Review
Fundamental Concepts and Techniques of Calculus

Detailed Solutions to Selected Exercises:
Differential Calculus

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1. Two-Sided Limits: Find the Following Limits if they Exist:

   (a) We first reduce the argument of the limit operator and obtain the expression

   $$\lim_{x\to c}\frac{2x-6}{x-3} = \lim_{x\to c}\frac{2(x-3)}{x-3} = \lim_{x\to c} 2$$

   whose limit clearly equals

   $$= 2$$

   by (R 3:3a).

   (b) Let $q(x) := x^2 - 6x - 9$. As can be easily verified, $q(3) = -18 \neq 0$, hence $3 \in D_r$, where $r(x) = (x-3)/(x^2-6x-9)$. Hence, we can evaluate the limit of $r(x)$ at the point $c = 3$ using (R 3:3d) and obtain

   $$\lim_{x\to 3}\frac{x-3}{x^2-6x-9} = \frac{3-3}{3^2-6\cdot 3-9} = 0.$$

   (c) Since the polynomial $q(x) = x^2 - 5x + 4$ is zero at $c = 1$, as can be easily verified, we cannot apply (R 3:3d) to compute the given limit. Therefore, we try to reduce the fraction:

   $$\lim_{x\to 1}\frac{x-1}{x^2-5x+4} = \lim_{x\to 1}\frac{x-1}{(x-1)(x-4)} = \lim_{x\to 1}\frac{1}{x-4}$$

   The denominator of the reduced fraction is different from zero at $c = 1$. Hence, we can apply (R 3:3d) and obtain

   $$= \frac{1}{1-4} = -\tfrac{1}{3}$$

   (d) Expanding the fraction by x yields

   $$\lim_{x\to 0}\frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \lim_{x\to 0}\frac{x\left(1-\frac{1}{x}\right)}{x\left(1+\frac{1}{x}\right)} = \lim_{x\to 0}\frac{x-1}{x+1}$$

   which satisfies the assumptions of (R 3:3d). Hence the limit can be determined by evaluating the argument function at $c = 0$

   $$= \frac{0-1}{0+1} = -1$$

   (e) By (R 3:3e) in conjunction with (R 3:2c), $\lim_{x\to 2}\sqrt{x-1} = \sqrt{2-1} = 1 \neq 0$, hence we can evaluate the given limit using the quotient rule for limits (R 3:2(a)iv)

   $$\lim_{x\to 2}\frac{3x-3}{\sqrt{x-1}} = \frac{\lim\limits_{x\to 2} 3x-3}{\lim\limits_{x\to 2}\sqrt{x-1}} = \frac{3\cdot 2-3}{1} = 3$$

   by (R 3:3c) and the initial computation.

   (f) We combine the fractions in the denominator and factor the denominator using the identity $x - 2 = \left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right)$, then follows

   $$\lim_{x\to 2}\frac{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{x}}}{x-2} = \lim_{x\to 2}\frac{\frac{\sqrt{x}-\sqrt{2}}{\sqrt{2x}}}{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right)} = \lim_{x\to 2}\frac{\sqrt{x}-\sqrt{2}}{\sqrt{2x}\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right)} = \lim_{x\to 2}\frac{1}{\sqrt{2x}\left(\sqrt{x}+\sqrt{2}\right)}$$

   The limit of the denominator can be computed using the product rule and summation rule for limits (R 3:2(a)iii) and (R 3:2(a)i) in conjunction with (R 3:3e) and (R 3:3c)

$$\lim_{x\to 2}\sqrt{2x}\left(\sqrt{x}+\sqrt{2}\right) =$$
$$= \lim_{x\to 2}\sqrt{2x}\left(\lim_{x\to 2}\sqrt{x}+\lim_{x\to 2}\sqrt{2}\right)$$
$$= \sqrt{\lim_{x\to 2}2x}\left(\sqrt{\lim_{x\to 2}x}+\sqrt{2}\right)$$
$$= \sqrt{2\cdot 2}\left(\sqrt{2}+\sqrt{2}\right) = 4\sqrt{2}\neq 0.$$

Hence, by the quotient rule (R 3:2(a)iv)

$$\lim_{x\to 2}\frac{1}{\sqrt{2x}\left(\sqrt{x}+\sqrt{2}\right)} =$$
$$= \frac{\lim_{x\to 2}1}{\lim_{x\to 2}\sqrt{2x}\left(\sqrt{x}+\sqrt{2}\right)} = \frac{1}{4\sqrt{2}}$$
$$= \tfrac{1}{8}\sqrt{2}.$$

