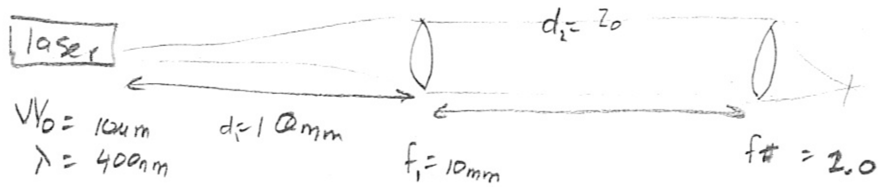
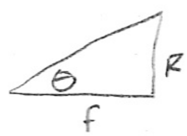
Use ABCD matrices for the following system



First do a geometrical optics analysis

Find divergence angle of initial beam

$$\theta = \frac{\lambda}{\pi W_0} = 0.0127 \text{ rad}$$



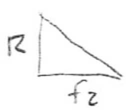
$$\frac{R}{f} = \theta$$

$$R = f\theta = (10\times10^{-3})(0.0127)$$

$$R = 127.3\ \mu m$$

$$\frac{f_2}{D} = 2$$

$$f_2 = 2\cdot 2\cdot R = (4)(127.3\times10^{-6})$$



$$\boxed{f_2 = 509.3\ \mu m}$$

$$\theta_2 = \frac{R}{f_2} = \frac{1}{2}\frac{D}{f_2} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{\lambda}{\pi W_0}$$

$$W_0 = \frac{4\lambda}{\pi} = \boxed{0.509\ \mu m = W_f}$$

$$Z_0 \approx W_1^2 \frac{\pi}{\lambda}$$

$$= (127.3\times10^{-6})^2 \frac{\pi}{400\times10^{-9}}$$

$$\boxed{d_2 = 0.1273 m}$$

Now using Gaussian beam analysis

$$M_1 = \begin{bmatrix} 1 & 10\times10^{-3} \\ 0 & 1 \end{bmatrix}$$

$$\frac{1}{q_0} = \frac{1}{\infty} - j\frac{\lambda}{\pi W_0^2}$$

$$q_0 = j\frac{\pi W_0^2}{\lambda} = j\,7.854\times10^{-4}$$

$$q_1 = \frac{q_0 + 10\times10^{-3}}{1} = 0.01 + j\,7.854\times10^{-4}$$

$$1/q_1 = 99.3869 - j\,7.8058$$

$$\frac{1}{R_1} = 99.3869$$

$$R_1 = 0.0101$$

$$\frac{\lambda}{\pi W_1^2} = 7.8058$$

$$W_1 = \sqrt{\frac{\lambda}{\pi\ 7.8058}}$$

$$\boxed{W_1 = 127.7\ \mu m}$$

After lens 1

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{10\times10^{-3}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 10\times10^{-3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.01 \\ -100 & 0 \end{bmatrix}$$

$$q_2 = \frac{q_0 + 0.01}{-100 q_0} = -0.01 + j\ 0.1273$$

$$\frac{1}{q_2} = -0.6131 - j\ 7.8058$$

$R_2 = -1.63$ slightly off collimated

$W_2 = 127.7\ \mu m$

Rayleigh range is approximately $Z_0 = \frac{\pi}{\lambda} W_2^2$

$$= \left(\frac{\pi}{0.4\times10^{-6}}\right)(127.7\times10^{-6})^2$$

$$Z_0 = 0.128 m$$

let's use $d_2 = 0.13 m$

$$M = \begin{bmatrix} 1 & 509\times10^{-6} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{509\times10^{-6}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.13 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{10\times10^{-3}} & 1 \end{bmatrix} \begin{bmatrix} 0 & 10\times10^{-3} \\ 0 & 1 \end{bmatrix}$$

$$q_3 = -1.0165\times10^{-6} + j\ 1.0832\times10^{-6}$$

$R_3 = -2.17\times10^{-6}$

$W_3 = 0.5093\ \mu m$