Chapter 1

Geometrical Optics

Start with an arbitrary field given by

$$\overline{E}(r) = \overline{E}_o(r) \exp\left[-jk_o\phi(r)\right] \tag{1.1}$$

and

$$\overline{H}(r) = \overline{H}_o(r) \exp\left[-jk_o\phi(r)\right] \tag{1.2}$$

The term $k_o\phi$ defines the planes of constant phase (the wavefront).

Apply Maxwell's equation

$$\nabla \times \overline{E} = -j\omega\mu\overline{H} \tag{1.3}$$

The electric field has both a vector and a scalar component

$$\overline{E}(r) = \underbrace{\overline{E}_o}_{\text{vector}} \underbrace{\exp\left[-jk_o\phi(r)\right]}_{\text{scalar}} \tag{1.4}$$

Use the vector identity

$$\nabla \times \left(f\overline{A}\right) = f\left(\nabla \times \overline{A}\right) + (\nabla f) \times \overline{A}, \tag{1.5}$$

to get

$$\nabla \times \overline{E} = \exp\left[-jk_o\phi(r)\right]\left(\nabla \times \overline{E}_o\right) + \nabla\left\{\exp\left[-jk_o\phi(r)\right]\right\} \times \overline{E}_o \tag{1.6}$$

The gradient of the exponential term becomes

$$\nabla\left\{\exp\left[-jk_o\phi(r)\right]\right\} = \hat{x}(-jk_o)\exp\left(-jk_o\phi\right)\frac{\partial\phi}{\partial x} \tag{1.7}$$

$$+ \hat{y}(-jk_o)\exp\left(-jk_o\phi\right)\frac{\partial\phi}{\partial y} \tag{1.8}$$

$$+ \hat{z}(-jk_o)\exp\left(-jk_o\phi\right)\frac{\partial\phi}{\partial z} \tag{1.9}$$

$$= -jk_o\exp\left(-jk_o\phi\right)\nabla\phi \tag{1.10}$$

Equation 1.6 becomes

$$\nabla \times \overline{E} = \exp\left(-jk_o\phi\right)\left(\nabla \times \overline{E}_o\right) + \left(-jk_o\right)\exp\left(-jk_o\phi\right)\nabla\phi \times \overline{E}_o \tag{1.11}$$

This equation is also equatl to $-j\omega\mu\overline{H}_o \exp\left(-jk_o\phi\right)$ resulting in

$$-j\omega\mu\overline{H}_o \exp\left(-jk_o\phi\right) = \exp\left(-jk_o\phi\right)\left(\nabla \times \overline{E}_o\right) + \left(-jk_o\right)\exp\left(-jk_o\phi\right)\nabla\phi \times \overline{E}_o \tag{1.12}$$

Eliminate the common exponential terms to yield

$$-j\omega\mu\overline{H}_o = \left(\nabla \times \overline{E}_o\right) + \left(-jk_o\right)\nabla\phi \times \overline{E}_o, \tag{1.13}$$

which can be rearranged to yield

$$\nabla\phi \times \overline{E}_o - \frac{\left(\nabla \times \overline{E}_o\right)}{jk_o} - \frac{-j\omega\mu}{j\omega\sqrt{\mu\epsilon}}\overline{H}_o = 0, \tag{1.14}$$

$$\nabla\phi \times \overline{E}_o + \frac{j}{k_o}\nabla \times \overline{E}_o - \eta\overline{H}_o = 0. \tag{1.15}$$

Follow a similar process the following equations can also be derived

$$\nabla\phi \times \overline{H}_o - \frac{j}{k_o}\nabla \times \overline{E}_o - \frac{n^2}{\eta}\overline{H}_o = 0 \tag{1.16}$$

$$\nabla\phi \cdot \overline{H}_o + \frac{j}{k_o}\nabla \cdot \overline{H}_o = 0 \tag{1.17}$$

$$\nabla\phi \cdot \overline{E}_o + \frac{j}{k_o n^2}\nabla \cdot \left(n^2\overline{E}_o\right) = 0 \tag{1.18}$$

$$\tag{1.19}$$

In the geometrical optics approximation $\lambda \to 0$ and $k_o \to \infty$ resulting in

