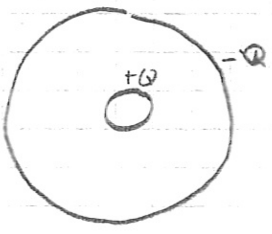
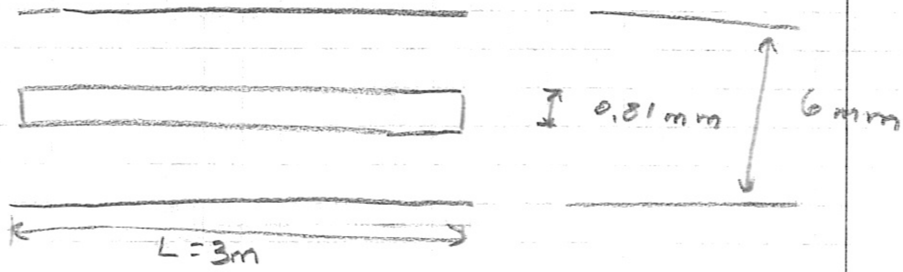
Electric field of a Charged section of RG-59 co-axial cable



end view



side view

step 1 : Determine symmetry of the system
→ $\vec{D}$ are radially away from conductors

$$\vec{D} = D_0(\rho)\,\hat{\rho}$$

step 2: choose surface such that $\vec{D}$ is either constant or zero

choose a cylinder of radius r where r varies from $0 \to \infty$

$\vec{D}$ is constant on tube surface
$\vec{D} \cdot \hat{z} = 0$ on the ends of the tube

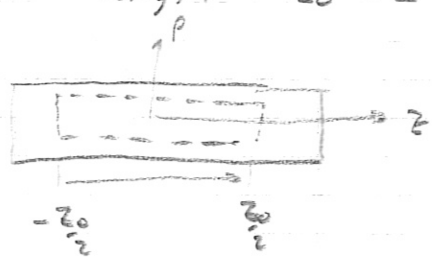
Step 3: set up integral

Break up into 3 regions
(1) $0 < r < 0.405$
(2) $0.405 < r < 3$
(3) $r > 3$

region 1 : Assume charge is equal spread throughout the center conductor

$$\rho_v = \frac{Q}{\pi (.405)^2 L} =$$

choose a tube of length $z_0 < L$ and radius r



$$\oint \vec{D} \cdot d\vec{s} = \int_v \rho_v \, dv$$

$$\underbrace{\int_0^{2\pi} \int_{-\frac{z_0}{2}}^{\frac{z_0}{2}} D_0 \hat{\rho} \cdot \rho \, d\rho \, d\phi \, \hat{\rho}}_{\rho = r} = \int_0^{2\pi} \int_{-\frac{z_0}{2}}^{\frac{z_0}{2}} \int_0^{r} \frac{Q}{\pi (.405)^2 L} \rho \, d\rho \, dz \, d\phi$$

use geometry to do LHS integral

$$D_0 \; 2\pi r \cancel{z_0} = 2\pi \cancel{z_0} \frac{Q}{\pi (0.405)^2 L} \int_0^r \rho \, d\rho$$

$$= \frac{2 \cancel{z_0} Q}{(.405)^2 L} \frac{1}{2} r^2$$

$$D_0 = \frac{Q}{2\pi (0.405)^2 L} r$$

region 2: enclosed charge doesn't change

$$Q = \int_0^{.405} \int_0^{2\pi} \int_0^{z_0} \frac{Q}{\pi (.405)^2 L} \rho \, dz \, d\phi \, d\rho$$

$$= \frac{Q}{\pi (.405)^2 L} (z_0)(2\pi) \frac{(.405)^2}{2}$$

$$= Q \left(\frac{z_0}{L}\right)$$

$$\oint_s \bar{D} \cdot ds = Q \left(\frac{z_0}{L}\right)$$

$$D_0 \; 2\pi r \cancel{z_0} = Q \frac{\cancel{z_0}}{L}$$

$$D_0 = \frac{Q}{2\pi r L}$$

region 3: total enclosed charge $= 0$

$$D_0 = 0$$

Step 4: write out $\bar{D}$

$$\vec{D} = \begin{cases} \dfrac{Q \, r}{2\pi (.405)^2 L} \hat{\rho} & 0 < r < .405 \\ \dfrac{Q}{2\pi r L} \hat{\rho} & .405 < r < 3 \\ 0 & r > 3 \end{cases}$$

Step 5 divide by ε to get $\vec{E}$