Chapter 10

Micro UAV Sensors

The objective of this chapter is to describe the on-board sensors typically used on a micro UAV and to quantify what they measure.

The following sensors may be found on a micro UAV:

- Rate gyros,
- Accelerometers,
- Pressure sensors,
- Magnetometers,
- GPS.

The following sections will discuss each of these sensors.

10.1 Rate Gyros

A MEMS rate gyro contains a small vibrating lever. When the lever undergoes an angular rotation, Coriolis effects change the frequency of the vibration, thus detecting the rotation. A brief description of the physics of rate gyros can be found at RateSensorAppNote.pdf.

The output of the rate gyro is given by

\[ y_{gyro} = k_{gyro} \Omega + \beta_{gyro} + \eta_{gyro}, \]

where \( y_{gyro} \) is in Volts, \( k_{gyro} \) is a gain, \( \Omega \) is the angular rate in radians per second, \( \beta_{gyro} \) is a bias term, and \( \eta_{gyro} \) is zero mean white noise. The gain \( k_{gyro} \) should be given on the spec sheet of the
sensor. However, due to variations in manufacturing it is imprecisely known. The bias term $\beta_{\text{gyro}}$ is strongly dependent on temperature and should be calibrated prior to each flight.

If three rate gyros are aligned along the $x$, $y$, and $z$ axes of the UAV, then the rate gyros measure the angular body rates $p$, $q$, and $r$ as follows:

$$
\begin{align*}
    y_{\text{gyro},x} &= k_{\text{gyro},x}p + \beta_{\text{gyro},x} + \eta_{\text{gyro},x} \\
    y_{\text{gyro},y} &= k_{\text{gyro},y}q + \beta_{\text{gyro},y} + \eta_{\text{gyro},y} \\
    y_{\text{gyro},z} &= k_{\text{gyro},z}r + \beta_{\text{gyro},z} + \eta_{\text{gyro},z}.
\end{align*}
$$

For simulation purposes, we may assume that the gains are identical and that the biases have been estimated and subtracted from the measurements to produce

$$
\begin{align*}
    \hat{y}_{\text{gyro},x} &= k_{\text{gyro}}p + \eta_{\text{gyro},x} \\
    \hat{y}_{\text{gyro},y} &= k_{\text{gyro}}q + \eta_{\text{gyro},y} \\
    \hat{y}_{\text{gyro},z} &= k_{\text{gyro}}r + \eta_{\text{gyro},z}.
\end{align*}
$$

MEMS gyros are analog devices that are sampled by the on-board processor. We will assume that the sample rate is given by $T_s$. The Kestrel autopilot samples the gyros at approximately 120 Hz.

### 10.2 Accelerometers

A MEMS accelerometer contains a small plate attached to torsion levers. The plate rotates under acceleration and changes the capacitance between the plate and the surrounding walls [9].

The output of the accelerometers is given by

$$
\begin{align*}
    y_{\text{acc}} &= k_{\text{acc}}A + \beta_{\text{acc}} + \eta_{\text{acc}},
\end{align*}
$$

where $y_{\text{acc}}$ is in Volts, $k_{\text{acc}}$ is a gain, $A$ is the acceleration in meters per second, $\beta_{\text{acc}}$ is a bias term, and $\eta_{\text{acc}}$ is zero mean white noise. The gain $k_{\text{acc}}$ should be given on the spec sheet of the sensor. However, due to variations in manufacturing it is imprecisely known. The bias term $\beta_{\text{acc}}$ is strongly dependent on temperature and should be calibrated prior to each flight.

Accelerometers measure the specific force in the body frame of the vehicle. A physically intuitive explanation is given in [1, p. 13-15]. Additional explanation is given in [5, p. 27]. Math-
Mathematically we have

\[
\begin{pmatrix}
a_x \\
a_y \\
a_z
\end{pmatrix} = \frac{1}{m} \left( \mathbf{F} - \mathbf{F}_{\text{gravity}} \right) = \dot{\mathbf{v}} + \mathbf{\omega} \times \mathbf{v} - \frac{1}{m} \mathbf{F}_{\text{gravity}}.
\]

