ECEN 370 Practice Final Exam

Solutions - Brian Jeffs

1. $E\{X\} = E\{E\{X|p\}\}$

$E\{X|p\} = np = 10p$

$E\{10p\} = 10E\{p\} = 10\left[\frac{0.4+0.5}{2}\right] = 4.5$

(b)

2. $E\{\cos(\theta+\frac{\pi}{2})\} = \int_0^{\pi/2} \cos(\theta+\frac{\pi}{2})\,\frac{2}{\pi}\,d\theta$

$= \frac{2}{\pi}\sin(\theta+\frac{\pi}{2})\Big|_0^{\pi/2} = \frac{-2}{\pi}$

(d)

3. Let $Z = XY$. $f_{(z)} = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{XY}(x, z/x)\,dx$

$f(z) = \int_0^1 \frac{1}{x}\, x\, e^{-x\, z/x}\, u(z/x)\,dx$

$= \int_0^1 e^{-z} u(z)\,dx = e^{-z} u(z)$

$E\{Z\} = \int_0^{\infty} z e^{-z}\,dz = (-z-1)e^{-z}\Big|_0^{\infty}$

$= +1$ (a)

4. For $Z \sim U(a,b)$ $Var(Z) = \frac{(b-a)^2}{12}$

So $Var(X) = \sigma_x^2 = \frac{1}{3}$, $Var(Y) = \sigma_Y^2 = \frac{1}{3}$

$\rho = \frac{m_{11}}{\sqrt{m_{20}m_{02}}} = \frac{\xi_{11} - \xi_1\xi_2}{\sqrt{m_{20}m_{02}}} = \frac{E\{XY\} - \mu_X\mu_Y}{\sigma_X\,\sigma_Y}$

$= \frac{8.1667 - (2)(4)}{1/3} = 0.5$ (d)

5. $P_X(x_i) = \sum_j P_{XY}(x_i, y_j)$.

$P_X(0) = 1/2, \quad P_X(1) = 1/8 + 1/8 = 1/4, \quad P_X(-1) = 1/8 + 1/8 = 1/4$

$E\{X\} = 1 \cdot 1/4 - 1 \cdot 1/4 + 0 \cdot 1/2 = 0$

$Var\{X\} = E\{(X-0)^2\} = 1^2 \cdot 1/4 + (-1)^2 1/4 + 0 \cdot 1/2 = 1/2$

(c)

6. $\lambda = (64 \times 10^3)(.001) = 64$ errors/sec.

$\tau = 10\,s$

$$P_X(k) = \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau} \quad \text{(poisson)}$$

$E\{X\} = \lambda\tau = 64 \cdot 10 = 640$

(e)

(Note, minus sign was missing on exam)

7. $f_T(t|A) = \frac{1}{\mu_A} e^{-t/\mu_A} u(t)$ etc. for B, C, D.

$$P(A) = \frac{1000}{1000+1000+1500+2000} = 0.1818 = \frac{1}{5.5}$$

$P(B) = \frac{1}{5.5}, \quad P(C) = \frac{1.5}{5.5} = \frac{3}{11}, \quad P(D) = \frac{2}{5.5}$

$E\{T\} = E\{T|A\}P(A) + E\{T|B\}P(B) + E\{T|C\}P(C) + E\{T|D\}P(D)$

$$= \frac{2}{5.5} + \frac{1.5}{5.5} + \frac{1 \times 3}{11} + \frac{.75 \cancel{\times 2}}{5.5} = 1.1818$$

(a) 1.18

8. $Y = X + V. \quad \mu_X = 1, \ \mu_V = 0, \ \mu_Y = \mu_X + \mu_V = 1$

$E\{XY\} = E\{X^2 + XV\} = (\sigma_X^2 + \mu_X^2) + (\mu_X)(\mu_V) = 3$

(due to independence)

$\sigma_Y^2 = \sigma_X^2 + \sigma_V^2 = 2 + 1 = 3$

$$\rho = \frac{E\{XY\} - \mu_X \mu_Y}{\sigma_X \sigma_Y} = \frac{2}{(\sqrt{2})(\sqrt{3})} = 0.816$$

(b)

9. $f_{XY}(x,y) = (e^{-x}u(x))(e^{-y}u(y)) = f_X(x)f_Y(y)$, so X & Y are independent, and we can use the general formula for transformations of single random variables. Also, since Z and W depend exclusively on X and Y respectively, W and Z are mutually independent, and $f_{ZW}(z,w) = f_Z(z)\, f_W(w)$.

$Z = g(X) = 3X$. $x_i = Z/3$ (single root).

$g'(x_i) = 3$. So $f_Z(z) = \frac{f_X(z/3)}{|3|} = \frac{1}{3} e^{-z/3} u(z)$.

Likewise, $f_W(w) = \frac{1}{2} e^{+w/2} u(-w)$.

$f_{ZW}(1,-1) = (\frac{1}{3} e^{-1/3})(\frac{1}{2} e^{-1/2}) = 0.0724$, (a)

10. $E\{Z\} = E\{2X^2+3\} = 2E\{X^2\} + 3$
$= 2(4 + (-2)^2) + 3 = 19$ (e).

11. $\Phi_X(\omega) = E\{e^{j\omega X}\} = (e^0)(1-p) + e^{j\omega} p$
$= 1 - p(e^{j\omega} - 1) = pe^{j\omega} + 1 - p$. (e)

12. Approach 1: $\Phi_X^{(10)}(\omega) = p(j)^{10} e^{j\omega}$
$\xi_{10} = \frac{1}{j^{10}} \Phi^{(10)}(0) = pe^{j0} = p = 0.6$ (c)

Approach 2: $\xi_{10} = 0^{10}(1-p) + (1)^{10} p = p = 0.6$