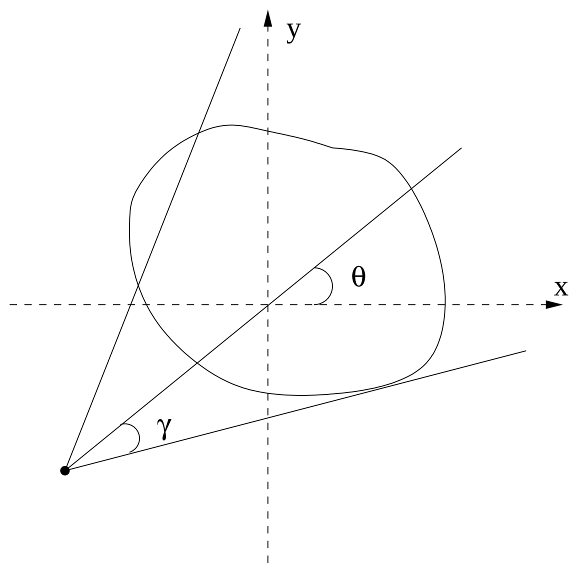
In this problem we will look at the modifications to the traditional Radon transform necessary to understand fan-beam reconstruction. The geometry for fan-beam reconstruction is shown in the following figure.



The data space is defined by the two variables (γ, θ). Define the emanation point of the beams to be at a distance r from the center of the region to be imaged. The variable γ runs from $-\pi$ to π while the variable θ runs from 0 to 2π. The projected data is computed for each γ and θ as the integral of the object along the line emanating from the source point (defined by r and θ) and inclined at an angle of γ from the line connecting the source point to the origin of the object. The data is therefore

$$g_r(\gamma, \theta) = R_F f(x, y)$$

where R_F is the linear fan-beam operator.

1. Find the kernel, K_F, of the fan-beam operator and it's adjoint such that

$$g_r(\gamma, \theta) = \int\int dxdy K_F(\gamma, \theta; x, y) f(x, y).$$

2. Find the operator $K_F^* K_F$. Is it also spatially invariant like in the radon-transform?