

## ECEn 555, Fall 2009

### Homework #6

Due November 23, 5:00 pm

**6.1** Calculate the threshold small signal gain coefficient for ruby given the following data: threshold population inversion  $5 \times 10^{22} \text{ m}^{-3}$ , refractive index 1.5, linewidth  $2 \times 10^{11} \text{ Hz}$ , Einstein A coefficient  $300 \text{ s}^{-1}$  and wavelength 694.3 nm.

**6.2** Calculate the threshold pumping power for an Nd:glass laser, given that the critical population inversion is  $9 \times 10^{21} \text{ m}^{-3}$ , the spontaneous lifetime is  $300 \mu\text{s}$  and that the upper laser level is at an energy of 1.4 eV.

**6.3** Estimate the efficiency of a GaAs laser operating well above threshold, given that  $n=3.6$  and that the length of the laser cavity is  $200 \mu\text{m}$ . Take the loss coefficient to be  $800 \text{ m}^{-1}$  and the internal quantum efficiency to be 0.8.

**6.4 Einstein coefficients and critical photon concentration.**  $\rho(h\nu)$  is the energy of the electromagnetic radiation per unit volume per unit frequency due to photons with energy  $h\nu = E_2 - E_1$ . Suppose that there are  $n_{ph}$  photons per unit volume. Each has an energy  $h\nu$ . The frequency range of emission is  $\Delta\nu$ . Then,

$$\rho(h\nu) = n_{ph} h\nu / \Delta\nu$$

Consider the Ar ion laser system. Given that the emission wavelength is at 488 nm and the linewidth in the output spectrum is about  $5 \times 10^9 \text{ Hz}$  between half intensity points, *estimate* the photon concentration necessary to achieve more stimulated emission than spontaneous emission.

### 6.5 Threshold gain and population inversion.

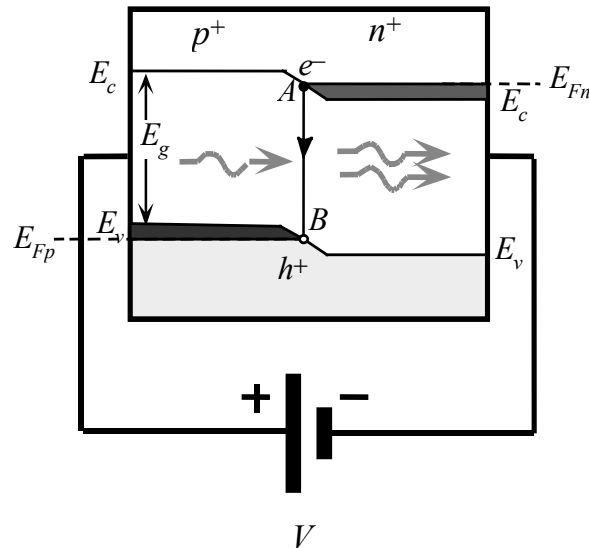
- (a) Consider a He-Ne gas laser operating at 632.8 nm. The tube length  $L=40 \text{ cm}$ , tube diameter is 1.5mm and mirror reflectances are approximately 99.9% and 98%. The linewidth  $\Delta\nu = 1.5 \text{ GHz}$ , the loss coefficient is  $\gamma \approx 0.05 \text{ m}^{-1}$ , spontaneous decay time constant  $\tau_{sp} = 1/A_{21} \approx 300 \text{ ns}$ , refractive index  $n \approx 1$ . What is the threshold gain and population inversion?
- (b) Consider a 488 nm Ar-ion gas laser. The tube length  $L = 1 \text{ m}$ , tube mirror reflectances are approximately 99.9% and 95%. The linewidth  $\Delta\nu = 3.0 \text{ GHz}$ , the loss coefficient is  $\gamma \approx 0.1 \text{ m}^{-1}$ , spontaneous decay time constant  $\tau_{sp} = 1/A_{21} \approx 10 \text{ ns}$ , refractive index  $n \approx 1$ . What is the threshold population inversion?
- (c) Consider a semiconductor laser operating at ( $\lambda$ ) 870 nm with a GaAs laser cavity with cleaved facets. The cavity length is  $50 \mu\text{m}$ . The refractive index ( $n$ ) of GaAs is 3.6. The loss coefficient  $\gamma$  at normal temperatures is of the order of  $\sim 10 \text{ cm}^{-1}$ . Estimate the required threshold gain. What is your conclusion?

**6.6 Population inversion in a GaAs laser diode.** Consider the energy diagram of a forward biased GaAs laser diode as in the figure below. For simplicity we assume a symmetrical device ( $n = p$ ) and we assume that population inversion has been just reached by A and B overlapping as illustrated in the figure which results in  $E_{Fn} - E_{Fp} = E_g$ . Estimate the minimum carrier concentration  $n = p$  for population inversion in GaAs

at 300K. The intrinsic carrier concentration in GaAs is of the order of  $10^7 \text{ cm}^{-3}$ . Assume for simplicity that

$$n = n_i \exp [(E_{Fn} - E_{Fi})/k_B T] \quad \text{and} \quad p = n_i \exp [(E_{Fi} - E_{Fp})/k_B T]$$

(Note: The analysis will only be an order of magnitude approximation as the above equations do not strictly hold in degenerate semiconductors.)



The energy band diagram of a degenerately doped  $p-n$  with with a sufficiently large forward bias to just cause population inversion where  $A$  and  $B$  overlap.

**6.7 InGaAsP-InP Laser.** Consider an InGaAsP-InP laser diode which has an optical cavity of length 250 microns. The peak radiation is at 1550 nm and the refractive index of InGaAsP is 4. The optical gain bandwidth (as measured between half intensity points) will normally depend on the pumping current (diode current) but for this problem assume that it is 2nm.

- What is the mode integer  $m$  of the peak radiation?
- What is the separation between the modes of the cavity?
- How many modes are there in the cavity?
- What is the reflection coefficient and reflectance at the ends of the optical cavity (faces of the InGaAsP crystal)?
- What determines the angular divergence of the laser beam emerging from the optical cavity?