In component form we have

\[
a_x = \dot{u} + qw - rv + g \sin \theta \\
a_y = \dot{v} + ru - pw - g \cos \theta \sin \phi \\
a_z = \dot{w} + pv - qu - g \cos \theta \cos \phi.
\]

The output of accelerometers is usually in units of \([g]\), therefore \(k_{\text{acc}} = 1/g\). Assuming that the biases have been removed, the output of the accelerometers is given by

\[
y_{\text{acc},x} = \frac{\dot{u} + qw - rv + g \sin \theta}{g} + \eta_{\text{acc},x} \\
y_{\text{acc},y} = \frac{\dot{v} + ru - pw - g \cos \theta \sin \phi}{g} + \eta_{\text{acc},y} \\
y_{\text{acc},z} = \frac{\dot{w} + pv - qu - g \cos \theta \cos \phi}{g} + \eta_{\text{acc},z}.
\]

From the dynamics of the UAV, we can also express the output of the accelerometers in terms of aerodynamic forces. Since

\[
(\mathbf{F} - \mathbf{F}_{\text{gravity}}) = \frac{1}{2} \rho V^2 S \left( \begin{array}{c}
C_{X_0} + C_{X_\alpha} \alpha + C_{X_q} \frac{\dot{q}}{V} + C_{X_{\delta_e}} \delta_e \\
C_{Y_0} + C_{Y_\beta} \beta + C_{Y_p} \frac{\dot{p}}{V} + C_{Y_r} \frac{\dot{r}}{V} + C_{Y_{\alpha}} \alpha + C_{Y_{\delta_a}} \delta_a + C_{Y_{\delta_r}} \delta_r \\
C_{Z_0} + C_{Z_\alpha} \alpha + C_{Z_q} \frac{\dot{q}}{V} + C_{Z_{\delta_e}} \delta_e
\end{array} \right)
+ \frac{1}{2} \rho S_{\text{prop}} \left( \begin{array}{c}
C_{X_p} [(k\delta_t)^2 - V^2] \\
0
\end{array} \right),
\]
we have

\[ y_{acc,x} = \frac{1}{2} \rho V^2 S \left( C_{X_0} + C_{X_\alpha} \alpha + C_{X_\delta_e} \delta_e \right) + \frac{1}{2} \rho \Sigma_{prop} C_{X_p} \left[ (k \delta_t)^2 - V^2 \right] + \eta_{acc,x} \]

\[ y_{acc,y} = \frac{1}{2} \rho V^2 S \left( C_{Y_0} + C_{Y_\beta} \beta + C_{Y_{bp}} \frac{bp}{2V} + C_{Y_{br}} \frac{br}{2V} + C_{Y_{\delta_a}} \delta_a + C_{Y_{\delta_r}} \delta_r \right) + \eta_{acc,y} \]

\[ y_{acc,z} = \frac{1}{2} \rho V^2 S \left( C_{Z_0} + C_{Z_\alpha} \alpha + C_{Z_q} \frac{q}{V} + C_{Z_\delta_e} \delta_e \right) + \eta_{acc,z} \]

MEMS accelerometers are analog devices that are sampled by the on-board processor. We will assume that the sample rate is given by \( T_s \). The Kestrel autopilot samples the accelerometers at approximately 120 Hz.

### 10.3 Pressure Sensors

#### 10.3.1 Altitude Sensor

Pressure is a measure of force per unit area or

\[ P = \frac{F}{A}. \]

We will be concerned with two types of pressure: (1) static pressure due to altitude, and (2) dynamic pressure due to airspeed.

The static pressure at a particular altitude is determined by the force exerted by a column of air at that altitude:

\[ P = \frac{m_{column} g}{A}, \]

where \( m_{column} \) is the mass of the column of air, \( g \) is the gravitational constant, and \( A \) is the area upon which the column is exerting pressure. The density of the is the mass per unit volume. Since the volume is given by the area times the height we get

\[ P = \rho hg, \]

where \( \rho \) is the density of air, and \( h \) is the altitude [10].

