NAME: ________________________________

EC En 370

Midterm

Fall Semester

October 26 - 31, 2007

Open Text Book and Notes. Time limit: 2.5 hours

Instructor: Brian D. Jeffs

1. Enter all answers on the scantron "bubble" answer sheet.
2. Assume answers given are accurate to <u>two significant digits</u>.
3. Each problem is worth five points.
4. Attach all your calculation worksheets to this test and return them to the testing center. Any requests of credit on incorrectly marked answers must be supported by a correct numerical answer on your calculation worksheets.
5. If you discover a mistake in the test text, make any assumptions needed to correct the problem statement, and note these in writing on this exam booklet.

1. Let $X = k$ be a binomial random variable with $b(k, 4, 1/3)$. Then $P[X = 3 \mid X \text{ is odd}] = ?$
(Round to one significant digit, and enter.)

2. A biased coin is flipped five times. The probability of getting a head is 1/4. The probability of *getting at least one head in the five flips* is:
(Enter the most significant digit without rounding.)

3. Photons from a dim nebula in space strike the CCD imaging array of a telescope at the average rate of 40,000 photons/second. What is the approximate probability that between 2 and 4 photons, inclusive, strike the CCD array during a 100 μs exposure time window?
a) 0.537; b) 0.764; c) 0.0; d) 1.0; e) 0.659; f) 0.291; g) 0.368.

4. Consider the function $f_X(x) = (\alpha e^{-x} + \beta e^{-3x})u(x)$. The only combination of constants in the list that would make this a legitimate pdf is:
a) $\alpha = 0.5, \beta = 0.5$; b) $\alpha = 3/4, \beta = 1/4$; c) $\alpha = 1.5, \beta = 0.5$;
d) $\alpha = 2, \beta = 5$; e) $\alpha = 1+j, \beta = -2j$; f) $\alpha = 0.5, \beta = 1.5$.

5. Let $X \sim N(\mu = 1, \sigma^2 = 4)$. Then $P[|X|>1] = ?$
a) 0.537; b) 0.764; c) 0.0; d) 1.0; e) 0.659; f) 0.291; g) 0.383.

6. A PDF is given by $F_X(x) = (1 - e^{-x^2})u(x)$. Then $P[-0.5 < X < 1.0] = ?$
(Calculate the answer to 3 significant digits. Enter the <u>second</u> most significant.)

7. An urn contains three balls of different colors: red, green, and blue. Three balls are removed, one at the time, their colors recorded, and returned to the urn. This is called sampling with replacement since each ball is returned before the next ball is removed. The probability of observing the sequence {blue, green, red}, in that order, is:
a) 2!/3! ; b) 1/3! ; c) 1/9; d) 1/3; e) 1/27; f) 1; g) 0; h) 3!; i) $\binom{3}{2}$.

8. The conditions for this problem are the same as the previous problem, except that once the ball is selected it is not returned to the urn. Then the probability of the sequence {blue, green, red}, in that order, is:
a) 2!/3! ; b) 1/3! ; c) 1/9; d) 1/3; e) 1/27; f) 1; g) 0; h) 3!; i) $\binom{3}{2}$.

9. A PDF is given by $F_X(x) = (1 - e^{-x})u(x)$. Then the conditional PDF $F_{X|B}(x|B)$ for $x = 3$ and $B = \{X > 2\}$ is: (do not round, and enter the most significant digit)

10. Pinochle is a card game that uses a modified 48 card deck containing exactly 8 of each of the following cards: {ace, 9, 10, jack, queen, king}. There are no other cards in the deck. A card is selected at random from a full Pinochle deck. Let *A* be the event of selecting any *face* card {jack, queen, or king} and let *B* be the event of selecting a "*counter*" card {ace, 10, king}. Only *counter* cards win points while playing Pinochle. Which of the following is true?

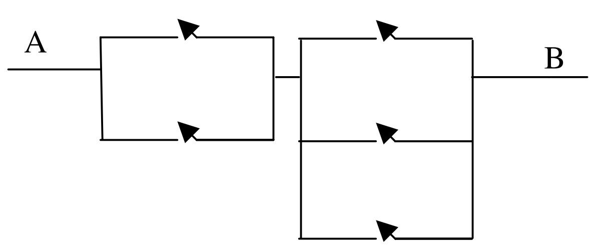
a) A and B are disjoint events; b) A and B are not defined on the same probability space; c) A and B are independent; d) A and B are dependent events; e) A and B form an exhaustive non-overlapping partition of Ω; f) $P[A \mid B] = P[A]$; g) (a) (d) and (e); h) (a) and (c).

