

Example Problem

An air core parallel plate waveguide has a thickness of $d = 40\text{mm}$ and an operating frequency of $f = 8\text{GHz}$.

(A) What are the first 4 TE mode cut off frequencies?

$$k_y = \frac{m\pi}{d} \quad k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2}$$

$$\frac{\omega}{c} \geq \frac{m\pi}{d}$$

$$\frac{2\pi f}{c} \geq \frac{m\pi}{d}$$

$$f \geq \frac{mc}{2d}$$

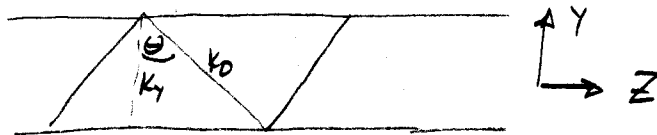
$$f_c = 3.75, 7.50, 11.25, 15 \text{ GHz}$$

(B) What is the frequency range for single mode operation?

$$3.75 \text{ GHz} < f < 7.5 \text{ GHz}$$

(C) What are the effective propagation angles for the various modes at the operating frequency? (8GHz)

There are 2 modes



$$\cos\theta = \frac{k_y}{k_0} = \left(\frac{m\pi/d}{\omega/c}\right) = \frac{m\pi c}{2\pi f d} = \frac{mc}{2fd}$$

$$\theta = \cos^{-1}\left(\frac{mc}{2fd}\right) = 62.04^\circ, 20.36^\circ$$

(D) What is the group velocity?

$$v_g = \left(\frac{dk_z}{d\omega}\right)^{-1} \quad \frac{dk_z}{d\omega} = \frac{d}{d\omega} \left\{ \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2} \right\}$$

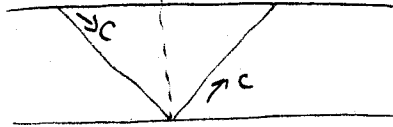
$$= \frac{1}{2} \left(\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2 \right)^{-1/2} \frac{2\omega}{c^2}$$

$$= \frac{\omega}{c^2 \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2}}$$

$$v_g = \frac{c^2}{\omega \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2}}$$

$$= 2.65 \times 10^8, 1.04 \times 10^8 \text{ m/s}$$

Can also use propagation angle



The ray speed is c

The mode speed is $c \sin \theta$
 $= 3 \times 10^8 \sin(62.09^\circ)$
 $3 \times 10^8 \sin(20.36^\circ)$

(E) A long pulse is transmitted down the waveguide. How far can the pulse travel before it becomes twice as long?

The travel time is $T = \frac{L}{v_g}$

The difference in travel time is

$$\Delta T = \frac{L}{v_{g2}} - \frac{L}{v_{g1}}$$
$$= L \left(\frac{1}{v_{g2}} - \frac{1}{v_{g1}} \right)$$

$$L = \frac{\Delta T}{\left(\frac{1}{v_{g2}} - \frac{1}{v_{g1}} \right)} = \frac{10 \times 10^{-7}}{\frac{1}{1.044 \times 10^8} - \frac{1}{2.65 \times 10^8}}$$

$$L \leq 1.7 \text{ m}$$