

Hints to the Exercises: Differential Calculus

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1. Two-Sided Limits: Find the Following Limits if they Exist:

- (a) First reduce the argument of the limit operator.
- (b) First check whether the denominator polynomial is 0 for $x = 3$. If not, apply (R 3:3d).
- (c) First reduce the argument of the limit operator. Then check, whether (R 3:3d) can be applied.
- (d) Expand the fraction by x .
- (e) Apply (R 3:3e) and (R 3:2(a)iv).
- (f) Reduce the fraction using

$$x - 2 = (\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2}).$$

- (g) Factor the numerator and denominator of the fraction and reduce it.
- (h) Write \tan in terms of \sin and \cos and use (R 3:3f), (R 3:3g) and (R 3:3j).
- (i) Use the substitution $y = x - \pi$, the addition formula for \sin and (R 3:3j).
- (j) Expand the argument of the limit operator by $2x$ and use (R 3:3j).
- (k) Coming soon ...
- (l) Use the inequality $-1 \leq \cos x \leq 1$ for $x \in \mathbb{R}$ in conjunction with the "Squeeze Play" (R 3:2e).
- (m) Coming soon ...

2. One-Sided Limits: Find the Following Limits if they Exist:

- (a) Coming soon ...
- (b) Expand the argument of the limit by \sqrt{x} and apply (R 3:3j) and (R 3:3e).
- (c) Coming soon ...
- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...

3. Limits at ∞ : Find the Following Limits if they Exist:

- (a) Coming soon ...
- (b) Factor out x^2 in the numerator and denominator of the argument of the limit operator and reduce. Then apply (R 3:2a) and (R 3:3l).
- (c) Coming soon ...
- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...
- (g) Coming soon ...
- (h) Coming soon ...
- (i) Coming soon ...

4. Squeeze Play, Pinching Theorem: Find the Following Limits:

- (a) Coming soon ...
- (b) Coming soon ...
- (c) Establish the inequality

$$0 \leq \frac{e^n}{n!} \leq \frac{e^3}{2} \frac{1}{n}$$

for all $n \in \mathbb{N}$ with $n \geq 4$ and apply the "Squeeze Play" (R 3:2e).

- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...

5. L'Hopital's Rule: Find the Following Limits:

- (a) Apply L'Hôpital's Rule and simplify the limit.
- (b) Rewrite

$$\left(1 + \frac{a}{x}\right)^x = e^{x \ln\left(1 + \frac{a}{x}\right)}$$

and use the Substitution Rule (R 3:2c) and L'Hôpital's Rule (R 3:2(g)i).

- (c) Coming soon ...
- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...

- (g) Coming soon ...
- (h) Coming soon ...
- (i) Coming soon ...
- (j) Coming soon ...
- (k) Coming soon ...
- (l) Coming soon ...
- (m) Coming soon ...
6. Continuity, Removable Discontinuities, and Jump Discontinuities:
- (a) Coming soon ...
- (b) Coming soon ...
- (c) Coming soon ...
- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...
- (g) Coming soon ...
7. Recall and interpret the Intermediate Value Theorem: The real valued, continuous function f is defined on the closed and bounded interval $[a, b]$. Which of the following must be true?
- (a) Coming soon ...
- (b) Coming soon ...
- (c) Coming soon ...
- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...
- (g) Coming soon ...
- (h) Coming soon ...
8. Pick the correct Statement of the Mean Value theorem:
- (a) Coming soon ...
- (b) Coming soon ...
- (c) Coming soon ...
- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...
- (g) Coming soon ...
- (h) Coming soon ...
- (i) Coming soon ...
- (j) Coming soon ...
9. Recall and Interpret the Extreme Value Theorem The real valued, continuous function f is defined on the closed and bounded interval $[a, b]$. Which of the following must be true?
- (a) Coming soon ...
- (b) Coming soon ...
- (c) Coming soon ...
- (d) Coming soon ...
- (e) Coming soon ...
- (f) Coming soon ...
- (g) Coming soon ...
- (h) Coming soon ...
- (i) Coming soon ...
10. Find the Equation for the Lines Tangent and Normal (Perpendicular) to the given Curve at the given Point
- (a) Compute the slope m of the graph of f at the point $(\frac{\pi}{6}, \frac{1}{2})$. Use the point-slope form of the line equation. The slope m^\perp of the normal line is $-1/m$.
- (b) Compute the slope m of the graph of f at the point $(3, 2)$. Use the point-slope form of the line equation. The slope m^\perp of the normal line is $-1/m$.
- (c) Compute the slope m of the graph of f at the point $(0, 0)$. Use the point-slope form of the line equation. The slope m^\perp of the normal line is $-1/m$.
- (d) Compute the slope m of the graph of f at the point $(3, \ln 3)$. Use the point-slope form of the line equation. The slope m^\perp of the normal line is $-1/m$.
11. The Derivative:
- (a)
- (b) Use the Definition to compute the Derivative of the following Functions at x_0 :
- i. Coming soon ...
- ii. Coming soon ...
- iii. Coming soon ...
- iv. Coming soon ...
- (c) Determine whether each Function is (α) differentiable (β) continuous at x_0 :
- i. Coming soon ...
- ii. Coming soon ...
- iii. Coming soon ...
12. Taking Derivatives Using the Differentiation Rules:
- (a) Polynomials and Rational Functions
- i. Coming soon ...

