

Quiz 8

ECEn 370

Name: _____KEY_____

1. 1. Let X be distributed uniform, i.e. $X \sim f_X(x) = 2(x-2)u(x-2)u(3-x)$.
 Let $Y = \ln(X)$. Use the general formula for transformations of one random variable shown below to find $f_Y(y)$.

$$\text{hints: } x = e^{\ln x} \rightarrow \frac{d}{dx} x = \frac{d}{dx} e^{\ln x} \rightarrow 1 = (e^{\ln x}) \left(\frac{d}{dx} \ln x \right) \quad f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|} + \dots + \frac{f_X(x_n)}{|g'(x_n)|}$$

$$\text{roots of } y = g(x): \quad y - \ln(x) = 0 \quad \text{for } 1 \leq x \leq 2$$

$$y = \ln(x) \rightarrow \text{a single root over this range.}$$

$$x_1 = e^y$$

$$g'(x_1) = \left. \frac{d}{dx} \ln(x) \right|_{x=x_1} = \frac{1}{x_1}$$

$$f_Y(y) = \frac{2(e^y - 2)u(e^y - 2)u(3 - e^y)}{\left| \frac{1}{e^y} \right|}$$

$$= \begin{cases} 2e^y(e^y - 2) & \ln(2) \leq y \leq \ln(3) \\ 0 & \text{otherwise} \end{cases}$$

$$\text{note: } \int_{\ln 2}^{\ln 3} 2e^y(e^y - 2)dy = e^{2y} - 4e^y \Big|_{\ln 2}^{\ln 3} = (3^2 - (4)(3)) - (2^2 - (4)(2)) = 1.$$