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ECEn 370

Probability Theory

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**BYU**

BRIGHAM YOUNG  
UNIVERSITY

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# Lecture 1:

## Why study probability?

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If this class was not required, would it be on your study list?  
**IT SHOULD BE IF YOU WANT TO BE AN ENGINEER!**

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# We **lied** to you in ECEn 212, 380, 360, and 224!

- Signals and systems of interest to engineers are not deterministic.
  - You cannot predict behavior precisely.
  - Deterministic analysis and design methods often don't work!
- Signals like:

$$x(t) = a \cos(\omega t + \phi), \quad y(t) = a u(t) e^{-\alpha t}$$

$$z[n] = \sum_{k=0}^{N-1} a_k e^{j \frac{kn2\pi}{N}}, \quad w[n] = \delta[n] + 4\delta[n-3]$$

are rare beasts in nature!

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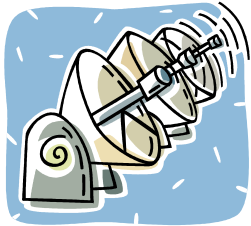
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    - ❑ Local oscillators in RF receivers.
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    - ❑ C.W. pilot tones and reference transmissions.
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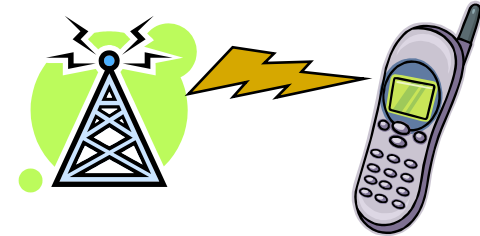
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  - ❑ a.c. power (maybe).
  - ❑ Some others, but BORING!

# Why is a deterministic model usually inadequate?

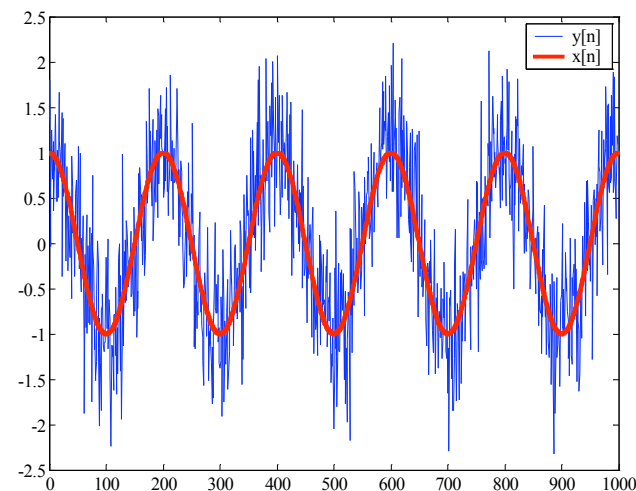
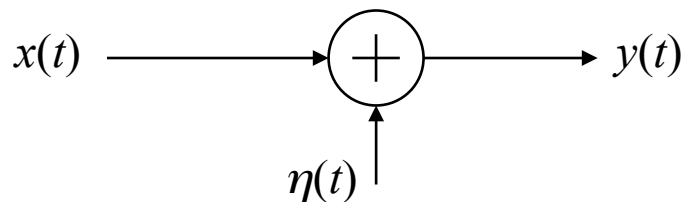
- EEs focus on information transmission and processing.
  - There is no information in deterministic signals.



Information = Uncertainty



- Let  $x(t)$  be a talk radio broadcast. How useful is it if  $x(t)$  is known?
- Noise is ubiquitous.



# Why is a deterministic model usually inadequate?

- Engineering systems may be deterministic in design (software, CPUs, circuits, etc.) but:
  - Must respond to unknown and unknowable random external events, input signals, demands, etc.
  - If these excitations were precisely known the design would be considerably simpler.
- We need a way to analyze performance and design systems in the presence of randomness.
  - How do you measure performance if signals are random?
    - Mean squared error, variance, power spectral density, probability of detection, probability of failure, etc, etc.
  - The study of probability and random processes gives us the tools!

# Random signal models

- More realistic mathematical representations look like (for example):

$$x(t) = A \cos(\omega t + \phi) + \eta(t), \quad y(t) = A u(t) e^{-\zeta t}$$

$$z[n] = \sum_{k=0}^{N-1} A_k e^{j\left(\frac{kn2\pi}{N} + \phi_k\right)}, \quad w[n] = B\delta[n] + C\delta[n - m]$$

where  $A, B, C, \phi, \eta, \zeta$  and  $m$  are random.

