

Review
Fundamental Concepts and Techniques of Calculus

Exercises:* Basic Techniques of Algebra

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1. Perform Arithmetic Operations with Fractions:

(a) Simplify: $(\frac{5}{2} - \frac{1}{2}) \div \frac{2}{7}$ HSD

(b) Simplify: $\frac{5}{8} - \frac{5}{12} + \frac{1}{6}$ S

(c) Simplify: $12 \div \frac{1}{4}$ HSD

(d) Simplify: $(\frac{10}{11} + \frac{6}{33} - \frac{13}{66}) \cdot \frac{3}{22}$ S

2. Simplify (and Rationalize) Expressions Involving Exponents and Radicals:

(a) $(5^{\frac{1}{2}})^{-2}$ S (g) $\sqrt{\sqrt[3]{10a^7b}}$ S

(b) $(-5z)^3$ S (h) $\sqrt[3]{16} + 3\sqrt[3]{54}$ S

(c) $(-\frac{125}{27})^{-1/3}$ HSD (i) $\frac{5}{\sqrt{10}}$ HSD

(d) $\sqrt[5]{-27^3}$ S (j) $\frac{5}{\sqrt[3]{10}}$ HSD

(e) $\sqrt{75x^2y^{-4}}$ S (k) $\frac{3}{\sqrt{5} + \sqrt{6}}$ HSD

(f) $\frac{x^{4/3}y^{2/3}}{(xy)^{1/3}}$ S

3. Simplify Expressions Involving Absolute Value:

(a) $|\frac{1}{5} - \frac{1}{\sqrt{5}}| - |-5|$ HSD

(b) $|-1| - |-2|$ S

(c) $\frac{|x+2|}{x+2}$ for $x < -2$ S

4. Simplify Expressions Involving Factorials:

(a) $5!$ S $3!$ S $4!$ S

(b) $\frac{8!4!}{3!5!}$ (c) $\frac{4!}{3!3!}$

5. Manipulate Logarithmic Expressions:

(a) Evaluate $\log_2 64$ S

(b) Evaluate $\log_{32} 8$ S

(c) Evaluate $\log_2 4^3$ HSD

(d) Evaluate $\log_4 2 + \log_4 32$ S

(e) Evaluate $\log_{10} 0.01$ S

(f) Simplify $\frac{1}{3} [\log_8 y + 2 \log_8 (y+4)] - \log_8 (y-1)$ HSD

(g) Simplify $4 \ln 6 - 3 \ln 2 + 2 \ln 3 - \ln 8$ HSD

(h) Simplify $5 \ln 6 - \ln 30 + \ln \frac{1}{5}$ HSD

6. Factor a Polynomial and Solve:

(a) $9x^2 - 4 = 0$ HSD

(b) $x^2 - 10x + 25 = 0$ S

(c) $4x^4 + 4x^2 + 1 = 0$ HSD

(d) $4x^2 + 12x + 9 = 0$ S

(e) $x^4 - 6x^2 + 8 = 0$ S

7. Complete the Square and find the Vertex:

(a) $y = x^2 - 4x + 1$ S

(b) $y = 3x^2 - 6x + 12$ HSD

(c) $y = 2x^2 + 20x + 3$ HSD

8. Solving Quadratic Equations by Factoring or Using the Quadratic Formula:

(a) $x^2 - 9 = 0$ S

(b) $x^2 - x - 2 = 0$ HSD

(c) $2x^2 - 5x - 3 = 0$ HSD

(d) $x^2 - x - 42 = 0$ S

(e) $x^2 - 6x + 9 = 0$ HSD

(f) $x^2 - 2x + 2 = 0$ HSD

(g) $x^2 - 2x + 5 = 0$ S

9. Solving Inequalities:

(a) $3x + 5 > \frac{1}{4}(x - 2)$ S

(b) $2x^2 + 9x + 6 \geq x + 2$ S

(c) $x(6x + 9)(x - 3) > 0$ S

*The letters "H","S", "D" in the status box indicate whether a *hint* (H), a *solution* (S) or a *detailed solution* (D) is available.

