NAME: ________________________________

EC En 370

Midterm

Fall Semester

November 2 - 6, 2006

Open Text Book and Notes.
Time limit: 2.5 hours

Instructor: Brian D. Jeffs

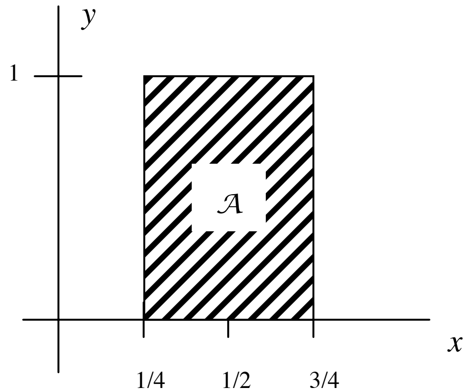
1. Enter all answers on the scantron "bubble" answer sheet.
2. Assume answers given are accurate to <u>two significant digits</u>.
3. Each problem is worth five points.
4. Attach all your calculation worksheets to this test and return them to the testing center. Any requests of credit on incorrectly marked answers must be supported by a correct numerical answer on your calculation worksheets.
5. If you discover a mistake in the test text, make any assumptions needed to correct the problem statement, and note these in writing on this exam booklet.

1. Let X and Y be jointly distributed as

$$f_{XY}(x,y) = \frac{4}{9}(x + xy + y + 1)u(x)u(1-x)u(y)u(1-y)$$

$$= \begin{cases} \frac{4}{9}(x + xy + y + 1) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases},$$

where $u(x)$ is the unit step function.
Find $P[(X,Y) \in \mathcal{A}]$ for the region as seen in the figure:



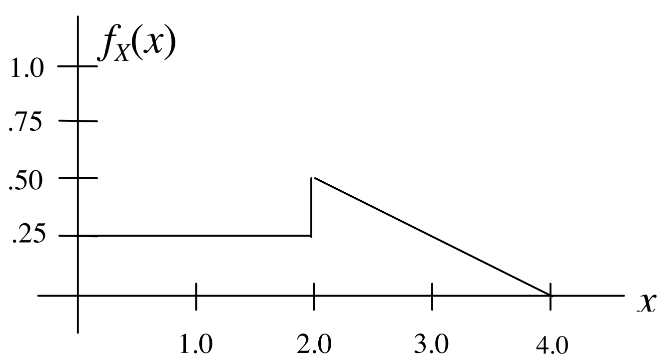
a) 0.0; b) 0.1; c) 0.333; d) 0.444; e) 0.5; f) 0.666; g) 0.707 h) 0.75; i) 1.0.

2. Using $f_{XY}(x,y)$ from the previous problem (one), find the marginal density $f_X(x)$.
a) $(4/9)x^2u(x)u(1\text{-}x)$; b) $(2/3)(x + 1)u(x)u(1\text{-}x)$; c) $2x\, u(x)u(1\text{-}x)$; d) $(1/2 + x)u(x)u(1\text{-}x)$;
e) $(5/3)x + (1/2)x^2\, u(x)u(1\text{-}x)$; f) $(2/3)x + (1/3)x^2\, u(x)u(1\text{-}x)$; g) $(x + y)\, u(x)u(1\text{-}x)$.

3. Are X and Y from problem one independent and identically distributed?
a) neither; b) both; b) independent but not identically distributed;
c) not independent but identically distributed; d) can't tell from given information.

4. Assume the average score on this exam will be 75, and that the distribution of scores will be Normal, with variance $\sigma^2 = 200$. What is the probability a randomly selected student will score 90 or better? (I'm sure your personal odds will be even higher)
a) 0.213; b) 0.123; c) 0.353; d) 0.673; e) 1.0; f) 0.147; g) 0.796
h) none of these.