11. As in the previous problem, consider drawing a single card from the deck. The possible outcomes are Ω = {ace, 9, 10, jack, queen, king}, and events A and B are as defined in the previous problem. Which event described below is not contained in the minimum sized (i.e. with fewest elements) sigma field, $\mathcal{F}$, on Ω which contains A and B.
   a) Ω; b) ϕ = { }; c) {king}; d) {jack};
   e) {9}; f) {jack, queen}; f) cannot be determined from the given information.
   g) none, because all possible events on Ω are in $\mathcal{F}$ since it is the power set.

12. Consider a (fictitious) test for cancer with the following properties:
   A = Event that the test indicates a person has cancer. B = Event that the person has cancer.
   It is known that $P[A|B] = P[A^c|B^c] = 0.94$ and $P[B] = 0.005$ in the general population.
   Find $P[B^c|A^c]$ (the probability a person who the test says does not have cancer actually does not).
   a) 0.0674; b) 0.0033; c) 0.578 d) 1.25×10^{-4}; e) 0.05; f) 0.0029; g) 7.34×10^{-4};
   h) none of these.

13. The average height for adult males in Tinyvania is 50.0 inches. Tinyvania is the shortest nation in the world. Assume that height is distributed Normal in this population with a standard deviation of 5.0 inches. Dr. Jeffs' cousin Dave is 60 inches tall, and travels to Tinyvania for a business meeting with two randomly selected male citizens of the country. What is the probability that both will be shorter than Dave?
   (Calculate to 3 significant digits and enter the second most significant).

14. In the circuit shown below, the probability that a given switch (shown as an arrow) is closed is 0.3. All switches operate independently. The probability that there is a closed circuit between A and B is:

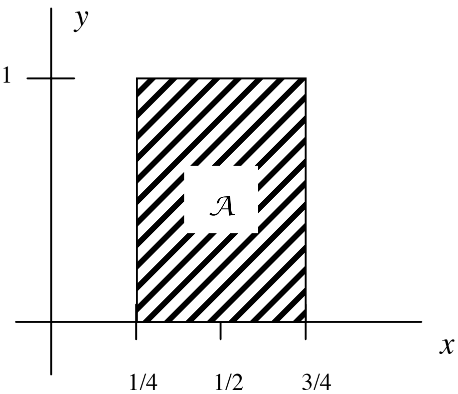


   a) 0.335; b) 0.214; c) 0.0266; d) 0.675; e) 0.04; f) 0.198; g) 0.564;
   h) none of these.

15. Let X and Y be jointly distributed as

$$f_{XY}(x,y) = \begin{cases} x+y & 0 \le x, y \le 1 \\ 0 & \text{otherwise} \end{cases},$$

Find $P[(X,Y) \in \mathcal{A}]$ for the region as seen in the figure:



a) 0.0; b) 0.1; c) 0.333; d) 0.444; e) 0.5; f) 0.666; g) 0.707 h) 0.75; i) 1.0.

16. Considering $f_{XY}(x,y)$ from the previous problem, which of the following is true?
a) X and Y are independent; b) $f_{X|Y}(x \mid y) = f_X(x)$; c) $f_X(x) = [(x^2 + x)/2]\ u(x)u(1\text{-}x)$;
d) $f_X(x) = (2/3)(x + 1)u(x)u(1\text{-}x)$; f) $f_X(x) = 2x\ u(x)u(1\text{-}x)$; g) none of the above.

17. During his late night show, David Letterman drops watermelons from the top of his building onto the street below. He is attempting to hit the origin of an x - y axis marked on the street. Let X and Y be the random coordinates on the axis where a melon falls. Assume X and Y are distributed jointly Gaussian, zero mean, with $\rho = 0$ and $\sigma_X = \sigma_Y = 1$ meter. Calculate the probability that the mellon will fall within a 1m radius circle centered on the x - y axis origin. (without rounding, enter the most significant digit.)

18. Three missiles are fired at a ship. It takes at least two missile hits to sink the ship. Given that the hit probability of a single missile is 0.3, the probability of the ship staying afloat is: (Calculate to 3 significant figures and enter the second-most significant.)

19. A fair six sided die is thrown six times. The probability of observing each of the six faces come up exactly once is:
a) 0.0154; b) 0.00154; c) 0.0000234; d) 0.167; e) 0.321.

20. A warehouse stocks memory chips made by two different companies, call them A, and B. If made from A, the probability that the chip is bad is 0.1. If made from B, the probability that the chip is bad is 0.2. Company A produces twice as many chips as company B. The probability that a purchased chip is bad is:

a) 0.333; b) 0.667; c) 0.133; d) 0.0667; e) 0.0333.