- (d) $\frac{x^2 - 9}{x + 1} > 0$
- (e) $\frac{1}{x-1} + \frac{4}{x-6} > 0$
- (f) $\frac{x^2 - 4x + 3}{x^2} > 0$

10. Solving Inequalities Involving Absolute Values:

- (a) $|x - 2| < 1$
- (b) $|x - \frac{3}{2}| \geq \frac{3}{2}$
- (c) $7 < |8x - 72|$
- (d) $|x - 3| - |x + 1| \leq x$

11. Solve Exponential and Logarithmic Equations:

- (a) $2^{x^2+3x} = 1024$
- (b) $6(2^{3x-1}) - 7 = 9$
- (c) $e^{2x} - 4e^x - 5 = 0$
- (d) $\frac{400}{1 + e^{-x}} = 350$
- (e) $\ln \sqrt{x+2} = 1$
- (f) $\ln x + \ln(x-2) = 1$
- (g) $\ln((2x+1)(x+2)) = 2 \ln(x+2)$
- (h) $2 \ln(x+2) - \frac{1}{2} \ln x^4 = 1$
- (i) $\log_4 x - \log_4(x-1) = \frac{1}{2}$
- (j) $\log_{10} 4x - \log_{10}(12 + \sqrt{x}) = 2$

12. Find the Equation of a Line:

- (a) through $(1, 1), (9, 7)$
- (b) through $(-4, 10), (4, -5)$
- (c) through $(-\frac{1}{2}, 1), (-\frac{5}{2}, \frac{4}{3})$
- (d) through $(1, 1)$ perpendicular to $2x + 8y = 2$
- (e) through $(2, 6)$ perpendicular to $-x + 2y = 1$

13. Find the Distance and Midpoint between Two Points:

- (a) $(1, 1), (9, 7)$
- (b) $(-4, 10), (4, -5)$
- (c) $(-\frac{1}{2}, 1), (-\frac{5}{2}, \frac{4}{3})$

14. Graph the Quadratic Equation (Conic):

- (a) $x^2 - 6x + 2y + 9 = 0$
- (b) $16x^2 + 16y^2 - 16x + 24y - 3 = 0$
- (c) $x^2 + y^2 - 2x - 31 = 0$
- (d) $-9x^2 - 24x + 4y^2 - 8y + 16 = 0$

15. Find the Composition of Two Functions:

- (a) Let $f(x) = \frac{2}{x}$ and $g(x) = 2x^3 + 2x^2 + 1$. Find and simplify $(g \circ f)(x)$ and $(f \circ g)(x)$.
- (b) Let $f(x) = \sqrt[3]{x-1}$ and $g(x) = x^3 + 1$. Find and simplify $(g \circ f)(x)$ and $(f \circ g)(x)$.

16. Find the Inverse of a Function:

- (a) Let $f(x) = \frac{1}{3} \frac{2x+3}{x+1}$. Find f^{-1} .
- (b) Let $f(x) = \sqrt{4-x^2}, 0 \leq x \leq 2$. Find f^{-1} .
- (c) Let $f(x) = \frac{x+1}{x-2}$. Find f^{-1} .
- (d) Let $f(x) = x^2 + 9x + 5$. Determine whether f has an inverse. If so, find it.
- (e) Let $f(x) = (4x-1)^3$. Determine whether f has an inverse. If so, find it.

17. Find the Domain and Range of a Function:

- (a) $f(x) = |x - 5|$
- (b) $f(x) = \sqrt{x-3} - 1$
- (c) $f(x) = \sqrt{4-x^2}$
- (d) $f(x) = \frac{x}{5x-3}$
- (e) $f(x) = \frac{\sqrt{9-x^2}}{x^2-3x+2}$
- (f) $f(x) = \frac{1}{\sqrt{1-4x^2}}$

18. Determine Whether a Function is Even, Odd or Neither:

- (a) $f(x) = x^6 - 2x^2 + 3$
- (b) $f(x) = x\sqrt{1-x^2}$
- (c) $f(x) = 4x^{2/3}$
- (d) $f(x) = \tan x$
- (e) $f(x) = \frac{3x^3 - 7x}{x^4 - 2x^2 - 7}$
- (f) $f(x) = x^3 - 5x$
- (g) $f(x) = x^3 - 5$
- (h) $f(x) = 1 + x^2 \cos 2x + x$

19. Graph Exponential and Logarithmic Functions:

- (a) $f(x) = \ln(x+5)$
- (b) $f(x) = 5^{x-2} + 4$
- (c) $f(x) = 1 - \ln(x-5)$

20. Manipulate Complex Numbers:

- (a) Simplify $(6 - 2i)(2 - 3i)$
- (b) Simplify $(2 + 3i)^2 + (2 - 3i)^2$
- (c) Simplify $\frac{6-7i}{1-2i}$
- (d) Simplify $\frac{-3-i}{-1-2i}$
- (e) Add and write in standard form: $\frac{i}{3-2i} + \frac{2i}{3+8i}$

21. Finding Powers and Roots of Complex Numbers Using D'Moivres Theorem:

- (a) Cube $-1 + \sqrt{3}i$
- (b) Cube $-1 - \sqrt{3}i$
- (c) Simplify i^{39}
- (d) Find the square roots of $16 + 12i$
- (e) Find the 6th roots of 64.