Hence,

$$\lim_{x\to 2}\frac{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{x}}}{x-2} = \tfrac{1}{8}\sqrt{2}.$$

(g) We first factor and reduce the argument of the limit operator:

$$\lim_{x\to 1}\frac{x^{2/3}-1}{x-1} =$$
$$= \lim_{x\to 1}\frac{\left(x^{1/3}-1\right)\left(x^{1/3}+1\right)}{\left(x^{1/3}-1\right)\left(x^{2/3}+x^{1/3}+1\right)}$$
$$= \lim_{x\to 1}\frac{x^{1/3}+1}{x^{2/3}+x^{1/3}+1}$$

Since by (R 3:2(a)iii), (R 3:3e) and (R 3:3c)

$$\lim_{x\to 1}\left(x^{2/3}+x^{1/3}+1\right) =$$
$$= \left(\lim_{x\to 1}x\right)^{2/3}+\left(\lim_{x\to 1}x\right)^{1/3}+\lim_{x\to 1}1$$
$$= 1+1+1 = 3\neq 0,$$

we can use the quotient rule (R 3:2(a)iv) to finish evaluating the limit above. Therefore, by another application of (R 3:3e) and (R 3:3c), we obtain

$$\lim_{x\to 1}\frac{x^{1/3}+1}{x^{2/3}+x^{1/3}+1} =$$
$$= \frac{\lim_{x\to 1}\left(x^{1/3}+1\right)}{\lim_{x\to 1}\left(x^{2/3}+x^{1/3}+1\right)}$$
$$= \frac{\left(\lim_{x\to 1}x\right)^{1/3}+1}{3} = \frac{2}{3}.$$

Hence,

$$\lim_{x\to 1}\frac{x^{2/3}-1}{x-1} = \tfrac{2}{3}.$$

(h) Clearly,

$$\lim_{x\to 0}\frac{\tan^2 x}{x} = \lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{\sin x}{\cos^2 x}\right)$$

and by the product rule (R 3:2(a)iii)

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{\cos^2 x}$$

since both limits exist. In fact, by (R 3:3j) $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and since by (R 3:3g) $\lim_{x\to 0}\cos x = 1\neq 0$, by (R 3:2(a)iv) and (R 3:2(a)iii)

$$\lim_{x\to 0}\frac{\sin x}{\cos^2 x} = \frac{\lim_{x\to 0}\sin x}{\left(\lim_{x\to 0}\cos x\right)^2} = \frac{0}{1} = 1.$$

The last limits were evaluated using (R 3:3f) and (R 3:3g) again. Continuing the computation from above, we see that

$$\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{\cos^2 x} =$$
$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{\cos^2 x} = 1\cdot 0 = 0,$$

hence

$$\lim_{x\to 0}\frac{\tan^2 x}{x} = 0.$$

(i) Rewriting the limit using the substitution $y = x-\pi$, we see that

$$\lim_{x\to\pi}\frac{\sin x}{x-\pi} = \lim_{y\to 0}\frac{\sin(y+\pi)}{y}$$

which, by the addition formula for $\sin$, equals

$$= \lim_{y\to 0}\frac{\sin y\cos\pi+\sin\pi\cos y}{y}$$

and, since $\sin\pi = 0$, $\cos\pi = -1$,

$$= \lim_{y\to 0}\frac{-\sin y}{y}$$

and

$$= -\lim_{y\to 0}\frac{\sin y}{y} = -1$$

by (R 3:2(a)ii) and (R 3:3j).

(j) Clearly,

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 3x} = \lim_{x\to 0}\frac{(\sin 2x)2x}{2x\sin 3x} = \lim_{x\to 0}\frac{(\sin 2x)3x\frac{2}{3}}{2x\sin 3x} = \lim_{x\to 0}\left(\frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}}\cdot\frac{2}{3}\right)$$

which by (R 3:2(a)iv and (R 3:2(a)ii)) equals

$$= \frac{2}{3}\frac{\lim\limits_{x\to 0}\frac{\sin 2x}{2x}}{\lim\limits_{x\to 0}\frac{\sin 3x}{3x}}$$

and

$$= \tfrac{2}{3}\cdot\frac{1}{1} = \tfrac{2}{3}$$

by (R 3:2c) and (R 3:3j). Note that we could only apply the quotient rule (R 3:2(a)iv) because the limit of the denominator is unequal to zero!