5. Consider the probability density function (pdf) plotted below for a random variable X. Calculate $F_X(3.0)$.



a) 0.875; b) 0.225; c) 0.333; d) 0.625; e) 0.5; f) 0.396; g) 0.75
h) none of these.

6. The government often makes studies of peak traffic loads on streets to determine if improvements are required (e.g. adding lanes, traffic lights, etc.) You have seen their meters on the road; one or two black tubes about ½ inch in diameter running out across the lane from a counter box on the road side. Assume that for a given road the *average* measured rate of cars passing the counter is 50 cars per minute during rush hours (6:30 - 9:30 a.m. and 4:30 - 6:30 p.m.) Assume there are no stop lights near the counter to artificially create bursts of traffic. Let n be the number of cars that pass the counter in a given 10 second interval during rush hours. What probability distribution would be a good model for n?
a) binomial; b) Gaussian; c) multinomial; d) Chi square; e) Laplace;
f) Rayleigh; g) uniform; h) Poisson.

7. For the scenario described in problem six, assume it is safe to cross the road in a crosswalk if fewer than three cars arrive during the 10 seconds it takes to walk across. Calculate the probability that a pedestrian entering the crosswalk during any 10 second window of the rush hour will be safe.
a) 0.0345; b) 0.225; c) 0.0106; d) 0.634; e) 0.005; f) 0.198; g) 0.474;
h) none of these.

8. The Marriott center uses expensive mercury vapor gas lights to illuminate the playing court. Some years ago the custodians replaced all existing lights. They found a very cheap supplier where they got 2/3 of the needed bulbs from brand X. These came with no warranty and have a failure probability per month, F, of $P[F \mid X] = 0.05$. The rest of the bulbs were purchased from brand Y at greater expense, but they come with a warranty and have a monthly failure probability of $P[F \mid Y] = 0.01$. Assume there are 30 bulbs total for the court. What is the probability that at least one bulb will fail during the first month after installation?
a) 1.0; b) 0.980; c) 0.3; d) 0.874; e) 0.4; f) 0.676; g) 0.104; h) none of these.

9. Use the scenario of question eight. Assume bulbs were randomly selected from brands X and Y when they were installed, but in the proportion stated. Consider one specific bulb (e.g. the bulb in the extreme North-East corner.) What is the probability it will both fail in the first month and be under warrantee?
a) 1.0; b) 0.033; c) 0.578 d) 0.01; e) 0.05; f) 0.0005; g) 0.0033; h) none of these.

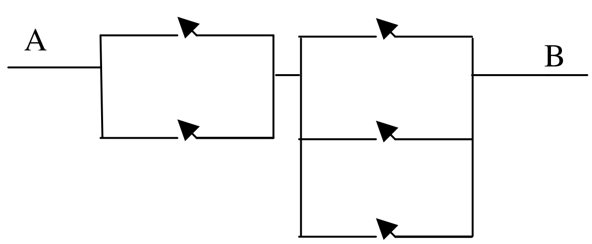
10. Consider a (fictitious) test for cancer with the following properties:
A = Event that the test indicates a person has cancer. B = Event that the person has cancer.
It is known that $P[A|B] = P[A^c|B^c] = 0.96$ and $P[B] = 0.003$ in the general population.
Find $P[B|A^c]$ (the probability a person who the test says <u>does not</u> have cancer actually <u>does</u> have cancer).
a) 0.0674; b) 0.0033; c) 0.578 d) 1.25×10^{-4}; e) 0.05; f) 0.0029; g) 7.34×10^{-4};
h) none of these.

