Review
Fundamental Concepts and Techniques of Calculus

Solutions to the Exercises: Integral Calculus

Last updated: 041012

1. Integration By Parts:

   (a) $xe^x - e^x + C$

   (b) $\frac{x^3 \cdot 3^3}{\ln 3} - \frac{3x^2 \cdot 3^x}{(\ln 3)^2} + \frac{6x \cdot 3^x}{(\ln 3)^3} - \frac{6 \cdot 3^x}{(ln3)^4} + C$

   (c) $\sin(x) - x\cos(x) + C$

   (d) $\frac{e^x}{2}\Big(\sin x - \cos x\Big) + C$

   (e) $x\ln(x) - x + C$

   (f) let $u = \ln n$ $\frac{x^{n+1}}{(n+1)^2}[(n+1)\ln n - 1]$

   (g) $\frac{1}{\pi^2}[\cos(\pi x) + \pi x \sin(\pi x) + C]$

   (h) $\frac{1}{2}\cos(\ln(x)) + \frac{1}{2}\sin(\ln(x)) + C$

   (i) $\frac{1}{2}x^2 \arctan(x^2) - \frac{1}{4}\ln(x^4+1) + C$

   (j) $\ln(x)\arcsin(\ln(x)) + \sqrt{1-(\ln(x))^2} + C$

   (k) $2\cos(\sqrt{x}) + 2\sqrt{x}\sin(\sqrt{x}) + C$

   (l) $\frac{1}{2}(x+1)^2\ln(x+1) - (x+1)\ln(x+1) - \frac{1}{4}x^2 + \frac{1}{2}x + \frac{3}{4} + C$

   (m) $\frac{1}{4}\arcsin^2(2x) + C$

   (n) $\frac{1}{2}x^2\sinh(2x) - \frac{1}{2}x\cosh(2x) + \frac{1}{4}\sinh(2x) + C$

   (o) $-\frac{1}{32(2x-1)^4} - \frac{1}{20(2x-1)^5} - \frac{1}{48(2x-1)^6} + C$

   (p) $\frac{1}{2}x^2\arctan(x^2) - \frac{1}{4}\ln(x^2+1) + C$

   (q) $x\arcsin(2x) + \frac{1}{2}\sqrt{1-4x^2} + C$

   (r) $\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\ln(x^2+1) + C$

   (s) $\frac{1}{2}\ln(x^2)\arcsin(\ln(x^2)) + \frac{1}{2}\sqrt{1-(\ln(x^2))^2} + C$

   (t) $2\sin(\sqrt{x}) - 2\sqrt{x}\cos(\sqrt{x}) + C$

2. Trigonometric Integrals:

   (a) $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

   (b) $\frac{1}{5}\sin^5(3t) + C$

   (c) $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$

   (d) $\frac{1}{12}\sec 6x \tan 6x + \frac{1}{12}\ln|\sec 6x + \tan 6x| + C$

   (e) $\frac{5}{16}x + \frac{15}{64}\sin 2x + \frac{3}{64}\sin 4x + \frac{1}{192}\sin 6x + C.$

   (f) $\frac{3}{128}x - \frac{1}{128}\sin(4x) + \frac{1}{1024}\sin(8x)$

   (g) $\frac{1}{12}\sin(6x) + x + \frac{1}{4}\cos(4x) - \frac{1}{2}\cos(2x) - \frac{1}{4}\sin(2x) + C$

   (h) $\frac{1}{3}\tan^3(x) - \tan(x) + x + C$

   (i) $\frac{2}{3}\sec^{\frac{3}{2}}(x) + C$

   (j) $\frac{1}{2}\sin(x) + \frac{1}{40}\sin(5x) + \frac{1}{8}\sin(3x) + C$

   (k) $-\frac{\cos(\frac{1}{2}x)}{\sin^2(\frac{1}{2}x)} + \ln|\csc(\frac{1}{2}x) - (\cot(\frac{1}{2}x))| + C$

   (l) $-\frac{1}{4}\cos(7x) - \frac{1}{6}\cos(3x) + C$

   (m) $\frac{\sin(x)}{4\cos^4(x)} + \frac{3\sin(x)}{8\cos^2(x)} + \frac{3}{8}\ln|\sec(x)+\tan(x)| + C$

   (n) $-\frac{\cos^3(x)}{3\sin^3(x)} + C$

3. Substitution:

   (a) $\frac{8}{15} + \frac{4}{15}(1+\sqrt{x})^{\frac{3}{2}}(3\sqrt{x-2}) + C$

   (b) $\frac{1}{7}t^4(1+t^2)^{\frac{3}{2}} - \frac{4}{35}t^2(1+t^2)^{\frac{3}{2}} + \frac{8}{105}(1+t^2)^{\frac{3}{2}} + C$

   (c) $x^2e^x - 2xe^x + 2e^x + C$

   (d) $-\ln|1+\cos(t)| + C$

   (e) $x + 2\sqrt{x+1} + C$

   (f) $-\ln|\cos(\ln(t))| + C$

   (g) $\frac{1}{4}\arcsin^2(2x) + C$

   (h)

