1. First, confirm $f_{XY}(x,y)$ is a valid pdf:

$$\int_0^1\int_0^1 f_{X,Y}(x,y)\,dy\,dx = \frac{4}{9}\left(\frac{x^2y}{2} + \frac{x^2y^2}{4} + \frac{xy^2}{2} + xy\right)\Big|_{y=0}^{1}\Big|_{x=0}^{1}$$

$$= \frac{4}{9}\left(\frac{x}{2} + \frac{x^2}{4} + \frac{x^2}{2} + x\right)\Big|_{x=0}^{1} = \frac{4}{9}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{2}+1\right) = \frac{4}{9}\left(\frac{9}{4}\right) = 1$$

$$P[(X,Y)\in A] = \frac{4}{9}\left(\frac{3}{2}x + \frac{3}{4}x^2\right)\Big|_{x=1/4}^{3/4}$$

$$= 4/9\left(\frac{3}{2}\,\frac{(3-1)}{4} + \frac{3}{4}\,\frac{(9-1)}{16}\right) = \frac{4}{9}\left(\frac{3}{4}+\frac{3}{8}\right) = \frac{1}{2}$$ (e)

2. $$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_0^1 f_{XY}(x,y)\,dy$$

$$= \frac{4}{9}\left(xy + \frac{xy^2}{2} + \frac{y^2}{2} + y\right)\Big|_{y=0}^{1} u(x)u(1-x)$$

$$= \frac{4}{9}\left(x + \frac{x}{2} + \frac{1}{2} + 1\right)u(x)u(1-x)$$

$$= \frac{2}{3}(x+1)\,u(x)\,u(1-x)$$ (b)

3. $$f_{XY}(x,y) = \frac{4}{9}(x+1)(y+1)\,u(x)u(1-x)u(y)u(1-y)$$

$$= \left[\frac{2}{3}(x+1)u(x)u(1-x)\right]\left[\frac{2}{3}(y+1)u(y)u(1-y)\right]$$

yes, i.i.d. since it can be factored into two equal functions (b)

4. $X \sim N(75, 200)$ $\quad P[X \geq 90] = \frac{1}{2} - \text{erf}\left(\frac{x-75}{\sqrt{200}}\right)$

$$= \frac{1}{2} - \text{erf}(1.061) \approx 0.147$$ (f)

5. $$F_X(x) = \int_0^x f_X(x)\,dx \qquad \begin{cases} \frac{1}{4}x & 0 \leq x \leq 2 \\ \frac{1}{2} + \frac{(x-2)}{2} - \frac{(x-2)^2}{8} & 2 < x \leq 4 \end{cases}$$

or, $F_X(3) = \frac{1}{2} + 1\left(\frac{1}{2}+\frac{1}{4}\right)/2 = 7/8 = 0.875$ (a)

6. Poisson, (h)

7. $\tau = \frac{1}{6}$ min, $\lambda = 50$ cars / min.

$$P[n<3] = e^{-\frac{50}{6}}\left(\frac{(50/6)^0}{0!} + \frac{(50/6)^1}{1!} + \frac{(50/6)^2}{2!}\right)$$

$$= 0.0106$$ (c)

8. All bulbs fail indepenbutly. Let F_i be the event bulb i fails.

$F_T = \bigcup_{i=1}^{30} F_i$. $P[F_T] = 1 - \underbrace{\prod_{i=1}^{30}(1-P[F_i])}_{\text{Probability of no failures}}$

$$P[F_T] = 1 - \left(\prod_{i=1}^{20}(1-P[F_i \mid X])\right)\left(\prod_{i=21}^{30}(1-P[F_j \mid Y])\right)$$

$$= 1-(.95)^{20}(.99)^{10} = 0.676$$ (f)

9. $P[F, Y] = P[F \mid Y]\ P[Y]$

$= (0.01)\left(\frac{10}{30}\right) = 0.0033$ (g)