We are interested in the altitude or heights above a ground station. Suppose that at the ground stations, the pressure sensor is calibrated to read

\[ P_{\text{ground}} = \rho h_{\text{ground}} g. \]
The output of the static pressure sensor is given by

\[ y_{\text{static pres}} = \rho gh + \eta_{\text{static pres}}(t) \]
\[ = \rho g(h - h_{\text{ground}}) + \rho gh_{\text{ground}} + \eta_{\text{static pres}}(t), \]

where \( h_{\text{ground}} \) is the altitude of the ground station. We will assume that the autopilot is calibrated to determine \( h_{\text{ground}} \). Therefore, we can model the output of the static pressure sensor in the simulator as

\[ y_{\text{static pres}} = \rho g(h - h_{\text{ground}}) + \eta_{\text{static pres}}(t). \]

### 10.3.2 Air Speed Sensor

When the UAV is in motion, the atmosphere exerts dynamic pressure on the UAV parallel to the direction of flow. The dynamic pressure is given by [10]

\[ P_d = \frac{1}{2} \rho V^2, \]

where \( V \) is the airspeed of the UAV. Bernoulli’s theorem states that [10]

\[ P_s = P_d + P_O, \]

where \( P_s \) is the total pressure, and \( P_O \) is the static pressure.

Therefore, the output of the differential pressure sensor is

\[ y_{\text{diff pres}} = P_s - \eta_{\text{diff pres}} \]
\[ = \frac{1}{2} \rho V^2_{\text{air}}. \]

### 10.3.3 Density of air.

The density of air \( \rho \) is dependent on the air temperature \( T \) and air pressure \( p_s \).

The air density is given by

\[ \rho = \frac{p_s}{RT}, \]

where \( R = 287.05 \text{[J/kgK]} \) is the specific gas constant. Notice that in this formula, temperature is expressed in units of Kelvin. The conversion from Fahrenheit to Kelvin is given by

\[ T[K] = \frac{5}{9}(T[F] - 32) + 273.15. \]
Pressure is expressed in $N/m^2$. Typical weather data reports pressure in inches of Mercury. The conversion factor is

$$p_s = 3385H,$$

where $H$ is the pressure in inches of Mercury. Therefore, to get accurate measurements of altitude and airspeed, we need to know air temperature and air pressure which can be obtained from the internet.

### 10.4 Magnetometers

The earth’s magnetic field is three dimensional. There is a component that points to magnetic north, but there is also a vertical component. A three axis magnetometer measures the strength of the magnetic field along each of its axes. Let $\mathbf{m}_0$ be the magnetic field vector that is fixed in the inertial or vehicle frame. Given the orientation of the airframe, the strength of the magnetic field along the body axes of the UAV are given by

$$\mathbf{m}(t) = R_{v\rightarrow b}\mathbf{m}_0$$

$$= \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix}\mathbf{m}_0.$$

Unfortunately, when the electric motor is activated, an additional magnetic field is activated due to the rotation of the motor coils. Fortunately, the magnetic field is sinusoidal, with a frequency approximately equal to the frequency of the rotating rotor, and can be removed with a low-pass filter. We will model the effect due to the motor as

$$\mathbf{m}_{\text{motor}} = \sin(\omega_{\text{motor}} t)\mathbf{m}_{\text{motor}}$$

which is measured in the body frame.

Including bias and white noise terms, the output of the magnetometers is given by

$$\mathbf{y}_{\text{mag}} = \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix}\mathbf{m}_0$$

$$+ \sin(\omega_{\text{motor}} t)\mathbf{m}_{\text{motor}} + \begin{pmatrix} \beta_{\text{mag},x} \\ \beta_{\text{mag},y} \\ \beta_{\text{mag},z} \end{pmatrix} + \begin{pmatrix} \eta_{\text{mag},x} \\ \eta_{\text{mag},y} \\ \eta_{\text{mag},z} \end{pmatrix}.$$

The magnetometers are sampled by the microcontroller and therefore run at about 120 Hz.
<table>
<thead>
<tr>
<th>Effect</th>
<th>Ave. Horizontal Error</th>
<th>Ave. Vertical Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere</td>
<td>5.5 meters</td>
<td>5.5 meters</td>
</tr>
<tr>
<td>Satellite Geometry (Ephemeris) data</td>
<td>2.5 meters</td>
<td>15 meters</td>
</tr>
<tr>
<td>Satellite clock drift</td>
<td>1.5 meters</td>
<td>1.5 meters</td>
</tr>
<tr>
<td>Multipath</td>
<td>0.6 meters</td>
<td>0.6 meters</td>
</tr>
<tr>
<td>Measurement noise</td>
<td>0.3 meters</td>
<td>0.3 meters</td>
</tr>
</tbody>
</table>

