

Hints to the Exercises: Basic Techniques of Algebra

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1. Perform Arithmetic Operations with Fractions:

- (a) Apply the rules for adding, subtracting, multiplying and dividing fractions!
- (b)
- (c) Note that the integer n is identified with the fraction $\frac{n}{1}$.
- (d)

2. Simplify (and Rationalize) Expressions Involving Exponents and Radicals:

- (a)
- (b)
- (c) Apply the laws of exponent systematically!
- (d)
- (e)
- (f)
- (g)
- (h)
- (i) Expand the fraction by $\sqrt{10}$.
- (j) Expand the fraction by $10^{2/3}$.
- (k) Expand the fraction by $\sqrt{5} - \sqrt{6}$ and use the identity $(a+b)(a-b) = a^2 - b^2$.

3. Simplify Expressions Involving Absolute Value:

- (a) First determine the sign of $(\frac{1}{5} - \frac{1}{\sqrt{5}})$.
- (b)
- (c)

4. Simplify Expressions Involving Factorials:

- (a)
- (b)
- (c)

5. Manipulate Logarithmic Expressions:

- (a)
- (b)
- (c) Consider the corresponding exponential equation and write 4 as power of 2.
- (d)
- (e)

- (f) Use the logarithmic rules to write the term as the logarithm of one term.
- (g) Use the logarithmic rules to express the logarithms in terms of $\ln 2$ and $\ln 3$.
- (h) Use the logarithmic rules to express the logarithms in terms of $\ln 5$ and $\ln 6$.

6. Factor a Polynomial and Solve:

- (a) Use the formula $a^2 - b^2 = (a - b)(a + b)$ to factor the polynomial!
- (b)
- (c) Use the formulae $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - b^2 = (a - b)(a + b)$ to factor the polynomial!
- (d)
- (e)

7. Complete the Square and find the Vertex:

- (a)
- (b) First factor out the 3
$$y = 3x^2 - 6x + 12 = 3(x^2 - 2x) + 12,$$
then use “completing the square”!
- (c) First factor out the 2 to obtain

$$y = 2x^2 + 20x + 3 = 2(x^2 + 10x) + 3,$$

then “complete the square”!

8. Solving Quadratic Equations by Factoring or Using the Quadratic Formula:

- (a)
- (b) Use the formula $x^2 + (a + b)x + ab = (x + a)(x + b)$ to factor the polynomial!
- (c) Use the formula $\alpha_1\alpha_2x^2 + (\alpha_1\beta_2 + \alpha_2\beta_1)x + \beta_1\beta_2 = (\alpha_1x + \beta_1)(\alpha_2x + \beta_2)$ to factor the polynomial!
- (d)
- (e) Use the Binomial formula.
- (f) Use the “quadratic formula” to solve the quadratic equations, since it does not have a rational zero!

- (g)
9. Solving Inequalities:
- (a)
- (b)
- (c)
- (d)
- (e) Consider the left side of the inequality as a rational function. Consider where this rational functions can change sign using the Intermediate-value Theorem.
- (f)
10. Solving Inequalities Involving Absolute Values:
- (a)
- (b)
- (c) Eliminate $|\cdot|$ by restricting the domain to the subintervals where $8x - 72 \geq 0$ and where $8x - 72 < 0$.
- (d) Decompose the domain $D = \mathbb{R}$ of this inequality into subdomains D_1, D_2, D_3 , defined by $x - 3 \geq 3$ and $x + 1 \geq 0$, $x - 3 < 0$ and $x + 1 \geq 0$, $x - 3 < 0$ and $x + 1 < 0$ and solve the inequality first on those subdomains. Then combine the solutions.
11. Solve Exponential and Logarithmic Equations:
- (a) Write 1024 as power of 2.
- (b) Simplify the equation and apply the respective logarithmic function to the equation.
- (c) First reduce the exponential equation to a quadratic equation by a suitable substitution.
- (d)
- (e)
- (f)
- (g)
- (h) First determine the domain of the logarithmic equation, then combine the logarithmic expressions and apply the corresponding exponential function to the compactified equation.
- (i)
- (j)
12. Find the Equation of a Line:
- (a)
- (b)
- (c)
- (d)
- (e)
13. Find the Distance and Midpoint between Two Points:
- (a)
- (b)
- (c)
14. Graph the Quadratic Equation (Conic):
- (a)
- (b) First transform the equation into normal form by completing the square. Then interpret the resulting equation.
- (c) First transform the equation into normal form by completing the square. Then interpret the resulting equation.
- (d) First transform the equation into normal form by completing the square. Then interpret the resulting equation.
15. Find the Composition of Two Functions:
- (a)
- (b)
16. Find the Inverse of a Function:
- (a)
- (b)
- (c)
- (d) First check whether f is injective (one-to-one). Can f be made injective by restricting it to a subset of its domain?
- (e)
17. Find the Domain and Range of a Function:
- (a)
- (b)
- (c)
- (d)
- (e) In order to determine the domain of f , eliminate those $x \in \mathbb{R}$ for which $f(x)$ is not defined (or real). To determine the range of f , investigate the behavior of f at its poles and find its global extrema (on each subinterval of its domain) and use the continuity of f .
- (f)
18. Determine Whether a Function is Even, Odd or Neither:
- (a)

- (b) Compute $f(-x)$ for an arbitrary $x \in \mathbb{R}$ and compare it with $\pm f(x)$.
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- (h) Compute $f(-x)$ for an arbitrary $x \in \mathbb{R}$ and compare it with $\pm f(x)$.

19. Graph Exponential and Logarithmic Functions:

- (a)
- (b)
- (c)

20. Manipulate Complex Numbers:

- (a)
- (b)
- (c) Expand the fraction by $1 + 2i$.
- (d) Simplify and expand with the conjugate of the denominator.
- (e) Choose a common denominator and add the two fractions, then transform the sum into standard form by expanding the fraction by the conjugate of its denominator.

21. Finding Powers and Roots of Complex Numbers Using D'Moivre's Theorem:

- (a)
- (b) First transform $-1 - \sqrt{3}i$ into polar form, then apply d'Moivre's Theorem.
- (c) Reduce the power by using the fact that $i^n = i^m$ if and only if 4 divides $m - n$, i.e. $4 \mid (m - n)$.
- (d) Transform $16 + 12i$ into polar form and then apply d'Moivre's Theorem.
- (e) What is the polar representation of 64?