Appendix

Time Harmonic Maxwell's Equations

$\nabla \times \vec{E} = -j\omega\mu \vec{H}$

$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon \vec{E}$

$\nabla \cdot \vec{B} = 0$

$\nabla \cdot \vec{D} = \rho_v$

$\vec{D} = \varepsilon \vec{E}$

$\vec{B} = \mu \vec{H}$

ε_o=8.85e-12 F/m
μ_o=4π*1e-7 H/m

Boundary Conditions

Tangential E:	$E_{1t} = E_{2t}$	$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$
Normal D:	$D_{1n} - D_{2n} = \rho_s$	$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$
Tangential H:	$H_{1t} = H_{2t} = J_s$	$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$
Normal B:	$B_{1n} - B_{2n} = 0$	$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$

General propagation constant: $$\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu \left(\varepsilon_o \varepsilon_r - j\frac{\sigma}{\omega} \right)$$

$$\gamma = -\alpha + j\beta$$

General plane wave equations: $$\vec{E} = \vec{E}_o \, e^{-\gamma z}$$

$$\vec{E} = -\eta \, \hat{k} \times \vec{H}$$

$$\vec{H} = \frac{1}{\eta} \hat{k} \times \vec{E}$$

Lossless propagation constant: $k = \omega\sqrt{\mu\varepsilon}$

Lossless propagation constant: $$k = \omega\sqrt{\mu\,\varepsilon} = \frac{2\pi}{\lambda}$$

Wavelength: $$\lambda = \frac{v}{f}$$

Phase velocity: $$v = \frac{1}{\sqrt{\mu\,\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}}$$

Intrinsic Impedance: $$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Skin depth: $\delta = \frac{1}{\alpha}$

Poynting Vector: $\vec{S} = \vec{E} \times \vec{H}$

Time average power density: $S_{av} = \frac{1}{2}\mathrm{Re}\left\{\vec{E} \times \vec{H}^*\right\}$

Snell's Law: $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

Reflection Coefficients:

$$\Gamma_\perp = \frac{\vec{E}_r}{\vec{E}_i} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\Gamma_{//} = \frac{\vec{E}_r}{\vec{E}_i} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Transmission coefficients

$$\tau_\perp = \frac{\vec{E}_t}{\vec{E}_i} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_{//} = \frac{\vec{E}_t}{\vec{E}_i} = \frac{\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_\perp = 1 + \Gamma_\perp$$

$$\tau_{//} = \left(1 + \Gamma_{//}\right)\frac{\cos\theta_i}{\cos\theta_t}$$

Reflectivity (power reflection) $R_\perp = \frac{P_r}{P_i} = \left|\Gamma_\perp\right|^2$ $R_{//} = \frac{P_r}{P_i} = \left|\Gamma_{//}\right|^2$

Antenna Radiation Equation: $\vec{A}(R) = \frac{\mu}{4\pi}\int \vec{J}\,\frac{e^{-jk\left|\vec{R}-\vec{R}'\right|}}{\left|\vec{R}-\vec{R}'\right|}\,dv'$

Far-Field Hertzian Dipole equations:

$$\vec{E} = \hat{\theta}\; j\frac{I_o\, l\, k\,\eta_o}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin(\theta)$$

$$\vec{H} = \hat{\phi}\; j\frac{I_o l\, k}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin(\theta)$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$