Homework Assignments

(Unless otherwise noted, all problems are from Stark and Woods)

Homework set #1 Due Sep. 10

1. Problem 1.3
2. Problem 1.7
3. Problem 1.8
4. Problem 1.10
5. Problem 1.11

Homework set #2 Due Sep. 17

1. Problem 1.13
2. Problem 1.15
3. Problem 1.18.
   Note that this exercise illustrates the difficulty with drug use screening tests (e.g. on the job, or Olympic athlete testing), or with cancer screening tests. Even with a low false positive rate, if the underlying true incidence of the condition being tested is low, the odds that a positive detection is wrong are very high. Only when the true incidence rate is high (e.g. perhaps steroid use in professional athletes) are the results reliable. Judges and juries don't understand this issue, and lots of people may be given unnecessary treatment for a disease they do not have. Yikes!
4. Problem 1.20
5. Problem 1.23

Homework set #3 Due Sep. 24

1. Problem 1.28, but change so $N = 4$ and the cumulative probability that a correct re-ordering occurs for the first time on the nth try is 0.80.
2. Problem 1.30
3. Problem 1.31

4. Problem 1.32, but change so for every 100 chips produced, 90 meet spec., 8 need rework, and 2 discarded.

5. Problem 1.33

6. Problem 1.35

Homework set #4 Due Oct. 1

1. Problem 1.38, but assume the switches are not equally likely to be closed. Let $P[A_2] = P[A_4] = 0.25$. All other switches have a probability of 0.6 for being closed.

2. Problem 1.42

3. Problem 1.45, but add a third terminal, C, located at the left node of the top, horizontal branch of the hexagon. Terminals pass packets through which are not addressed to them. Compute the probability of successful transmission from C to A, and also from C to B.

4. Problem 1.47

5. Problem 1.48

6. Problem 1.50

Homework set #5 Due Oct. 8

1. Problem 2.9

2. Problem 2.11

3. Problem 2.2, except let

$$F_X(x) = \begin{cases} 0 & -\infty \le x < 0 \\ \left(\frac{1}{9}\right) & 0 \le x \le 3 \\ \left(\frac{x^2}{27}\right) & 3 < x \le 4 \\ \left(\frac{4x}{27}\right) & 4 < x \le \frac{27}{4} \\ 1 & x > \frac{27}{4} \end{cases}$$

4. Problem 2.5, except rather than using $k\sigma$ do the following:
Noise voltage power (variance) in a resistor is given by $\sigma^2 = 4k_B BTR$ where $k_B = 1.38\times10^{-23}$ is Boltzmann's constant, T is the resistor's absolute temperature in degrees

Kelvin, *B* is the bandwidth in Hertz of the measurement, and *R* is the resistance in Ohms. Consider an audio line level input to a 24 bit analog to a digital converter (A/D) that has an input impedance of 600Ω and a bandwidth of 20 kHz. Assume the full-scale voltage for the A/D converter is +/-5V. What is the probability that for a single time sample, the 600Ω input resistor thermal noise will produce a voltage that exceeds the 1/2 LSB (A/D) threshold? In other words, what is the probability that the noise voltage would cause the least significant bit of the A/D to toggle on. This occurs if the voltage exceeds $5/(2^{24})$ V. Assume room temperature of 300 K.

5. Problem 2.8, except replace the impulse at $x = 2$ has area 1/3, and the impulse at x = 3 has area 1/8. Add: f) compute $F_X(2.5)$.

Homework set #6 Due Oct. 15

1. Problem 2.26

2. Problem 2.11

3. Problem 2.13, except assume *X* is distributed uniform on [0,3], not [0,5].

4. Problem 2.16, except *X* is distributed uniform on [-2,2], not [-1, 1].

5. Problem 2.18

Homework set #7 Due Oct. 22

1. Problem 2.17, except the arrival time is distributed Gaussian, with mean, $\mu = 9{:}00$ a.m., and standard deviation, $\sigma = 15$ minutes.