(k) Coming soon . . .

(l) We will use the "Squeeze Play" (R 3:2e): Since $-1 \le \cos x \le 1$ for all $x \in \mathbb{R}$, it follow by multiplying the double-inequality by $|x| \ge 0$ that

$$-|x| \le |x|\cos x \le |x| \quad \text{for all } x \in \mathbb{R}.$$

Since by (R 3:3e) and (R 3:2(a)iii)

$$\lim_{x\to 0}|x| = \lim_{x\to 0}\sqrt{x^2}\sqrt{\left(\lim_{x\to 0}x\right)^2} = \sqrt{0} = 0,$$

it follow by (R 3:2e) that also the limit of the expression in the middle of the double-inequality above has to be zero, i.e.

$$\lim_{x\to 0}|x|\cos x = 0.$$

(m) Coming soon . . .

2. One-Sided Limits: Find the Following Limits if they Exist:

(a) Coming soon . . .

(b) Clearly,

$$\frac{\sin x}{\sqrt{x}} = \frac{\sin x}{x}\cdot\sqrt{x}$$

for all $x \in \mathbb{R}$, $x \ge 0$. Hence by (R 3:2(a)iii)

$$\lim_{x\to 0^+}\frac{\sin x}{\sqrt{x}} = \lim_{x\to 0^+}\left(\frac{\sin x}{x}\cdot\sqrt{x}\right) = \lim_{x\to 0^+}\frac{\sin x}{x}\lim_{x\to 0^+}\sqrt{x}$$

which by (R 3:3j) and (R 3:3e) equals

$$= 1\cdot\sqrt{\lim_{x\to 0^+}x} = 1\cdot 0 = 0$$

by (R 3:3b).

(c) Coming soon . . .

(d) Coming soon . . .

(e) Coming soon . . .

(f) Coming soon . . .

3. Limits at ∞: Find the Following Limits if they Exist:

(a) Coming soon . . .

(b) Clearly,

$$\lim_{x\to\infty}\frac{x+1}{x-3} = \lim_{x\to\infty}\frac{x^2(1+\frac{1}{x^2})}{x^2(2+\frac{4}{x^2})} = \lim_{x\to\infty}\frac{1+\frac{1}{x^2}}{2+\frac{4}{x^2}}$$

which in turn by (R 3:2(a)iv), (R 3:2(a)i) and (R 3:2(a)iii) equals

$$= \frac{\lim\limits_{x\to\infty}1 + \left(\lim\limits_{x\to\infty}\frac{1}{x}\right)^2}{\lim\limits_{x\to\infty}2 + \left(\lim\limits_{x\to\infty}\frac{1}{x}\right)^2}$$

and by (R 3:3a) and (R 3:3l) finally

$$= \frac{1+0^2}{2+0^2} = \tfrac{1}{2}.$$

(c) Coming soon . . .

(d) Coming soon . . .

(e) Coming soon . . .

(f) Coming soon . . .

(g) Coming soon . . .

(h) Coming soon . . .

(i) Coming soon . . .

4. Squeeze Play, Pinching Theorem: Find the Following Limits:

(a) Coming soon . . .

(b) Coming soon . . .

(c) Clearly, $e/n \leq 1$ for all $n \in \mathbb{N}$ with $n \geq 3$. Hence

$$\begin{aligned}\frac{e^n}{n!} &= \frac{1}{e}\frac{e}{2}\frac{e}{3}\frac{e}{4}\cdots\frac{e}{n-1}\frac{e}{n}\\ &= \frac{e^3}{2}\frac{e}{3}\frac{e}{4}\cdots\frac{e}{n-1}\frac{1}{n}\\ &\leq \frac{e^3}{2}\cdot 1\cdot 1\cdots 1\cdot\frac{1}{n} = \frac{e^3}{2}\frac{1}{n}\end{aligned}$$

for all $n \in \mathbb{N}$ with $n \geq 4$. Therefore, we have established that

$$0 \leq \frac{e^n}{n!} \leq \frac{e^3}{2}\frac{1}{n}$$

for all $n \in \mathbb{N}$ with $n \geq 4$. Since

$$\lim_{n\to\infty}\frac{e^3}{2}\frac{1}{n} = \frac{e^3}{2}\lim_{n\to\infty} = \frac{e^3}{2}\cdot 0 = 0$$

by (R 3:2(a)ii) and (R 3:3l), we can apply the "Squeeze Play" (R 3:2e) and obtain that the middle expression in the double-inequality above also has to be zero, i.e.

$$\lim_{n\to\infty}\frac{e^n}{n!} = 0.$$

(d) Coming soon ...

