1. $P[\{\text{at least 1 head}\}] = 1 - P[\{\text{no heads}\}]$

$= 1 - \binom{5}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5 = 0.868$ a)

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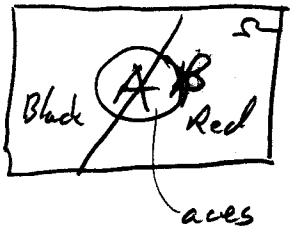
2. 

$P[A] = \frac{4}{52} = 1/13$

$P[B] = \frac{26}{52} = 1/2$

$P[AB] = \frac{2}{52} = 1/26$

$P[AB] = P[A]P[B] = \frac{1}{13}\cdot\frac{1}{2} = \frac{1}{26}$ c)

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3. $P[3 \le X \le 4] = \sum_{k=3}^{4} e^{-\lambda\tau}\frac{(\lambda\tau)^k}{k!}, \quad \lambda\tau = 4$

$= e^{-4}\sum_{k=3}^{4}\frac{4^k}{k!} = (0.0183)\left(\frac{4^3}{3!} + \frac{4^4}{4!}\right)$

$= 0.3907$ f)

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4. $E \cup F^c = \{3, 5, 6\}$. This (and many other events in the sigma-field) does not appear in a-d) f)

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5. $P[|X| > 2] = 2\int_2^\infty 0.5\, e^{-x}\, dx$

$= -e^{-x}\Big|_2^\infty = e^{-2} = 0.135$ b)

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6. $Z = \left(\frac{X}{\sqrt{1/2}}\right) = \sqrt{2}X. \quad X = 1 \rightarrow Z = \sqrt{2}$

$P[|X| > 1] = 1 - 2\,\text{erf}(\sqrt{2})$

$= 1 - 2(.4192)$

$= 0.162$ c)

---

7. $Z = \left(\frac{X+2}{2}\right)$, $X_1 = -3 \rightarrow Z_1 = -.5$ $X_2 = -1 \rightarrow Z_2 = .5$

$P[-3 < X < -1] = 2\,erf(.5^-)$

$= 2(.1915) \approx 0.383$ b)

---

8. $P[B/A] = \frac{P[A/B]\,P[B]}{P[A/B]P[B] + P[A/B^c]P[B^c]}$

$= \frac{(0.92)(0.01)}{(0.92)(0.01) + (1-0.92)(1-0.01)}$

$= 0.104$ a)

---

9. $(1/3)^3 = \frac{1}{27}$ e)

---

10. There are 3! possible sequences. $P[2,3,1] = \frac{1}{3!} = \frac{1}{6}$ a)

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11. $k\int_0^\infty x e^{-4x^2} dx = 1$, $k\left[\frac{-1}{8} e^{-4x^2}\Big|_0^\infty\right] = 1$

$k = 8/(1-0) = 8$ d)

---

12. $Z = \left(\frac{X-4.35}{0.59}\right)$, $X_1 = 4 \rightarrow Z_1 = -0.593$,

$X_2 = 5 \rightarrow Z_2 = 1.102$,

$P[4 < X < 5] = erf(1.102) + erf(.593) = 0.59$ c)

13. $P[B] = 1 - F(2) = 1 - [1 - e^{-1}] = 0.368$

$$F_{X|B}(x|B)\Big|_{x=3} = \frac{P[X \le x, B]}{P[B]}\Big|_{x=3} = \frac{F_X(3) - F_X(2)}{0.368}$$

$$= \frac{(1 - e^{-3/2}) - (1 - e^{-1})}{0.368} = 0.393 \quad b)$$

---

14. $E = \{$Event a closed path $A \to B$ exists$\}$

$C = \{$Event left switch is closed$\}$

$D = \{$Event top switch is closed$\}$

$F = \{$Event bottom switch is closed$\}$

$E = C(D \cup F)$. $P[E] = P[C]\,P[D \cup F]$

$P[E] = P[C](P[D] + P[F] - P[D]P[F])$

$= .9(.9 + .9 - .9^2)$

$= 0.891$ e)

---

15. $P[X < 0.5^-, Y < 0.25^-] = \int_0^{y=0.25^-}\int_0^{x=0.5^-} 6e^{-2x-3y}\,dx\,dy$

$= (e^{-3y})\Big|_0^{.25}(e^{-2x})\Big|_0^{.5} =$

$= (e^{-3/4} - 1)(e^{-1} - 1) = 0.334$ a)

---

16. $f_Y(y) = \int_{-\infty}^{\infty} (x+y)\,u(x)\,u(1-x)\,u(y)\,u(1-y)\,dx$

$= \int_0^1 (x+y)\,u(y)\,u(1-y)\,dx$ d)

$= (\frac{x^2}{2} + xy)u(y)u(1-y)\Big|_0^1 = (\frac{1}{2} + y)u(y)u(1-y)$

17. $P[\text{not black}] = 1 - P[x=0 \cup x=1 \cup x=2]$

$= 1 - P[k=0; 72/24] - P[k=1; 72/24] - P[k=2; 72/24]$

$= 1 - \frac{3^0}{0!}e^{-3} - \frac{3}{1!}e^{-3} + \frac{3^2}{2!}e^{-3}$

$= 0.577$ 6)

---

18. $P[\text{sink}] = b(2; 3, .4) + b(3; 3, .4)$

$= \binom{3}{2}(.4)^2(.6)^1 + \binom{3}{3}(.4)^3(.6)^0$

$= (3)(.16)(.6) + (1)(.064)(1) = 0.352$ d)

---

19. $P[\text{Interrupt in} \le 2 \text{ days}] = P[C \cup D] = P[C]P[D]$

$C = \{\text{Event top fuse fails in} \le 2 \text{ days}\}$

$D = \{\text{Event bottom fuse fails in} \le 2 \text{ days}\}$

$P[C] = P[D] = \int_0^2 f_T(t)\,dt = -e^{-t/2}\Big|_0^2 = 1 - e^{-1}$

$P[\text{Interrupt in} \le 2 \text{ days}] = (1-e^{-1})^2 = 0.399$ a)

---

20. $\int_0^{y=1}\int_0^{x=2}(ax+by)\,dy\,dx = 1$

$\left(\frac{ax^2y}{2} + \frac{by^2x}{2}\right)\Big|_{y=0}^{1}\Big|_{x=0}^{2} = 1$

$\left(\frac{ax^2}{2} + \frac{bx}{2}\right)\Big|_{x=0}^{2} = 1$

$\left(\frac{a(4)}{2} + \frac{b(2)}{2}\right) = 1, \quad 2a + b = 1$

6)