Review
Fundamental Concepts and Techniques of Calculus

Solutions to the Exercises: Trigonometry

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1. Apply the Basic Trigonometric Identities to Simplify Expressions:
   (a) $\frac{1}{2}\big(1+\cos(2x)+\sin(2x)\big)$
   (b) $\frac{1}{2}\sin(4x)$
   (c) 1
   (d) $\frac{1}{2}\sec(x)$
   (e) $\dfrac{3\tan(x)}{\sin x-\cos x}$
   (f) $\sin(2\theta)$
   (g) $\frac{1}{2}\big(1-\cos x\big)$
   (h) $\frac{1}{2}\cos(2\beta)-\frac{1}{2}\cos(2\alpha)]$
   (i) 1
   (j) $1+\tan\theta$
   (k) $\csc^2 x$

2. Recall the Sum and Difference Formulas for Sine and Cosine:
   (a) $\sin(x+y)=\sin x\cos y+\sin y\cos x$
   (b) $\cos(x+y)=\cos x\cos y-\sin x\sin y$
   (c) $\sin(x-y)=\sin x\cos y-\sin y\cos x$
   (d) $\cos(x-y)=\cos x\cos y+\sin x\sin y$

3. The three Pythagorean identities:

$$\cos^2 x+\sin^2=1$$
$$1+\tan^2 x=\sec^2 x$$
$$1+\cot^2 x=\csc^2 x$$

4. Recall the Double and Half Angle Formulas for Sine and Cosine:
   (a) $\sin 2x=2\cos x\sin x$
   (b) $\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$
   (c) $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$
   (d) $\cos^2 x=\frac{1}{2}\big(1+\cos 2x\big)$
   (e) $\sin^2 x=\frac{1}{2}\big(1-\cos 2x\big)$
   (f) $\sin\frac{x}{2}=\pm\sqrt{\dfrac{1-\cos x}{2}}$
   (g) $\cos\frac{x}{2}=\pm\sqrt{\dfrac{1+\cos x}{2}}$
   (h) $\tan\frac{x}{2}=\dfrac{\sin x}{1+\cos x}=\dfrac{1-\cos x}{\sin x}$

5. Recall the Law of Sines and Law of Cosines; Determine which Law applies; Solve Triangle:
   (a) $a=7$, $\beta=\arcsin\big(\frac{3}{14}\sqrt{3}\big)$, $\gamma=\frac{1}{3}\pi-\beta$.
   (b) $b=\sqrt{274-210\cos 62°}$, $\alpha=\arcsin\left(\frac{15}{b}\sin 62°\right)$, $\gamma=118°-\alpha$.
   (c) $\alpha=\arccos\left(\frac{42.29}{50.84}\right)$ (Law of Cosine), $\beta=\arccos(\frac{-8.67}{29.52})$ (Law of Cosine), $\gamma=\arccos\left(\frac{34.59}{44.64}\right)$ (Law of Cosine)
   (d) $\beta=\arcsin(\frac{34\sin 36.5°}{24})$ (Law of Sines), $\gamma=118-\arcsin(\frac{34\sin 36.5°}{24})$ (Law of Sines), $c=\sqrt{1732-1632\cos\arcsin(\frac{34\sin 36.5°}{24})}$ (Law of Cosine)
   (e) $\beta=\frac{5\pi}{12}$, $b=\frac{8\sin\frac{5\pi}{12}}{\sqrt{2}}$ (Law of Sines), $c=\frac{4\sqrt{3}}{\sqrt{2}}$ (Law of Sines)

6. Recall the Trigonometric Values of Angles which are Multiples of $\frac{\pi}{4}$ And $\frac{\pi}{6}$, and find the exact Value:
   (a) 0
   (b) $\frac{1}{2}$
   (c) -1
   (d) $\frac{-\sqrt{2}}{2}$
   (e) $\sqrt{2}$
   (f) $\frac{\sqrt{3}}{3}$
   (g) $\frac{1}{512}$

