

**Review**  
Fundamental Concepts and Techniques of Calculus

**Hints to the Exercises: Integral Calculus**

Last updated: 041012

1. Integration By Parts:

- (a) Use integration by parts (R 4:6c) and set  $u = x$  and  $v' = e^x$ .
- (b)
- (c) Use integration by parts (R 4:6c) and set  $u = x$  and  $v' = \sin x$
- (d) Integrate by parts (R 4:6c) twice such that the remaining integral is a multiple of the original integral. Combine those integrals on the left side of the equation.
- (e)
- (f)
- (g)
- (h)
- (i)
- (j)
- (k)
- (l)
- (m)
- (n)
- (o)
- (p)
- (q)
- (r)
- (s)
- (t)

2. Trigonometric Integrals:

- (a) Use the Second Pythagorean identity and rewrite
$$\tan^3 x \sec^3 x = \sec^2 x \tan^2 x (\sec x \tan x)$$
$$= \sec^2 (\sec^2 x - 1) (\sec x \tan x).$$
Then apply (R 4:5b).
- (b)

- (c) Use the first Pythagorean identity and rewrite

$$\cos^3 x = \cos^2 x \cos x = (1 - \sin^2 x) \cos x.$$

Then apply (R 4:5b).

- (d)
- (e) You can use the trigonometric identity  $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$  to reduce the power of  $\cos$  in the integrand or apply the recursion formula (R 4:5q).
- (f) Use the trigonometric identities

$$\sin x \cos x = \frac{1}{2} \sin(2x)$$
$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$
$$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

to reduce the number of factors.

- (g)
- (h)
- (i)
- (j)
- (k)
- (l)
- (m)
- (n)

3. Substitution:

- (a) Try the substitution (R 4:6b) given by  $y = 1 + \sqrt{x}$ .
- (b)
- (c)
- (d)
- (e)
- (f)
- (g) Try the substitution (R 4:6b) given by  $y = \sin^{-1}(2x)$ .
- (h)
- (i)
- (j)

- (k)
- (l)
- (m)
- (n)
- (o)
- (p)
- (q)
- (r)
- (s) Reduce the integrand by the substitution (R 4:6b) given by  $y = e^x$ .
- (t) Complete the square and use trigonometric substitution (R 4:6(b)iii).
- (u)
- (v)
- (w)
- (x)
- (y) Use trigonometric substitution (R 4:6(b)iii) and set  $x = a \tan \theta$ .
- (z) let  $u = -x^3$

4. Partial Fractions:

- (a)
- (b)
- (c)
- (d)
- (e)
- (f)
- (g)
- (h)
- (i)
- (j)
- (k)
- (l)
- (m)
- (n)
- (o)
- (p)
- (q)
- (r)
- (s)
- (t)
- (u)
- (v)

- (w)
- (x)
- (y)

5. Improper Integrals:

- (a)
- (b)
- (c)
- (d)
- (e)
- (f)

6. Applications of Integration:

(a) *Area Between Two Curves:*

- i.
- ii.
- iii.

(b) *Volume By Slicing:*

- i.
- ii.
- iii.
- iv.
- v.

(c) *Volume By Shells:*

- i.
- ii.
- iii.
- iv.
- v.

(d) *Arc Length:*

- i.
- ii.

(e) *Surface Area:*

- i.
- ii.
- iii.
- iv.

(f) *Mass From Density:*

- i.
- ii.
- iii.
- iv.

(g) *Center of Mass and Centroid:*

- i.
- ii.
- iii.

- iv.
- v.
- vi.
- vii.
- viii.
- ix.

(h) *Fluid Pressure:*

- i.
- ii.

(i) *Work:*

- i.
- ii.
- iii.
- iv.

7. Parametric Curves:

(a) Express the Parametric Curve by an Equation in  $x$  and  $y$ :

- i.
- ii.
- iii.
- iv.
- v.

(b) Find a Parametrization  $x = x(t)$ ,  $y = y(t)$ ,  $t \in [0, 1]$  for

- i.
- ii.
- iii.
- iv.

(c)

(d) Find the slope of the given curve at the given point and give an equation of the tangent line:

- i.
- ii.

(e) Find the Length of the Parametric Arc:

- i.
- ii.

8. Polar Coordinates:

(a) Write the Equation in Polar Coordinates:

- i.
- ii.
- iii.

(b) Write the Equation in Cartesian Coordinates:

- i.
- ii.
- iii.

(c) Sketch the Polar Curves:

- i.
- ii.
- iii.
- iv.
- v.
- vi.

(d) Calculate the Area enclosed by the Polar Curve:

- i.
- ii.
- iii.
- iv.
- v.

(e) Find the Slope of the Polar Curve:

- i.
- ii.
- iii.

(f) Find the Length of the Polar Curve:

- i.
- ii.