10. $$P[B \mid A^c] = \frac{P[A^c \mid B]\ P[B]}{P[A^c \mid B]P[B] + P[A^c \mid B^c]P[B^c]}$$

$$= \frac{(1-.96)(0.003)}{(1-.96)(0.003)+(0.96)(1-.003)} = 1.25\times10^{-4}$$ (d)

11. a) + b) $\in \mathcal{F}$ by denition. c) $= F^c \in \mathcal{F}$

d) $= E^cF^c \in \mathcal{F}$, e) $= E \cup F \in \mathcal{F}$

f) - Cannot be formed by unions, intersections or complements of E, F, ϕ, Ω. (f)

12. $p([1,0,2], 3, [\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{6}])$ (Multinomial)

$$= \frac{3!}{(1!)(0!)(2!)} (\tfrac{1}{2})^1 (\tfrac{1}{3})^0 (\tfrac{1}{6})^2$$ a

$$= \frac{6}{2}\left(\frac{1}{2}\right)\left(\frac{1}{36}\right) = \frac{1}{24}$$

13. These are disjoint, dependent events that fully partition Ω. g

14. Let C be event of closed circuit path from A to B.

$$P[C^c] = 1 - (1-(1-0.7)^2)(1-(1-0.7)^3)$$ h

$$= 0.115$$

Note: Let E, F, & G be the events that each of 3 parallel switches are closed. The probability of a closed circuit (on any branch) is $1 - P[E^c F^c G^c] = 1 - (1-P[E])(1-P[F])(1-P[G])$ for independent events.

15. $P[|X|^3 < 1] = P[$ Denominator should be sqrt(3). Final answer is 0.42

$= P[-1 < X < 0] +$ ... $]$ b

$= -\mathrm{erf}\left(\frac{-1-(-.5)}{\cancel{2}}\right)$ ... $5)\Big)$

$= \mathrm{erf}(\cancel{0.25}) + \mathrm{erf}(\cancel{0.75})$

$= \cancel{0.09871 + .27337 = 0.372}$

16. $P[E] = 1 - P[E^c] = 1 - (P[A_0] + P[A_1])$

where A_k is the event k messages were received.

$P[E] = 1 - \binom{7}{0}(.4)^0(.6)^7 - \binom{7}{1}(.4)^1(.6)^6$

$= 1 - (.6)^7 - 7(.4)(.6)^6$

$= 0.84$

(g)

17. $$F_{X|B}(x|B)\Big|_{x=\frac{\pi}{2}} = \frac{P[X \le x;\ \frac{\pi}{4} \le X \le \pi]\Big|_{x=\frac{\pi}{2}}}{P[\frac{\pi}{4} \le X \le \pi]}$$

$$= \frac{P[\frac{\pi}{4} \le X \le \pi/2]}{P[\frac{\pi}{4} \le X \le \pi]}$$

$$= \frac{F_X(\pi/2) - F_X(\pi/4)}{F_X(\pi) - F_X(\pi/4)} = \frac{\frac{1}{2} - \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right)}{1 - \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right)}$$

(e)

$$= \frac{\sqrt{2}/4}{\frac{1}{2} + \frac{\sqrt{2}}{4}} = \frac{\sqrt{2}}{2+\sqrt{2}} = \frac{1}{\sqrt{2}+1} = 0.414$$

18. This is a one-sided Gaussian, so:

$\frac{1}{2\sigma^2} = 3$, $\sigma^2 = \frac{1}{6}$, $f_X(x) = \frac{2}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} u(x)$

so $c = \frac{2}{\sqrt{2\pi\sigma^2}} = \frac{2}{\sqrt{2\pi/6}} = 1.95$

(b)

19. $P[\{10, 9, 8, 7\}] = \left(\frac{1}{10}\right)^4 = 10^{-4}$

(c)

20. $P[\{10, 9, 8, 7\}] = \left(\frac{1}{10}\right)\left(\frac{1}{9}\right)\left(\frac{1}{8}\right)\left(\frac{1}{7}\right) = \frac{(10-4)!}{10!}$

(a)

See text p.25

$= 1.98 \times 10^{-4}$