7. Simplify the Trigonometric Expression and find the exact Value: (Hint: Use Right-Triangle Construction)
   (a) $-\pi/4$
   (b) $\sqrt{3}/2$
   (c) $\frac{1}{2}$
   (d) $\frac{1}{\sqrt{1+x^2}}$

(e) $\frac{\sqrt{4+x^2}}{2}$

(f) $\sqrt{1-x^2}$

(g) $\frac{1}{\sqrt{1-x^2}}$

8. Graph the Trigonometric Equations:

   (a) Function increases $\frac{-\pi \pm n\pi}{2} < x < \frac{\pi \pm n\pi}{2}$, with vertical asymptotes at $\frac{\pi \pm n\pi}{2}$, n is any integer

   (b) Function decreases $0 \pm n\pi < x < \pi \pm n\pi$, with vertical asymptotes at $\pi \pm n\pi$, n is any integer

   (c) Function decreases $\frac{-3\pi \pm 2n\pi}{2} < x < \frac{-\pi \pm 2n\pi}{2}$ and increases $\frac{-\pi \pm 2n\pi}{2} < x < \frac{\pi \pm 2n\pi}{2}, n$ is any real integer

   (d) Function increases $-\pi \pm 2n\pi < x < 0 \pm 2n\pi$ and decreases $0 \pm 2n\pi < x < \pi \pm 2n\pi$, n is any integer

   (e) Function has vertical asymptotes at $x = \frac{\pi}{2} \pm n\pi$; n is any integer.

   (f) Function has vertical asymptotes at $x = \pi \pm n\pi$; n is any integer.

   (g) Function is similar to $y = \sin(x)$, but with twice the amplitude and twice the period.

   (h) Function is similar to $y = \cos(x)$, but is shifted $\frac{\pi}{2}$ units to the right and is flipped over the x-axis.

   (i) Function is similar to $y = \sin(x)$, but is shifted π units to the right and 2 units down.

   (j) Function is similar to $y = \cos(x)$, but is shifted $\frac{\pi}{2}$ units to the left and has half the period and half the amplitude.

9. Solve for x in the Trigonometric Equations:

   (a) $\frac{\pi}{4}$

   (b) $\frac{\pi}{6}$

   (c) $\frac{\pi}{2}$

   (d) $\frac{\pi}{2}$

   (e) $\frac{11\pi}{6}$

   (f) $\frac{\pi}{4}$

   (g) $\pi - \arccos(3 \pm \sqrt{5})$

   (h) $0, \pi$

   (i) $\pm \arctan(\sqrt{6} \mp 1)$

10. Convert Cartesian Coordinates to Polar Coordinates:

    (a) $(1, \frac{\pi}{2})$

    (b) $(3, \pi)$

    (c) $(2\sqrt{2}, \frac{7\pi}{4})$

    (d) $(\sqrt{2x}, \frac{\pi}{4})$

    (e) $(\sqrt{6}, \frac{5\pi}{3})$

    (f) $(2, \frac{11\pi}{6})$

11. Convert Polar Coordinates to Cartesian Coordinates:

    (a) $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

    (b) $(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$

    (c) $(-2, 0)$

    (d) $(\frac{-3}{2}, \frac{3\sqrt{3}}{2})$

    (e) $(0, -3)$

    (f) $(-2, 0)$

    (g) $(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2})$

    (h) $(\frac{-1}{6}, \frac{\sqrt{3}}{6})$

12. Define the Hyperbolic Trigonometric Functions in terms of the Exponential Functions:

    (a) $\frac{1}{2}(e^x - e^{-x})$

    (b) $\frac{1}{2}(e^x + e^{-x})$

    (c) $\frac{\sinh(x)}{\cosh(x)}$

    (d) $\frac{\cosh(x)}{\sinh(x)}$

    (e) $\frac{1}{\cosh(x)}$

    (f) $\frac{1}{\sinh(x)}$

13. Apply the Basic Hyperbolic Trigonometric Identities to simplify Expressions:

    (a) 1

    (b) $\sinh(x+y)$

    (c) $\cosh(x+y)$

    (d) $\frac{\sinh(2x)}{2}$

    (e) $\cosh(2x)$

    (f) $\coth^2(x)$