

Review
Fundamental Concepts and Techniques of Calculus

Exercises: * Trigonometry

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1. Apply the Basic Trigonometric Identities to Simplify Expressions:

(a) $\sin x \cos x + \cos^2 x$ HSD

(b) $2 \sin x \cos^3 x - 2 \sin^3 x \cos x$ HSD

(c) $\frac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x}$ HSD

(d) $\frac{2 \sin^2 x}{\cos^3 x} \cdot \left(\frac{\cos x}{2 \sin x}\right)^2$ HSD

(e) $\frac{3 \sin x}{\cos^2 x} \cdot \frac{\cos^2 x + \cos x \sin x}{\sin^2 x - \cos^2 x}$ HSD

(f) $\frac{2 \tan \theta}{1 + \tan^2 \theta}$ HSD

(g) $\frac{\tan x - \sin x}{2 \tan x}$ HSD

(h) $\sin(\alpha + \beta) \sin(\alpha - \beta)$ HSD

(i) $\left(\frac{\cot x}{\csc x}\right)^2 + \frac{1}{\csc^2 x}$ HSD

(j) $\frac{\cos \theta + \sin \theta}{\cos \theta}$ HSD

(k) $(\cos^2 x) \frac{1 + \tan^2 x}{1 - \cos^2 x}$ HSD

2. Recall the Sum and Difference Formulas for Sine and Cosine:

(a) Give a formula for $\sin(x + y)$ in terms of $\cos x$, $\sin x$, $\sin y$ and $\cos y$. HSD

(b) Give a formula for $\cos(x + y)$ in terms of $\cos x$, $\sin x$, $\sin y$ and $\cos y$. HSD

(c) Give a formula for $\sin(x - y)$ in terms of $\cos x$, $\sin x$, $\sin y$ and $\cos y$. HSD

(d) Give a formula for $\cos(x - y)$ in terms of $\cos x$, $\sin x$, $\sin y$ and $\cos y$. HSD

3. Recall the Three Pythagorean Identities HSD

4. Recall the Double and Half Angle Formulas for Sine and Cosine:

(a) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$ HSD

(b) Express $\cos 2x$ in terms of $\sin x$ and $\cos x$ HSD

(c) Express $\tan 2x$ in terms of $\tan x$ HSD

(d) Give a formula for $\cos^2 x$ in terms of $\cos 2x$ HSD

(e) Give a formula for $\sin^2 x$ in terms of $\cos 2x$ HSD

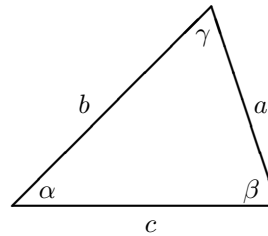
(f) Give a formula for $\sin \frac{x}{2}$ in terms of $\cos x$ HSD

(g) Give a formula for $\cos \frac{x}{2}$ in terms of $\cos x$ HSD

(h) Give a formula for $\tan \frac{x}{2}$ in terms of $\cos x$ HSD

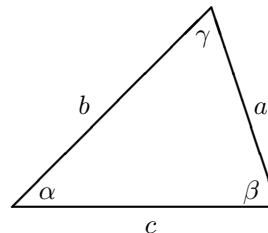
5. Recall the Law of Sines and Law of Cosines; Determine which Law applies; Solve Triangle:

(a) $c = 5$, $b = 3$ and $\alpha = \frac{2}{3}\pi$.



HSD

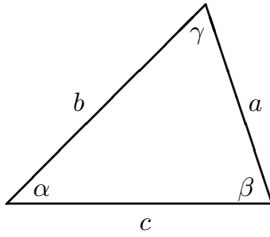
(b) $a = 15$, $c = 7$, $\beta = 62^\circ$.



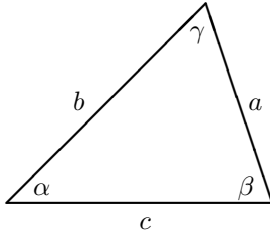
HSD

(c) $a = 3.6$, $b = 6.2$, $c = 4.1$

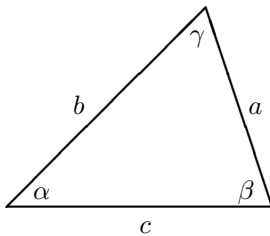
*The letters "H","S", "D" in the status box indicate whether a *hint* (H), a *solution* (S) or a *detailed solution* (D) is available.



(d) $\alpha = 36.5^\circ$, $a = 24$, $b = 34$



(e) $\alpha = \frac{1}{4}\pi$, $\gamma = \frac{1}{3}\pi$, $a = 4$



6. Recall the Trigonometric Values of Angles which are Multiples of $\frac{\pi}{4}$ And $\frac{\pi}{6}$, and find the exact Value:

- | | | | |
|-----------------------------|--------------------------------|--|--------------------------------|
| (a) $\sin \pi$ | <input type="text" value="S"/> | (e) $\csc(\frac{3\pi}{4})$ | <input type="text" value="S"/> |
| (b) $\cos(-\frac{\pi}{3})$ | <input type="text" value="S"/> | (f) $\tan(\frac{\pi}{6})$ | <input type="text" value="S"/> |
| (c) $\tan(\frac{11\pi}{4})$ | <input type="text" value="S"/> | (g) $\sin^9(\frac{\pi}{6}) \cos(4\pi)$ | <input type="text" value="S"/> |
| (d) $\sin(\frac{5\pi}{4})$ | <input type="text" value="S"/> | | |