Table 10.1: GPS Error Budget

10.5 GPS

There are several sources of GPS error. The following table lists the sources of error and the respective error budget. This data was taken from [www.montana.edu/places/gps/lres357/slides/GPSaccuracy.ppt](http://www.montana.edu/places/gps/lres357/slides/GPSaccuracy.ppt).

The current weather effects the speed of light in the atmosphere. However, this inaccuracy should be relatively constant for a given day. We will model the effect of the atmosphere by a constant random variable drawn from a Gaussian distribution with a standard deviation equal to 5 meters.

The geometry of the Satellites viewed by the receiver is used to triangulate the location of the GPS receiver. Triangulation is much more effective in the horizontal plane than in the vertical direction. The satellite geometry is slowly changing in time. Therefore we will measure the effect of satellite geometry as a sinusoid with amplitude equal to $2.5\sqrt{2}$ (RMS=2.5), with a frequency equal to $\omega_{\text{geometry}} = 10^{-6}$ and a phase that is a constant random variable drawn from a uniform distribution over $[-\pi, \pi]$.

We will assume that the clock drift is relatively constant over time. Therefore, we will model the clock drift by a constant random variable drawn from a Gaussian distribution with standard deviation of 1.5 meters.

The multipath is a function of the position of the UAV. Therefore we will assume that the error is a sinusoidal signal with a magnitude of $0.6\sqrt{2}$, a frequency equal to $\omega_{\text{multipath}} = 10^{-3}$ and a constant random phase drawn from a uniform distribution over $[-\pi, \pi]$.

We will model the measurement noise as a stochastic Gaussian process with zero mean and variance equal to 0.3 meters.
The model for GPS is therefore given by

\[ y_{GPS,N}(t) = p_N + \nu_{N,\text{atmosphere}} + \nu_{\text{clock}} + \eta_{N,\text{measurement}}(t) + 2.5\sqrt{2}\sin(\omega_{\text{geometry}}t + \nu_{N,\text{geometry}}) + 0.6\sqrt{2}\sin(\omega_{\text{multipath}}t + \nu_{N,\text{multipath}}) \]

\[ y_{GPS,E}(t) = p_E + \nu_{E,\text{atmosphere}} + \nu_{E,\text{clock}} + \eta_{E,\text{measurement}}(t) + 2.5\sqrt{2}\sin(\omega_{\text{geometry}}t + \nu_{E,\text{geometry}}) + 0.6\sqrt{2}\sin(\omega_{\text{multipath}}t + \nu_{E,\text{multipath}}) \]

\[ y_{GPS,h}(t) = h + \nu_{h,\text{atmosphere}} + \nu_{h,\text{clock}} + \eta_{h,\text{measurement}}(t) + 15\sqrt{2}\sin(\omega_{\text{geometry}}t + \nu_{h,\text{geometry}}) + 0.6\sqrt{2}\sin(\omega_{\text{multipath}}t + \nu_{h,\text{multipath}}), \]

where \( p_N, p_E, \) and \( h \) are the actual earth coordinates and altitude above sea level respectively.

The GPS receiver also computes estimated ground speed and heading from the measurements listed above. Accordingly, we have

\[ y_{GPS,\text{ground}} = \sqrt\left(\frac{y_{GPS,N}(t + T_s) - y_{GPS,N}(t)}{T_s}\right)^2 + \left(\frac{y_{GPS,E}(t + T_s) - y_{GPS,E}(t)}{T_s}\right)^2 \]

\[ y_{GPS,\text{heading}} = \tan^{-1}\left(\frac{y_{GPS,E}(t + T_s) - y_{GPS,E}(t)}{y_{GPS,N}(t + T_s) - y_{GPS,N}(t)}\right). \]

The update rate of a GPS receiver is typically on the order of \( T_s = 1 \) second. However, the update rate can vary between \( 0.5 - 2 \) seconds.