11. Consider a roll of a 8 sided die, with faces numbered 1 through 8. Let ζ be the outcome of the experiment (i.e. ζ is the number rolled). Define two events:
$E = \{ \zeta \mid \zeta = 5\} = \{5\}$ and $F = \{ \zeta \mid \zeta \text{ even}\} = \{2, 4, 6, 8\}$.
Which event described below is not contained in the minimum sized (i.e. with fewest elements) sigma field, $\mathcal{F}$, on Ω which contains E and F.
a) $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$; b) $\phi = \{ \}$; c) $\{1, 3, 5, 7\}$; d) $\{1, 3, 7\}$;
e) $\{2, 4, 5, 6, 8\}$; f) $\{2, 8\}$; f) cannot be determined from the given information.
g) none, because all possible events on Ω are in $\mathcal{F}$ since it is the power set.

12. Consider a loaded three sided die with probabilities for rolling a 1, 2, or 3 given by $p_1 = 1/2$, $p_2 = 1/3$, $p_3 = 1/6$ respectively. Find the probability of rolling a 1 and two threes (e.g. {3,1, 3}) in any order, in three rolls.
a) 1/24; b) 1/72; c) 1/3 d) 1/18; e) 1/2; f) 2/9; g) 1/12; h) none of these.

13. A card is selected at random from a standard deck of 52 cards. Let A be the event of selecting a *face* card (jack, queen, or king) and let B be the event of selecting a *numbered* card (1, or ace, through 10). There are 12 face cards and 40 number cards in the standard deck. Then:
a) A and B are disjoint events; b) A and B are not defined on the same probability space;
c) A and B are independent; d) A and B are dependent events; e) A and B form an exhaustive non-overlapping partition of Ω; f) $P[A \mid B] = P[A]$; g) (a) (d) and (e);
h) (a) and (c).

14. In the circuit shown below, the probability that a given switch (shown as an arrow) is closed is 0.7. All switches operate independently. The probability that there is <u>no</u> closed circuit between A and B is:



a) 0.0345; b) 0.225; c) 0.0106; d) 0.634; e) 0.005; f) 0.198; g) 0.474;
h) none of these.

15. Let $X \sim N(-0.5, 3)$. Then $P[\,|X|^3 < 1]$ is
a) 0.19; b) 0.37; c) 0.251; d) 0.73; d) 0.45; e) 0.04.

16. Assume that in a disaster 7 separate messages are sent by radio and TV to city residents to warn them to evacuate. The probability that a resident receives any one of these messages is 0.4. Since residents are stubborn, it takes at least two received warnings to convince them to

evacuate. What is the probability that a resident will evacuate the city?

a) 0.901; b) 0.663; c) 0.580; d) 0.352; e) 0.169 f) 0.276; g) none of these.

17. Let X have a PDF $F_X(x) = \frac{1}{2}(1 - \cos(x))u(x)u(\pi - x) + u(x - \pi)$. Define event $B = \left\{\varsigma : \frac{\pi}{4} \le X(\varsigma) \le \pi\right\}$. Then the conditional PDF $F_{X|B}(x|B)$ evaluated at $x = \pi/2$ has value:

a) 0.25; b) 0.39; c) 0.61; d) 0.5; e) 0.414; f) 0.723; g) none of these.

18. The correct value of c so that $f(x) = c\exp(-3x^2)u(x)$ is a valid pdf is:

a) 0.125; b) 1.95; c) 1.5; d) 8.0 e) 16; f) 6.28; g) 3.414 h) 3.99.

19. An urn contains 10 balls numbered 1 through 10. Four balls are removed, *one at a time*, their number recorded and then *returned* to the urn. This is called sampling with replacement since each ball is returned before the next ball is removed. The probability of observing the sequence 10, 9, 8, 7, in that order is:

a) 2.0×10^{-4}; b) 1/10!; c) 1×10^{-4} d) 1/4; e) 1/27; f) 1.0×10^{-3}; g) none of these.

20. The conditions for this problem are the same as the previous problem except that once the ball is selected it is not returned to the urn. Then the probability of the sequence 10, 9, 8, 7 is:

a) 2.0×10^{-4}; b) 1/10!; c) 1×10^{-4} d) 1/4; e) 1/27; f) 1.0×10^{-3}; g) none of these.