

Quiz 12

ECEEn 370

Name: _____

1. Let $X \sim$ exponential with $f_X(x) = e^{-x}u(x)$.
- a) Compute the moment generating function, $\theta(t)$, for X .
(Hint: $\theta(t)$ is unbounded for $t \geq 1$, so assume $t < 1$.)

$$\begin{aligned}\theta(t) &= E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f_X(x) dx = \int_0^{\infty} e^{x(t-1)} dx \\ &= \frac{1}{t-1} e^{x(t-1)} \Big|_{x=0}^{\infty} = 0 - \frac{1}{t-1} = \frac{1}{1-t} \text{ for } t < 1.\end{aligned}$$

- b) Use $\theta(t)$ to compute $\xi_1 = E[X]$. (If you are uncertain about your answer for part a) you will receive some credit for correctly stating the formula for computing the first moment using the moment generating function.)

$$\begin{aligned}\xi_1 &= \theta^{(1)}(t) \Big|_{t=0} \\ &= \frac{1}{(1-t)^2} \Big|_{t=0} = 1.\end{aligned}$$