(e) Coming soon ...

(f) Coming soon ...

5. L'Hopital's Rule: Find the Following Limits:

(a) Since $f(x) := 3^x$ and $g(x) := \ln x$ are differentiable in a neigborhood of $c = \infty$ and $\lim_{x\to]infty} 3^x = \lim_{x\to\infty} e^{x\ln x} = \infty$ by (R 3:3n) and $\lim_{x\to\infty}\ln x = \infty$ by (R 3:3o), we can apply L'Hôpital's Rule (R 3:2(g)ii) and conclude

$$\begin{aligned}\lim_{x\to\infty}\frac{3^x}{\ln x} &= \lim_{x\to\infty}\frac{3^x\ln 3}{\frac{1}{x}}\\ &= \lim_{x\to\infty}\left(x3^x\ln 3\right)\\ &= \infty.\end{aligned}$$

(b) We first rewrite the argument of the limit operator in terms of the natural exponential function:

$$\left(1+\frac{a}{x}\right)^x = e^{x\ln(1+\frac{a}{x})}.$$

We now apply the Substitution Rule (R 3:2c) to this expression and first compute the limit of the interior function $f(x) := x\ln(1+\frac{a}{x})$:

$$\lim_{x\to\infty} x\ln(1+\frac{a}{x}) = \lim_{x\to\infty}\frac{\ln(1+\frac{a}{x})}{\frac{1}{x}}$$

which satisfies the assumptions of L'Hôpital's Rule (R 3:2(g)i) as can be easily verified and therefore equals

$$\begin{aligned}&= \lim_{x\to\infty}\left(\frac{\frac{x}{x+a}(-\frac{a}{x^2})}{-\frac{1}{x^2}}\right)\\ &= \lim_{x\to\infty}\frac{ax}{x+a}\\ &= \lim_{x\to\infty}\frac{x\cdot a}{x(1+\frac{a}{x})}\\ &= \lim_{x\to\infty}\frac{a}{1+\frac{a}{x}}\\ &= \frac{a}{1+0} = a\end{aligned}$$

by (R 3:2(a)iv), (R 3:2c) and (R 3:3l). By (R 3:3h),

$$\lim_{y\to a} e^y = e^a,$$

hence by the Substitution Rule (R 3:2c)

$$\lim_{x\to\infty} e^{x\ln(1+\frac{a}{x})} = e^a.$$

Therefore,

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a.$$

(c) Coming soon ...

(d) Coming soon ...

(e) Coming soon ...

(f) Coming soon ...

(g) Coming soon ...

(h) Coming soon ...

(i) Coming soon ...

(j) Coming soon ...

(k) Coming soon ...

(l) Coming soon ...

(m) Coming soon ...

6. Continuity, Removable Discontinuities, and Jump Discontinuities:

(a) Coming soon ...

(b) Coming soon ...

(c) Coming soon ...

(d) Coming soon ...

(e) Coming soon ...

(f) Coming soon ...

(g) Coming soon ...

7. Recall and interpret the Intermediate Value Theorem: The real valued, continuous function f is defined on the closed and bounded interval $[a, b]$. Which of the following must be true?

   (a) Coming soon ...

   (b) Coming soon ...

   (c) Coming soon ...

   (d) Coming soon ...

   (e) Coming soon ...

   (f) Coming soon ...

   (g) Coming soon ...

   (h) Coming soon ...

8. Pick the correct Statement of the Mean Value theorem:

   (a) Coming soon ...

   (b) Coming soon ...

   (c) Coming soon ...

   (d) Coming soon ...

   (e) Coming soon ...

   (f) Coming soon ...

   (g) Coming soon ...

   (h) Coming soon ...

   (i) Coming soon ...

   (j) Coming soon ...

9. Recall and Interpret the Extreme Value Theorem The real valued, continuous function f is defined on the closed and bounded interval $[a, b]$. Which of the following must be true?

   (a) Coming soon ...

   (b) Coming soon ...

   (c) Coming soon ...