7. Simplify the Trigonometric Expression and find the exact Value: (Hint: Use Right-Triangle Construction)

- | | |
|---------------------------------------|--------------------------------|
| (a) $\sin^{-1}(\sin(\frac{7\pi}{4}))$ | <input type="text" value="S"/> |
| (b) $\sin(\cos^{-1}(-\frac{1}{2}))$ | <input type="text" value="S"/> |
| (c) $\cos(\sec^{-1}(2))$ | <input type="text" value="S"/> |
| (d) $\cos(\tan^{-1}(x))$ | <input type="text" value="S"/> |
| (e) $\csc(\cot^{-1}(\frac{x}{2}))$ | <input type="text" value="S"/> |
| (f) $\sin(\cos^{-1}(x))$ | <input type="text" value="S"/> |
| (g) $\sec(\sin^{-1}(x))$ | <input type="text" value="S"/> |

8. Graph the Trigonometric Equations:

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|------------------|--------------------------------|--|--------------------------------|
| (a) $y = \tan x$ | <input type="text" value="S"/> | (g) $y = 2 \sin(\frac{1}{2}x)$ | <input type="text" value="S"/> |
| (b) $y = \cot x$ | <input type="text" value="S"/> | (h) $y = -\cos(x - \frac{\pi}{2})$ | <input type="text" value="S"/> |
| (c) $y = \sin x$ | <input type="text" value="S"/> | (i) $y = \sin(x - \pi) - 2$ | <input type="text" value="S"/> |
| (d) $y = \cos x$ | <input type="text" value="S"/> | (j) $y = \frac{1}{2} \cos(2x + \frac{\pi}{2})$ | <input type="text" value="S"/> |
| (e) $y = \sec x$ | <input type="text" value="S"/> | | |
| (f) $y = \csc x$ | <input type="text" value="S"/> | | |

9. Solve for x in the Trigonometric Equations:

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|--------------------------------------|--------------------------------|
| (a) $\sin 2x = 1$ | <input type="text" value="S"/> |
| (b) $\sin x = \frac{1}{2}$ | <input type="text" value="S"/> |
| (c) $\tan(\frac{1}{2}x) = 1$ | <input type="text" value="S"/> |
| (d) $\sqrt{\sin^2 x} = 1$ | <input type="text" value="S"/> |
| (e) $\sin 2x = -\frac{1}{2}\sqrt{3}$ | <input type="text" value="S"/> |
| (f) $\cos x = \frac{1}{2}\sqrt{2}$ | <input type="text" value="S"/> |
| (g) $\cos^2 x + 6 \cos x + 4 = 0$ | <input type="text" value="S"/> |
| (h) $\sin 2x \cos x + \sin x = 0$ | <input type="text" value="S"/> |
| (i) $\sec^2 x + 2 \tan x = 6$ | <input type="text" value="S"/> |

10. Convert Cartesian Coordinates to Polar Coordinates:

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|---------------|--------------------------------|----------------------------|--------------------------------|
| (a) $(0, 1)$ | <input type="text" value="S"/> | (d) (\sqrt{x}, \sqrt{x}) | <input type="text" value="S"/> |
| (b) $(-3, 0)$ | <input type="text" value="S"/> | (e) $(3, -3\sqrt{3})$ | <input type="text" value="S"/> |
| (c) $(2, -2)$ | <input type="text" value="S"/> | (f) $(\sqrt{3}, -1)$ | <input type="text" value="S"/> |

11. Convert Polar Coordinates to Cartesian Coordinates:

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|-----------------------------|--------------------------------|-------------------------------------|--------------------------------|
| (a) $(1, \frac{1}{3}\pi)$ | <input type="text" value="S"/> | (e) $(3, -\frac{1}{2}\pi)$ | <input type="text" value="S"/> |
| (b) $(3, \frac{3}{4}\pi)$ | <input type="text" value="S"/> | (f) $(2, 3\pi)$ | <input type="text" value="S"/> |
| (c) $(-2, 0)$ | <input type="text" value="S"/> | (g) $(-1, \frac{1}{4}\pi)$ | <input type="text" value="S"/> |
| (d) $(-3, -\frac{1}{3}\pi)$ | <input type="text" value="S"/> | (h) $(\frac{1}{3}, \frac{2}{3}\pi)$ | <input type="text" value="S"/> |

12. Define the Hyperbolic Trigonometric Functions in terms of the Exponential Functions:

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|-----------------------------|--------------------------------|
| (a) $\sinh x$ | <input type="text" value="S"/> |
| (b) $\cosh x$ | <input type="text" value="S"/> |
| (c) $\tanh x$ | <input type="text" value="S"/> |
| (d) $\coth x$ | <input type="text" value="S"/> |
| (e) $\operatorname{sech} x$ | <input type="text" value="S"/> |
| (f) $\operatorname{csch} x$ | <input type="text" value="S"/> |

13. Apply the Basic Hyperbolic Trigonometric Identities to simplify Expressions:

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|-----------------------------|--------------------------------|
| (a) $\cosh^2 x - \sinh^2 x$ | <input type="text" value="S"/> |
|-----------------------------|--------------------------------|

(b) $\sinh x \cosh y + \sinh y \cosh x$

(c) $\cosh x \cosh y + \sinh x \sinh y$

(d) $\sinh x \cosh x$

(e) $\cosh^2 x + \sinh^2 x$

(f) $\frac{\cosh^2 x - \sinh^2 x}{\tanh^2 x}$