ECEn 672 – DETECTION AND ESTIMATION THEORY

Winter 2006 Homework 3

Due Tues. Mar. 21

1. 12.3-6
2. 12.3-7
3. 12.3-13
4. Let $\mathbf{y} \sim \mathcal{N}(\mathbf{x}, \sigma^2 \mathbf{I})$. Write the mean as $\mathbf{x} = \beta \mathbf{u}_x$, where $\beta = \|\mathbf{x}\|$ (the norm of $\mathbf{x}$) and $\mathbf{u}_x$ is a unit-vector in the direction of $\mathbf{x}$. If possible, find the maximum likelihood estimates of:
   (a) β when$\mathbf{u}_x$ is known (σ^2 can be be unknown, but it doesn't matter)
   (b) $\mathbf{u}_x$ (β and σ^2 can be known or unknown)
   (c) β and $\mathbf{u}_x$ (σ^2 can be known or unknown)
   (d) σ^2 when β and $\mathbf{u}_x$ are known
   (e) σ^2 and β when $\mathbf{u}_x$ is known
   (f) σ^2 and $\mathbf{u}_x$ when β is known
   (g) σ^2, β, and $\mathbf{u}_x$.
5. 12.6-9
6. 12.6.21
7. Let $\mathbf{x} \in \Re^N$ be a random vector with normal distribution $\mathcal{N}(\theta \mathbf{e}, \sigma^2 \mathbf{I})$ where θ is a scalar and $\mathbf{e}$ is a vector of ones. The parameter θ is also normally distributed: $\theta \sim \mathcal{N}(m, \sigma_\theta^2)$.
   (a) Find the conditional density of θ given $\mathbf{x}$.
   (b) Find the conditional mean and variance of θ given $\mathbf{x}$.
   (c) Compare the mean and variance of the two estimators $E(\theta|\mathbf{x})$ and $\hat{\theta}_{ML}$.
8. Let $N(t)$ denote the number of particles emitted by a radioactive source over the time interval $[0, t)$. Assume a Poisson distribution:

   $$P[N(t) = n] = e^{-\lambda t} \frac{(\lambda t)^n}{n!}.$$

   Let $\{t_1, t_2, \ldots, t_n\}$ denote a sequence of times at which $N(t)$ is measured.
   (a) Find the MMSE estimator of $N(t_n)$ given $N(t_i)$ for $i = 1, 2, \ldots, n-1$.
   (b) Find the mean of the MMSE estimator.
   (c) Find the mean-squared error of the MMSE estimator.