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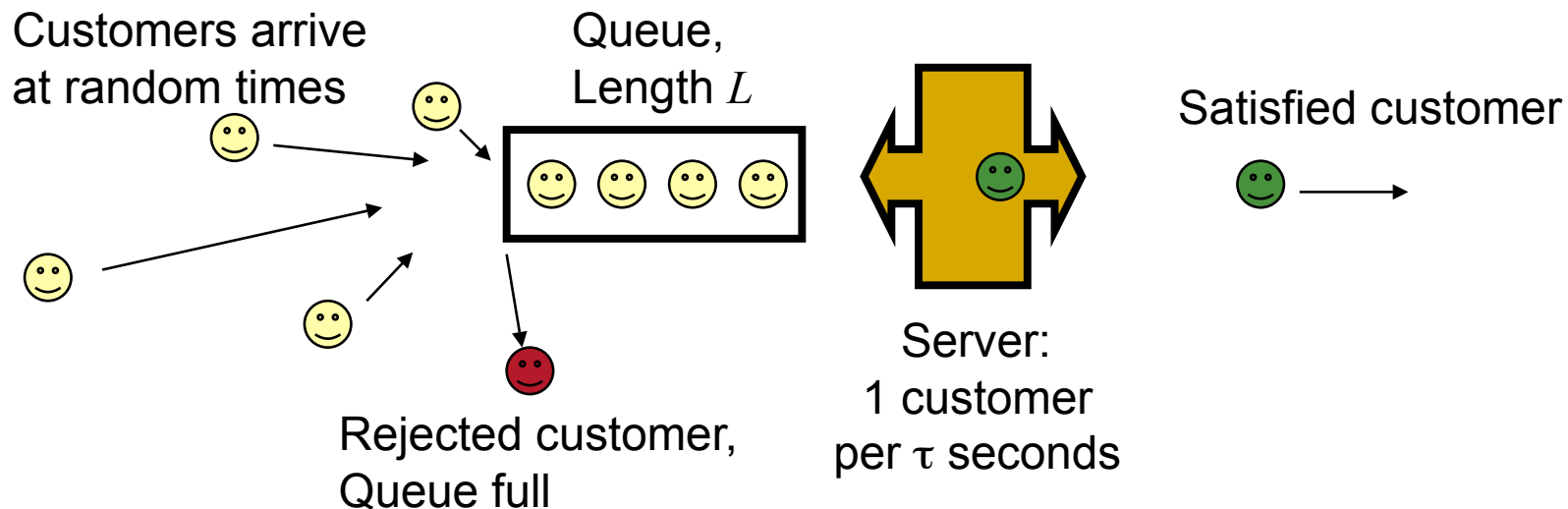
# The need for probabilistic analysis,

## Example one: White Noise

- ❑ See MATLAB demo whiteNoiseDemo.m
  - ❑ Illustrates thermally generated Gaussian white noise.
  - ❑ This is the most common noise signal in sensor measurement and communications.
    - We would like to know noise power level, spectral content, its correlation from sample to sample, etc.
    - This is a real signal, but it changes in an unknown fashion from sample to sample.
  - ❑ White noise is broadband, but the conventional deterministic FFT fails to evaluate its spectrum.
  - ❑ Lowpass or bandpass filtering makes it look more deterministic, but FFT still fails.
  - ❑ Statistical analysis based on sample second moments, the averaged periodogram, is needed to analyze this signal.
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# The need for probabilistic analysis, Example two: Server Queue

- Consider a server process
  - e.g. internet packet switcher, HDTV frame decoder, bank teller line, instant messenger video display, IP phone, multitasking operating system, hard disk drive controller, etc., etc.



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# The need for probabilistic analysis, Example two: Server Queue

- Classroom exercise: Candy server.
  - Coin tosses with three successive heads in 30 seconds authorizes you to queue for candy. Simulates a Poisson arrival distribution.
  - Server gives at most one piece of candy in 10 seconds.
  - Queue size is 6 people long. If queue is full you get no candy.

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- What would we like to know about performance to guide us in design? All these require probability theory tools!
  - Details about arrivals: rate, arrival interval spread, its random distribution over time, how arrival distribution affects server performance, etc.
  - Rate of dropping service requests.
  - Queue occupancy, average and variation, excess capacity.
  - Typical values and variations in throughput.
  - Typical values and variations in delay through queue, i.e. latency, wait time.