ECEn 672 – DETECTION AND ESTIMATION THEORY

Winter 2006 Homework 2

Due Tues. Feb. 21

1. 11.2-1
2. 11.2-2
3. 11.2-4
4. 11.2-6
5. 11.2-9
6. 11.2-10
7. 11.2-11
8. 11.2-13
9. 11.5-18
10. 11.10-30
11. 11.10-24
12. 11.10-31
13. 11.10-32
14. A transmitter sends one of two discrete waveforms represented by the vectors

$$\begin{aligned} \mathbf{s}_0 &= [s_0(1) \ \cdots \ s_0(N)]^T \\ \mathbf{s}_1 &= [s_1(1) \ \cdots \ s_1(N)]^T \ . \end{aligned}$$

At the receive end, the vector $\mathbf{x} = \mathbf{s}_i + \mathbf{n}$ is obtained, where $\mathbf{n}$ is a noise vector distributed as $\mathcal{N}(0, \sigma^2\mathbf{I})$. The problem is to determine which of the two waveforms was sent (assume hypothesis H_0 signifies that $\mathbf{s}_0$ was sent, while H_1 signifies that $\mathbf{s}_1$ was sent). Assume that $\mathbf{s}_i^T\mathbf{s}_i = E_s$ and that $\mathbf{s}_0^T\mathbf{s}_1 = \rho E_s$, where $-1 \leq \rho \leq 1$.

   (a) Find the likelihood ratio detector for this problem.

   (b) Compute and plot the ROC curve (β versus α) when $E_s/\sigma^2 = 10$ and $\rho = 0$. Repeat for $\rho = 0.5$.

   (c) Show operating points on the ROC curves for (i) the Neyman-Pearson detector with $\alpha = 0.01$, (ii) the Bayes detector when $p_0 = 7/16, p_1 = 9/16$ and $L_{01} = L_{10} = 1$, (iii) the minimax detector when $L_{01} = 4L_{10}$, also find the least favorable prior, and (iv) the balanced detector when $p_0 = 15/16$.

   (d) For each of the operating points in (c), determine the thresholds.

15. In an optical communications system, a laser transmits n photons on the baud interval $[0, t)$ with Poisson probability $P[N(t) = n] = e^{-\lambda_i t}(\lambda_i t)^n/n!$. The rate parameter λ_i is equal to λ_0 for a binary digit 0, or is equal to λ_1 for the binary digit 1. Each photon is detected with probability p. You are to observe $M(t)$, the number of photons detected on $[0, t)$, and test $H_0 : i = 0$ versus $H_1 : i = 1$.

    (a) What is the distribution of $M(t)$?

    (b) Find the likelihood ratio detector for testing H_0 versus H_1.

    (c) Compute and plot the ROC curve for $\lambda_0 p = 1$ and $\lambda_1 p = 2$ when t is chosen so that $\beta = 0.5$ at $\alpha = 0.01$.

    (d) Show operating points on the ROC curve for (i) the Neyman-Pearson detector with $\alpha = 0.1$, (ii) the Bayes detector when $p_0 = 1/4, p_1 = 3/4$ and $L_{01} = 4L_{10}$, (iii) the minimax detector when $L_{01} = 4L_{10}$, also find the least favorable prior, and (iv) the balanced detector when $p_0 = 1/4$.