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Coordinated Control of Multisatellite Systems

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The problem of formation and reconfiguration of multiple microsatellite systems is addressed. A control scheme called the perceptive frame is adopted that integrates the decentralized feedback of each satellite with online sensor information to achieve the goal of formation keeping and intersatellite coordination. The proposed design algorithm has the advantage of relative position keeping and easy formation reconfiguration.

I. Introduction

IN recent years, the innovative idea of distributing the functionality of large satellites among smaller, cooperative satellites has been seriously considered for numerous space missions. For instance, one possible use for microsatellites is clusters of satellites flying in formation for high-resolution, synthetic-aperture imaging. In this case, groups of microsatellites are operated cooperatively to act as a sparse aperture with an effective dimension larger than can be achieved by a single, larger satellite.

A critical component in all multisatellite applications is a reliable formation keeping controller. To keep multiple satellites flying in close formation, a feedback controller is used to make the satellites track the desired trajectories. Because disturbance forces deteriorate the formation, keeping a satellite on a given orbit is not good enough. The target relative position at any given moment must be kept within an acceptable range. This requires intersatellite coordination.

Another topic specifically important to formation problems is reconfiguration. In Refs. 1–3, formations are designed based on the optimization of the imaging performance metric function. Performance metric functions are defined to meet the mission requirements. Various primary mission requirements, such as achieving the best image quality or gaining the highest probability of detecting moving targets, led to different performance metric functions. As a result, image quality is closely related to the formation baseline and the distribution of the satellites in the formation. The performance of moving target indication system depends on the number of satellites and the footprint. To meet multiple mission requirements, it is important for the formation controller to have the capability of easy reconfiguration. Furthermore, if one satellite has a malfunction in the middle of a mission, the adjustment of the satellite distribution to keep the system working, or the replacement of the satellite with malfunction, requires reconfiguration of the formation. The reconfiguration considered in this paper includes the adjustment of the relative distance between satellites, reassignment of the leader of a formation, the changing of the numbers of satellite in a formation, and the combination of two formations flying closely.

A control scheme called the perceptive frame is introduced. Controllers designed in a perceptive frame have the capability of tracking, formation keeping, and online formation and control reconfiguration. The perceptive frame approach is a method that integrates the tracking feedback law of each satellite with online sensor information. It has the advantages of both stable tracking and easy formation reconfiguration.

Formations of multiple vehicles have been studied by many researchers for various kinds of vehicles (see Refs. 4–12 and the references therein). The method developed in this paper is different from the existing centralized or decentralized controller design of multiple vehicles. It is known that a decentralized controller (which follows the master–slave strategy) has the advantage of easy reconfiguration. However, the tracking error is hard to control in the presence of disturbances. The performance is sensitive to the tracking error of the team leader. The leader vehicle cannot be removed or replaced without major control replanning. The centralized control design treats the multiple vehicles as one large dynamic with a single performance function. Controllers designed in this way usually have smaller tracking errors. However, control reconfiguration is difficult when a vehicle is added into or removed from the formation.

The developed design method combines both the advantages of accurate tracking and of easy reconfiguration. The tracking feedback law of subsystems is part of the controller. The sensor information is integrated with the tracking feedback in the perceptive frame. The final control input is determined by both the tracking feedback law designed offline and the online sensor information. While the tracking feedback drives the system approaching the desired trajectory, the online sensor information is filtered to determine the perceptive status of the formation. Then the perceptive status is fed into the tracking feedback to determine the next control input. The way of filtering the online sensor information determines how the satellites are coordinated with each other. Simple changes in the way of managing the sensor information result in the control reconfigurations such as assigning a new team leader, adding new satellites to a formation, and connecting two or more formations to form a new formation. All of these can be done without changing the tracking feedback law.

We also want to emphasize that the purpose of this paper is to demonstrate a general control architecture for the formation control of multisatellite systems. The architecture has to be used with meaningful, low-thrust formations and the fuel-efficient lower-level feedbacks.

II. Control Architecture and Its Design

Most existing work on satellite formation keeping focused on the problem of stabilizing a satellite at the desired position in a given formation. In this paper, the attention is focused on both formation keeping and coordination. A control architecture called the perceptive frame is introduced. It has the advantage of easy reconfiguration. In the following, the method in Refs. 13 and 14 is introduced without proof. The model of a complex system with multiple subsystems (vehicles) is given by the following equations:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = f_i(x_i, u_i, r_i), \qquad 1 \le i \le k$$

$$y_i = h_i(x_i) \tag{1}$$

where k is the total number of subsystems. The variable $x_i \in I\!R^{n_i}$ is the state of the ith subsystem. The function r_i represents the coupling of subsystems. It is a function of (x_j, u_j) for $j \neq i$. The input

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$u_i \in I\!R^{m_i}$ is the control variable for the ith subsystem. The output function $h_i(x_i)$ represents the performance variable. For instance, in the formation of multiple satellites, h_i is the position of the ith satellite in the space. The model of the subsystems could be different. However, the formation is defined in a common space. Thus, it is assumed that $y_i \in I\!R^p$, where p is a constant for all subsystems.