$$\nabla\phi \times \overline{E}_o = n\eta_o\overline{H}_o \tag{1.20}$$

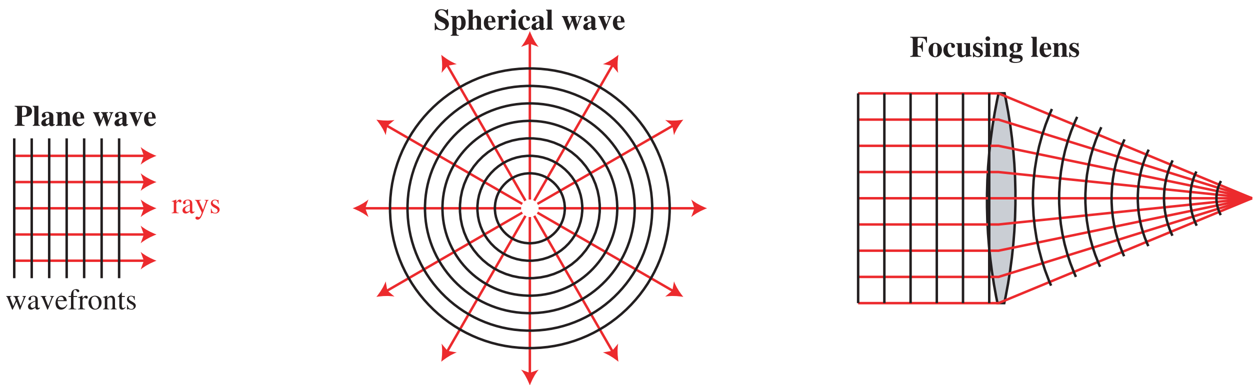
$$\nabla\phi \times \overline{H}_o = -\frac{n^2}{\eta}\overline{E}_o \tag{1.21}$$

$$\nabla\phi \cdot \overline{H}_o = 0 \tag{1.22}$$

$$\nabla\phi \cdot \overline{E}_o = 0 \tag{1.23}$$

$$\tag{1.24}$$

This means that $\overline{E}_o$ and $\overline{H}_o$ are both perpendicular to $\nabla\phi$. Therefore, $\nabla\phi$ defines the ray direction which is also the power flow direction.



Are the rays essentially narrow beams of light (like laser beams)?

No!

Very narraow beams of light have very rapid divergence because of diffraction.

Rays are simply used to characterize the propagation of a wavefront.

Even though rays are not actually narrow beams of light, the can still be used to analyze irradaince by looking at ray density.

Example: A line source has a power of $P = 1W/m$. A linear detector with a width of $D = 1mm$ is placed $R = 1m$ away from the line source. How mauch power is received by the detector?

If you trace N equally spaced rays out of the line source the angular separation between the rays is

$$\Delta\theta = \frac{2\pi}{N} \tag{1.25}$$

The angle to the detector is

$$\theta = \pm \tan^{-1}\left(\frac{0.510^{-3}}{1}\right) = 0.510^{-3} \tag{1.26}$$

The number of received rays is

$$10^{-3}\left(\frac{N}{2\pi}\right) \tag{1.27}$$

For $N = 10^5$ this becomes

$$10^{-3}\left(\frac{10^5}{2\pi}\right) \approx 16 \text{ rays} \tag{1.28}$$

The received power is then

$$P = (16)\left(\frac{1}{10^5}\right) = 160\mu W \tag{1.29}$$

The actual power is

$$P = (1W)\,\frac{10^{-3}}{2\pi} = 159.2\mu W \tag{1.30}$$

The concept of calculating irradiance using ray tracing enables irradiance calcualtion for arbitrary shapes and optical systems.

1.1 Basic Postulates of Geometrical Optics

1. Rays are normal to the wavefront and vice versa.
2. Rays satisfy the laws of reflecton and refraction.
3. Th optical path length along any ray between two wavefronts are equal
4. Irradiance at any point is proportional to the ray density at that point.

Other resulting corollaries are:

1. In a uniform medium, light travels along striaght lines.
2. The optical paths are reversible.
3. The optical path difference between two neighboring rays is zero.

Ray trasing can be used to:

1. Analyze optical systmes
   Optimize lenses
   Determine focal length
   Deterine magnification
   Determine image quality
   etc.
2. Illumination analysis/design
   Head light reflectors
   Light pipes
   etc. (see
3. Image artifacts
   Multiple reflections off of lens surfaces
   Scatter of light off of surfaces