   (d) Coming soon ...

   (e) Coming soon ...

   (f) Coming soon ...

   (g) Coming soon ...

   (h) Coming soon ...

   (i) Coming soon ...

10. Find the Equation for the Lines Tangent and Normal (Perpendicular) to the given Curve at the given Point

(a) Clearly, the derivative of f is given by

$$f'(x) = \cos x$$

for $x \in \mathbb{R}$. The slope of f at $(\frac{\pi}{6}, \frac{1}{2})$, which by definition is the slope of the tangent at $(\frac{\pi}{6}, \frac{1}{2})$, is therefore $m = f'\big(\frac{\pi}{6}\big) = \cos\frac{\pi}{6} = \frac{1}{2}\sqrt{3}$. With the point-slope form of the line equation, which is given by

$$y - y_0 = m(x - x_0),$$

we can now easily compute he tangent line at the point $(\frac{\pi}{6}, \frac{1}{2})$:

$$y - \tfrac{1}{2} = \tfrac{1}{2}\sqrt{3}\big(x - \tfrac{\pi}{6}\big) = \tfrac{1}{2}\sqrt{3}x - \tfrac{\pi}{12}\sqrt{3}.$$

Hence the tangent line is given by

$$T(x) = \big(\tfrac{1}{2}\sqrt{3}\big)x + \big(\tfrac{1}{2} - \tfrac{\pi}{12}\sqrt{3}\big)$$

for all $x \in \mathbb{R}$.
The normal line of f at $(\frac{\pi}{6}, \frac{1}{2})$, can be computed the same way. Recall that the slope of the normal at $(\frac{\pi}{6}, \frac{1}{2})$ is given by

$$\begin{aligned} m^{\perp} &= -\frac{1}{m} = -\frac{1}{f'\big(\frac{\pi}{6}\big)} \\ &= -\frac{1}{\frac{1}{2}\sqrt{3}} = -\frac{2}{\sqrt{3}} = -\tfrac{2}{3}\sqrt{3}. \end{aligned}$$

We can now compute the normal line of f at the point $(\frac{\pi}{6}, \frac{1}{2})$:

$$y - \tfrac{1}{2} = -\tfrac{2}{3}\sqrt{3}\big(x - \tfrac{\pi}{6}\big) = -\tfrac{2}{3}\sqrt{3}x + \tfrac{\pi}{9}\sqrt{3}$$

and obtain

$$N(x) = \big(-\tfrac{2}{3}\sqrt{3}\big)x + \big(\tfrac{1}{2} + \tfrac{\pi}{9}\sqrt{3}\big)$$

for all $x \in \mathbb{R}$.

(b) Since the derivative of f is

$$f(x) = \frac{1}{2\sqrt{x+1}}$$

the slope of f at $(3, 2)$ is

$$m = f'(3) = \frac{1}{3\sqrt{3+1}} = \tfrac{1}{6}.$$

Moreover, the slope of the normal line at $(3, 2)$ is given by

$$m^{\perp} = -\frac{1}{m} = -\frac{1}{\frac{1}{6}} = -6.$$

With the point-slope form $y-y_0 = m(x-x_0)$ of the line equation, we obtain $y-2 = \frac{1}{6}(x-3) = \frac{1}{6}x - \frac{1}{2}$ and $y-2 = -6(x-3) = -6x+18$. Thus, the tangent line of f at $(3,2)$ is given by

$$T(x) = \tfrac{1}{6}x + \tfrac{3}{2}$$

while the normal line of f at $(3,2)$ is

$$N(x) = -6x + 16$$

for $x \in \mathbb{R}$.

(c) The derivative of f is $f'(x) = e^{-x}$, hence the slope of the tangent of f at the point $(0,0)$ is

$$m = f'(0) = e^{-0} = 1$$

while the slope of the normal of f at $(0,0)$ is

$$m^{\perp} = -\frac{1}{m} = -\frac{1}{1} = -1.$$

We can therefore compute the equations of the tangent and normal line with point-slope form of the line equation and obtain $y-0 = 1\cdot(x-0) = x$ and $y-0 = -1\cdot(x-0) = -x$, respectively. Hence, the tangent line and normal line of f at $(0,0)$ are given by

$$T(x) = x$$

and

$$N(x) = -x,$$

respectively.

