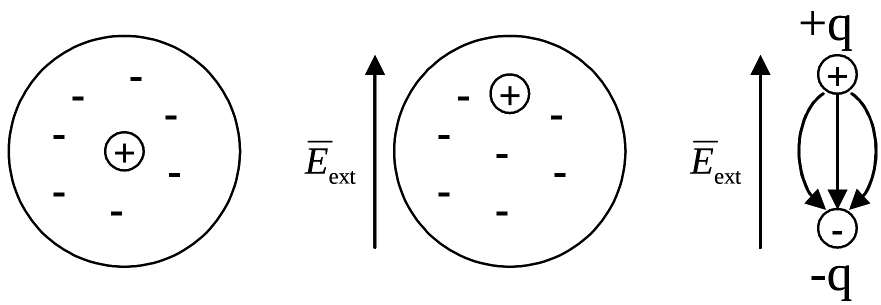
Let's examine dielectrics a little bit more. We will assume we are dealing with perfect dielectrics with $\sigma = 0$ and therefore $\overline{J} = 0$. If we look at the microscopic level, we consider an atom or molecule with a positively charged nucleus and negatively charged electron cloud. The cloud center is coincident with the nucleus center, leading to a neutral charge configuration. If an external electric field $\overline{E}_{ext}$ is applied to the material, the center of the electron cloud will be displaced from its equilibrium value. While we still have charge neutrality, we can consider that there will be an electric field emanating from the positively charged nucleus and ending at the negatively charged cloud center. This process of creating electric *dipoles* within the material by applying an electric field is called *polarizing* the material.

The induced electric field created by our new dipole is referred to as a *polarization* field, and it is weaker and in the opposite direction to $\overline{E}_{ext}$. We actually express this using the flux density

$$\overline{D} = \epsilon_0\overline{E} + \overline{P} \tag{1.83}$$

where $\overline{P}$ is referred to as the *Polarization* of the material. This quantity is proportional to the applied field strength, since the separation of the charges is more pronounced for stronger external fields. We can therefore write

$$\begin{aligned}
\overline{P} &= \epsilon_0\chi_e\overline{E} && (1.84)\\
\overline{D} &= \epsilon_0\overline{E} + \epsilon_0\chi_e\overline{E} = \epsilon_0(1+\chi_e)\overline{E} = \epsilon\overline{E} && (1.85)\\
\epsilon &= \epsilon_0\underbrace{(1+\chi_e)}_{\epsilon_r} && (1.86)
\end{aligned}$$



1.8 Capacitance

Capacitance is capacity to store charge.

$$C = \frac{Q}{V} \tag{1.87}$$

with units of Farads = C/V. V is the potential difference between the conductor with charge $+Q$ and the conductor with charge $-Q$.

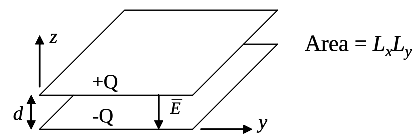
Steps:

1. Assume a charge on the conductors
2. Find $\overline{D}$ from Gauss's Law
3. Find $\overline{E} = \overline{D}/\epsilon$.
4. Find V from $\overline{E}$
5. Find C from $C = Q/V$

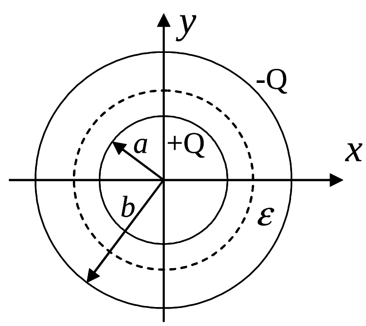
1.8.1 Parallel Plate Capacitor

We assume that the charge will evenly distribute itself over the conducting plates.

$$
\begin{aligned}
\oint \overline{D} \cdot d\overline{s} &= Q & (1.88)\\
\int_0^{L_y} \int_0^{L_x} D_o dx dy &= Q & (1.89)\\
D_o &= \frac{Q}{L_x L_y} = \frac{Q}{A} & (1.90)\\
\overline{D} &= -\frac{Q}{A}\hat{z} & (1.91)\\
\overline{E} &= -\frac{Q}{A\epsilon}\hat{z} & (1.92)\\
V &= -\int_0^d -\frac{Q}{A\epsilon}\hat{z} \cdot \hat{z} dz = \frac{Qd}{A\epsilon} & (1.93)\\
C &= \frac{Q}{V} = \frac{\epsilon A}{d} & (1.94)
\end{aligned}
$$



1.8.2 Spherical Shell Capacitor



The charge will evenly distribute itself over the conducting spherical shells.

$$\oint \overline{D} \cdot d\overline{s} = Q \tag{1.95}$$

$$\int_0^{2\pi} \int_0^{\pi} D_o R^2 \sin\theta \; d\theta d\phi = Q \tag{1.96}$$

$$4\pi R^2 D_o = Q \tag{1.97}$$

$$D_o = \frac{Q}{4\pi R^2} \tag{1.98}$$

$$\overline{D} = \frac{Q}{4\pi R^2}\hat{R} \tag{1.99}$$

$$\overline{E} = \frac{Q}{4\pi\epsilon R^2}\hat{R} \tag{1.100}$$

$$V_{ab} = V_a - V_b = -\int_b^a \frac{Q}{4\pi\epsilon R^2}\hat{R} \cdot \hat{R} dR \tag{1.101}$$

$$= \left.\frac{Q}{4\pi\epsilon R}\right|_b^a = \frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right) \tag{1.102}$$

$$C = \frac{Q}{V_{ab}} = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}} \tag{1.103}$$

Question: Why does larger ϵ increase the capacitance?