2. Problem 2.23, except the Poisson rate is 1×10^6 photons per second.

3. Problem 2.20 & 2.21, except change $x_A = 0.75$, and $P[M] = 10^{-4}$.

4. In this problem you will evaluate the MATLAB Gaussian random number generation algorithm. Using MATLAB functions "randn," "hist," and "erf," to do the following.
a) Generate a vector, x, of 1000 independent samples of a Gaussian random variable, *x*, with unit variance, zero mean.
b) Compute and plot (as a line plot, not a bar graph) the histogram of X, normalized to be an approximation of the pdf, $p(x)$. Normalization involves dividing the bin counts by the total number of samples, and by the width of a bin. In computing the histogram, use 100 bins, and the form [N,X] = hist(x,100). On the same plot include the true theoretical Gaussian pdf curve for $p(x)$ for comparison. Comment on how good the match is between the histogram and true pdf.

c) Repeat step b for 1e4, 1e5, and 1e6 samples in x. How does the comparison change, and why?
d) Use the erf function to compute the probability $P(x<1.5)$. Please note the difference in your text and MATLAB definitions for erf(x). See footnote page 69. Also approximate this using the N and X vectors from the hist function with 1e6 samples. Explain how you did this.

5. Problem 2.24

Homework set #8 Due Oct. 29

1. a) Problem 3.4. This is the model used in our ECEn 688 class when we study X-ray computed tomography.
b) Using the MATLAB randn function, generate random Gaussian samples, x, with $\mu = .5$ and $\sigma^2 = 2$. Use the above transform to obtain $y = e^z$ and plot the histogram of y and the pdf you computed in part a) on the same graph. This can serve as a validation for your answer to part a).

Homework set #9 Due Nov. 5

1. Problem 3.6.

2. Problem 3.9, except X is uniform on [0,1].

3. Problem 3.7.

4. Problem 3.16, except the desired pdf for X is: $f_X(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$.
Use the MATLAB rand function to generate the uniform samples, transform them using your solution, and plot a histogram to confirm the pdf is right. (Hint: look at the synthTrianglepdf.m demo).

5. Problem 3.13, except $Z = Y\text{-}3X$.

Homework set #10 Due Nov. 12

1. Problem 3.17, except instead of summing uniform RVs, Us the transform method $x = F_X^{-1}(y)$ where $y \sim U(0,1)$ and $F_X^{-1}(\,)$ is the inverse distribution function for the desired distribution. (hint: use the matlab function "erfinv." Make sure to account for MATLAB's different definition of erf(), and take care of the fact that erf only integrates from 0 to x.

2. Problem 3.21

3. Problem 3.24

4. Let $X \sim f_X(x) = 2xu(x)u(1-x)$. Find $E\{X\}$.

1. Let $X \sim f_X(x) = \alpha e^{-\alpha x}u(x)$. Find $E\{X\}$.

Homework set #11 Due Nov. 19

5. Problem 4.5, except let *Y*= 1/*X*.

1. Problem 4.8. You can do this with a single statement, one line proof.

2. Problem 4.12, except
   $f_\Theta(\theta) = e^{-\theta}, \quad 0 \le \theta \le \infty$

3. Problem 4.13 (a through e) except use the moment generating function rather than direct integration to derive the result. [Hints: For the Poisson distribution utilize the fact that $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$. The Rayleigh moment generating function is
   $\theta(t) = (1 + \sqrt{\pi/2}\ t)e^{\sigma^2 t^2/2}$]

4. Problem 4.14

Homework **set #12** Due Tues. Nov. 24

1. Problem 4.16

2. Problem 4.22

3. Problem 4.23

Homework set #13 Due Thurs. Dec. 3

1. Problem 4.24

2. Problem 4.25

3. Problem 4.28

4. The Cauchy pdf has some odd properties, one of which is that a Cauchy distributed R.V. has *no* moments, i.e. the moments do not exist because the defining integrals do not converge.
   a) Consider problem 4.30. Write down the integral equation that must be evaluated to find the characteristic function of the Cauchy pdf. To solve this requires complex plane contour integration and partial fraction expansion. If you are brave and want 5 points extra credit, prove that the characteristic function is $\Phi_X(\omega) = e^{-\alpha|\omega|}$.
   b) Using the characteristic function show that a Cauchy distributed R.V. has no

moments.
c) Work problem 4.31. This demonstrates another odd property. Computing the sample mean yields a Cauchy R.V. with the same distribution as a single sample. The averaging does *not* reduce sample mean estimation error variance, i.e. the sample mean does not converge to the mean (which does not exist anyway) as N gets large.

Homework set #14 Due Dec. 10

1. Problem 4.34

5. Problem 4.39 (a) through (d) (not (e)). For part c) calculate the characteristic function for Z_n, sample it, then use the inverse FFT.

6. Problem 4.40

7. Problem 4.45