(d) The derivative of f is $f'(x) = \frac{1}{x}$, hence the slope of the tangent and normal of f at the point $(3, 1+\ln 3)$ is

$$m = f'(3) = \tfrac{1}{3}$$

and

$$m^{\perp} = -\frac{1}{m} = -\frac{1}{\frac{1}{3}} = -3,$$

respectively. With the point-slope form of the line equation, we again obtain $y-(1+\ln 3) = \frac{1}{3}(x-3) = \frac{1}{3}x - 1$ and $y-(1+\ln 3) = -3(x-3) = -3x+9$. Hence

$$T(x) = \tfrac{1}{3}x + \ln 3$$

is the equation of the tangent line of f at the $(3, 1+\ln 3)$, while the eation of the normal line of f at the same point is given by

$$N(x) = -3x + (10 + \ln 3)$$

for $x \in \mathbb{R}$¿

11. The Derivative:

    (a) Coming soon . . .

    (b) Use the Definition to compute the Derivative of the following Functions at x_0:

    i. Coming soon . . .
    ii. Coming soon . . .
    iii. Coming soon . . .
    iv. Coming soon . . .

    (c) Determine whether each Function is (α) differentiable (β) continuous at x_0:

    i. Coming soon . . .
    ii. Coming soon . . .
    iii. Coming soon . . .

12. Taking Derivatives Using the Differentiation Rules:

    (a) Polynomials and Rational Functions

    i. Coming soon . . .
    ii. Coming soon . . .
    iii. Coming soon . . .
    iv. Coming soon . . .
    v. Coming soon . . .
    vi. Coming soon . . .
    vii. Coming soon . . .
    viii. Coming soon . . .
    ix. Coming soon . . .
    x. Coming soon . . .

    (b) Trigonometric Functions

    i. Coming soon . . .
    ii. Coming soon . . .
    iii. Coming soon . . .
    iv. Coming soon . . .
    v. Coming soon . . .
    vi. Coming soon . . .
    vii. Coming soon . . .
    viii. Coming soon . . .
    ix. Coming soon . . .
    x. Coming soon . . .
    xi. Coming soon . . .
    xii. Coming soon . . .

    (c) Exponential and Logarithmic Functions

    i. Coming soon . . .
    ii. Coming soon . . .
    iii. Coming soon . . .

iv. Coming soon . . .

(d) Misc.

i. Coming soon . . .
ii. Coming soon . . .
iii. Coming soon . . .
iv. Coming soon . . .
v. Coming soon . . .
vi. Coming soon . . .
vii. Coming soon . . .
viii. Coming soon . . .
ix. Coming soon . . .
x. Coming soon . . .
xi. Coming soon . . .
xii. Coming soon . . .
xiii. Coming soon . . .
xiv. Coming soon . . .
xv. Coming soon . . .

13. Implicit Differentiation

(a) Find dy/dx in terms of x and y:

i. Coming soon . . .
ii. Coming soon . . .
iii. Coming soon . . .
iv. Coming soon . . .
v. Coming soon . . .

(b) Find Equations for the Tangent and Normal Lines at the indicated Point, respectively:

i. Coming soon . . .
ii. Coming soon . . .
iii. Coming soon . . .

14. Find the Critical Numbers and Classify the Extreme Values (as Local/Global):

(a) Coming soon . . .
(b) Coming soon . . .
(c) Coming soon . . .
(d) Coming soon . . .
(e) Coming soon . . .
(f) Coming soon . . .
(g) Coming soon . . .
(h) Coming soon . . .
(i) Coming soon . . .

15. Describe the Concavity of the Graph of f and find the Points of Inflection:

(a) Coming soon . . .
(b) Coming soon . . .
(c) Coming soon . . .
(d) Coming soon . . .
(e) Coming soon . . .
(f) Coming soon . . .

16. Find the Intervals on which f increases and the Interval on which f decreases:

(a) Coming soon . . .
(b) Coming soon . . .
(c) Coming soon . . .
(d) Coming soon . . .

17. Sketch the Graph of the Function f:

(a) Coming soon . . .
(b) Coming soon . . .
(c) Coming soon . . .
(d) Coming soon . . .
(e) Coming soon . . .
(f) Coming soon . . .

18. Maximun/Minimun Problems

(a) Coming soon . . .
(b) Coming soon . . .
(c) Coming soon . . .
(d) Coming soon . . .
(e) Coming soon . . .
(f) Coming soon . . .
(g) Coming soon . . .
(h) Coming soon . . .