

Quiz 10

ECEn 370

Name: _____

Let X be distributed $U(0,1)$, and let Y be distributed $U(1,5)$. Let $Z = X + 2Y$.

1. Find $f_{Z|Y}(z|y)$.

$f_{Z|Y}(z|y)$ is simply $f_X(z-2y)$, i.e. the mean is shifted by $2y$. This follows from treating y as a constant, and applying the transformation of RV formula:

$$f_{Z|Y}(z|y) = \frac{f_X(x_1)}{|g'(x_1)|}. \quad x_1 = z - 2y, \quad g'(x) = 1, \quad \text{so}$$

$$f_{Z|Y}(z|y) = f_X(z - 2y) = U(2y, 2y + 1).$$

2. Compute $E[Z | Y = y]$.

$$E[Z | Y = y] = (2y + 1 + 2y) / 2 = 2y + 1/2.$$

3. Compute $E[E[Y | X]]$.

$$E[E[Y | X]] = E[Y] = (5 + 1) / 2 = 3.$$