   (i)

   (j)

   (k)

   (l) $\frac{1}{4}(x^3+9x+1)^{\frac{4}{3}} + C$

   (m) $2e^{\sqrt{x}} + C$

   (n)

   (o)

   (p)

   (q) $2\ln|x+1|\sqrt{x+1} - 4\sqrt{x+1} + C$

   (r) $-\frac{3(x+1)}{2\sqrt[3]{x^2(x+1)}} + C$

   (s) $\sin^{-1}\big(\frac{1}{3}e^x\big) + C$

   (t) $\sqrt{x^2+4x+13} + \ln\big|x+2+\sqrt{x^2+4x+13}\big| + C$

   (u) $-\sqrt{4-x^2} + C$

   (v) $-\frac{1}{4}(6-2x)\sqrt{6x-x^2-8} + \frac{1}{2}\arcsin(x-3) + C$

   (w) $\frac{2}{3}(x-1)^{\frac{3}{2}} + 2\sqrt{x-1} + C$

   (x) $-\ln|x-\tan(x)| + C$

(y) $-\frac{\sqrt{a^2+x^2}}{a^2x}+C$

(z) $-\frac{1}{3}e^{-x^3}+c$

4. Partial Fractions:

(a) $\ln|x-2|-\ln|x+5|+C$

(b) $-\frac{3}{2}\ln|x+3|-\frac{1}{2}\ln|x+1|+2\ln|x+2|+C$

(c) $-\frac{1}{2(x-1)^2}+C$

(d) $\frac{1}{4}x^4+\frac{4}{3}x^3+6x^2+32x-\frac{32}{x-2}+80\ln|x-2|+C$

(e) $-3\ln|x|+5\ln|x-1|+\frac{3}{x}+C$

(f) $x+7\ln|x-2|-4\ln|x-1|+C$

(g) $-\frac{9}{7}\ln|x+3|-\frac{12}{7}\ln|x-4|+C$

(h) $\frac{1}{2}\ln|x^2+4x+1|+\frac{5\sqrt{3}}{3}\text{arctanh}(\frac{\sqrt{3}}{3}x+\frac{2\sqrt{3}}{3})+C$

(i) $-\frac{1}{6x+1}+\frac{1}{6}\ln|6x+1|+C$

(j) $\arctan(x)+\frac{1}{2}\ln|x^2+3|+C$

(k) $\frac{1}{2}\ln|x|-\ln|x-1|+\frac{3}{2}\ln|x+2|+C$

(l) $\frac{1}{2}x^2+2\ln|x|-2\ln|x-1|-\frac{1}{x}+C$

(m) $\frac{\tan^3(e^{2x})}{6}+C$

(n) $\ln|\tan^2(\frac{1}{2}x)+1|-2\ln|\tan(\frac{1}{2}x)+1|-\frac{2}{\tan(\frac{1}{2}x)+1}+C$

(o) $5\ln|x+3|+\frac{1}{10}\ln|5x^2+3|+\frac{1}{5}\arctan|\frac{x}{\sqrt{\frac{3}{5}}}|+C$

(p)

(q)

(r)

(s)

(t)

(u)

(v)

(w)

(x)

(y)

5. Improper Integrals:

(a)

(b)

(c)

(d)

(e)

(f)

6. Applications of Integration:

(a) *Area Between Two Curves:*

i.

ii.

iii.

(b) *Volume By Slicing:*

i.

ii.

iii.

iv.

v.

(c) *Volume By Shells:*

i.

ii.

iii.

iv.

v.

(d) *Arc Length:*

i.

ii.

(e) *Surface Area:*

i.

ii.

iii.

iv.

(f) *Mass From Density:*

i.

ii.

iii.

iv.

(g) *Center of Mass and Centroid:*

i.

ii.

iii.

iv.

v.

vi.

vii.

viii.

ix.

(h) *Fluid Pressure:*

i.

ii.

(i) *Work:*

i.

ii.

iii.

iv.

7. Parametric Curves:

(a) Express the Parametric Curve by an Equation in x and y:

i.

ii.

iii.

iv.

v.

(b) Find a Parametrization $x = (t)$, $y = y(t)$, $t \in [0, 1]$ for

i.

ii.

iii.

iv.

(c)

(d) Find the slope of the given curve at the given point and give an equation of the tangent line:

i.

ii.

(e) Find the Length of the Parametric Arc:

i.

ii.

8. Polar Coordinates:

(a) Write the Equation in Polar Coordinates:

i.

ii.

iii.

(b) Write the Equation in Cartesian Coordinates:

i.

ii.

iii.

(c) Sketch the Polar Curves:

i.

ii.

iii.

iv.

v.

vi.

(d) Calculate the Area enclosed by the Polar Curve:

i.

ii.

iii.

iv.

v.

(e) Find the Slope of the Polar Curve:

i.

ii.

iii.

(f) Find the Length of the Polar Curve:

i.

ii.