

Quiz 7

ECEn 370

Name: _____

1. Let X and Y be jointly distributed as

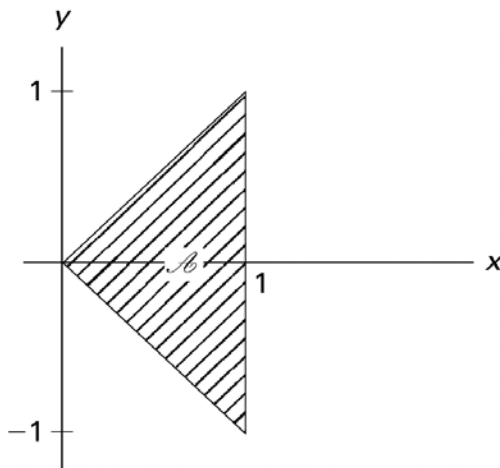
$$f_{XY}(x,y) = (x^2y)u(x)u(1-x)u(y)u(1-y) = \begin{cases} x^2y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases},$$

where $u(x)$ is the unit step function.

- a) Are X and Y independent? Why?

Yes, $f_{XY}(x,y)$ can be factored so $f_{XY}(x,y) = f_X(x)f_Y(y)$.

- b) Find $P[(X,Y) \in \mathcal{A}]$ as seen in the figure below



The region \mathcal{A}

$$\begin{aligned} P[(X,Y) \in \mathcal{A}] &= \int_{x=0}^{x=1} \int_{y=0}^{y=x} x^2y \, dydx = \int_{x=0}^{x=1} x^2 \int_{y=0}^{y=x} y \, dydx = \int_{x=0}^{x=1} x^2 \left(\frac{y^2}{2} \right) \Big|_0^x dx \\ &= \int_{x=0}^{x=1} x^2 \left(\frac{x^2}{2} \right) dx = \frac{x^5}{10} \Big|_0^1 = \frac{1}{10} \end{aligned}$$