- ii. Coming soon ...
 - iii. Coming soon ...
 - iv. Coming soon ...
 - v. Coming soon ...
 - vi. Coming soon ...
 - vii. Coming soon ...
 - viii. Coming soon ...
 - ix. Coming soon ...
 - x. Coming soon ...
- (b) Trigonometric Functions
- i. Coming soon ...
 - ii. Coming soon ...
 - iii. Coming soon ...
 - iv. Coming soon ...
 - v. Coming soon ...
 - vi. Coming soon ...
 - vii. Coming soon ...
 - viii. Coming soon ...
 - ix. Coming soon ...
 - x. Coming soon ...
 - xi. Coming soon ...
 - xii. Coming soon ...
- (c) Exponential and Logarithmic Functions
- i. Coming soon ...
 - ii. Coming soon ...
 - iii. Coming soon ...
 - iv. Coming soon ...
- (d) Misc.
- i. Coming soon ...
 - ii. Coming soon ...
 - iii. Coming soon ...
 - iv. Coming soon ...
 - v. Coming soon ...
 - vi. Coming soon ...
 - vii. Coming soon ...
 - viii. Coming soon ...
 - ix. Coming soon ...
 - x. Coming soon ...
 - xi. Coming soon ...
 - xii. Coming soon ...
 - xiii. Coming soon ...
 - xiv. Coming soon ...
 - xv. Coming soon ...
13. Implicit Differentiation
- (a) Find dy/dx in terms of x and y :
- i. Coming soon ...
 - ii. Coming soon ...
 - iii. Coming soon ...
 - iv. Coming soon ...
 - v. Coming soon ...
- (b) Find Equations for the Tangent and Normal Lines at the indicated Point, respectively:
- i. Coming soon ...
 - ii. Coming soon ...
 - iii. Coming soon ...
14. Find the Critical Numbers and Classify the Extreme Values (as Local/Global):
- (a) Coming soon ...
 - (b) Coming soon ...
 - (c) Coming soon ...
 - (d) Coming soon ...
 - (e) Coming soon ...
 - (f) Coming soon ...
 - (g) Coming soon ...
 - (h) Coming soon ...
 - (i) Coming soon ...
15. Describe the Concavity of the Graph of f and find the Points of Inflection:
- (a) Coming soon ...
 - (b) Coming soon ...
 - (c) Coming soon ...
 - (d) Coming soon ...
 - (e) Coming soon ...
 - (f) Coming soon ...
16. Find the Intervals on which f increases and the Interval on which f decreases:
- (a) Coming soon ...
 - (b) Coming soon ...
 - (c) Coming soon ...
 - (d) Coming soon ...
17. Sketch the Graph of the Function f :
- (a) Coming soon ...
 - (b) Coming soon ...
 - (c) Coming soon ...
 - (d) Coming soon ...
 - (e) Coming soon ...
 - (f) Coming soon ...
18. Maximun/Minimun Problems

- (a) Coming soon . . .
- (b) Coming soon . . .
- (c) Coming soon . . .
- (d) Coming soon . . .

- (e) Coming soon . . .
- (f) Coming soon . . .
- (g) Coming soon . . .
- (h) Coming soon . . .