A formation is defined in a coordinate frame moving with the desired trajectory. Let $y_d(s)$ be any curve in $I\!R^p$ with parameter s. Let

$$\mathcal{F}(s) = [\boldsymbol{e}_1(s), \boldsymbol{e}_2(s), \ldots, \boldsymbol{e}_p(s)]$$

be p orthonormal vectors in $I\!R^p$, which forms a moving frame. The origin of the moving frame is $y_d(s)$. A formation consists of k points in $\mathcal{F}$, denoted by $F = \{P_1, P_2, \ldots, P_k\}$, where

$$P_i = \sum_{j=1}^{p} \alpha_{ij} \boldsymbol{e}_j$$

In general, α_{ij} is a function of the parameter s or the time t. In this section, a feedback control algorithm is developed for controllers $u_i(x)$, $i = 1, 2, \ldots, k$. The purpose of the controller is to keep y_i, $i = 1, \ldots, k$, at the position of P_i in the moving frame $\mathcal{F}$ and to drive the origin of $\mathcal{F}$ moving along the desired path $y_d(s)$. Another important issue is the coordination of the multiple vehicles. For instance, if additional vehicles are to be attached to the formation or some vehicles are to be removed from the formation, it is desired that the controller reconfiguration be simple and efficient.

A perceptive frame includes an action reference and a reference projection. The concept of an action reference represents a key parameter determined by the task of a controller. In the formation control problem, a convenient choice for action reference is s, the parameter used for the desired path $y_d(s)$. The concept of a reference projection represents a mapping that calculates the value of the action reference based on the online sensor information. How to define a reference projection to meet the coordination requirement is discussed in Sec. II.A. In a standard signal tracking approach, the tracking control law is a function of state x and time t. Because t is independent of the formation, the time-varying feedback is not robust to unexpected environmental changes. In the following, non-time-based control laws are developed in which the driving parameter is the action reference directly related to the task.

A. Controller Design in a Perceptive Frame

In the literature of complex system control, centralized and decentralized controllers have been studied by many researchers. Centralized controllers usually have better performance, but less flexibility for reconfiguration. On the other hand, most decentralized controllers use the master–slave design. It is easy to adjust the control configuration if the number of slave vehicles in the formation is changed. However, achieving accurate relative position between the vehicles is difficult. The chain instability results in larger tracking errors in the presence of disturbances. Furthermore, if the master vehicle fails to follow the desired path, the entire formation cannot be kept in the desired orbit without major replanning. In the following, the controller design in a perceptive frame is introduced for the formation of multiple vehicles.

The method introduced in this section is a different approach. The controllers designed in a perceptive frame are based on separately designed feedbacks. The vehicles are coordinated using the information of action reference computed through the reference projection. Except for the value of the action reference, the computation of the control value is decentralized. Hence, it can be easily implemented in a distributed computational environment, and the reconfiguration is easy when the number of subsystems is changed. On the other hand, the local feedbacks are coordinated through the action reference and the reference projection. The performance is smooth and the stability is proven for the entire system. The control design using a perceptive frame has the following four steps.

Step 1 is to generate the desired trajectory for each subsystem in the formation. Given a desired path $y_d(s)$, and given a formation $\{P_1, \ldots, P_k\}$ in the moving frame $\mathcal{F}$, the path for each subsystem is generated by

$$y_{di}(s) = y_d(s) + \sum_{j=1}^{p} \alpha_{ij} \boldsymbol{e}_j(s) \tag{2}$$

The action reference is the parameter s. The speed of the formation moving along $y_d(s)$ is determined by the task. It is defined by a strictly increasing function

$$s = v(t)$$

The desired trajectories in terms of time variable t are $y_{di}[v(t)]$.

Step 2 involves feedback design for subsystems. The feedback law $u_i = \alpha_i(x, t)$, $1 \le i \le k$, for each subsystem is designed separately using an existing method of path tracking. The feedback satisfies

$$\lim_{t \to \infty} \{y_i(t) - y_{di}[v(t)]\} = 0 \tag{3}$$

Furthermore, if the initial position is on the desired path, then the trajectory of the controlled system follows the path. More specifically, there exists an initial condition of the system $x_0 = (x_{01}, x_{02}, \ldots, x_{0k})^T$ such that the trajectory starting from x_0 satisfies $h_i[x(t)] = y_{di}(t)$. Denote this path by $x_{di}(s)$ or $x_{di}[v(t)]$.

How to find $\alpha_i(x, t)$ is up to the designer. Any exponentially stable feedback is applicable. The freedom of feedback selection allows the designer to select from a large family of existing design algorithms such as linear quadratic regulator (LQR) and H_∞ control. In step 2, the subsystems may adopt different feedback design algorithms. For systems with triangular coupling functions [i.e., $r_i(x)$ is independent of (x_j, u_j) for $j > i$], the design is straightforward. The feedback $u_1 = \alpha_1(x, t)$ is found first, then the second feedback $u_2 = \alpha_2(x, t)$ is found based on the known u_1, continuing the process until all of the feedback laws are found. If the coupling functions r_i, $i = 1, 2, \ldots, k$, do not have a triangular structure, the techniques of feedback decoupling for nonlinear systems could be applied first to decouple the system. Then feedback laws of the subsystems are designed for the decoupled system.

The feedbacks $\alpha_i(x, t)$ are not directly applied to the system. The final design of the controller adds another feedback loop to $\alpha_i(x, t)$. Theoretically, the control laws $u_i = \alpha_i(x, t)$ drive the system in formation along y_d because they are designed to satisfy Eq. (3). However, the feedbacks are designed separately. No coordination between the subsystems has been taken into consideration. Because of sensor and actuation errors, the controller cannot keep the formation for long periods of time. The coordination between the subsystems in the presence of disturbances cannot be achieved by these decentralized feedback laws. To improve the performance and the coordination of the controller, a projection mapping is introduced in the next step.

Step 3 involves the definition of the reference projection. The projection is a set of transformations $s = \gamma_i(x)$ for the ith subsystem satisfying

$$\gamma_i[x_d(s)] = s \tag{4}$$

for $1 \le i \le k$. The current value of x is obtained from sensor information. The value of $\gamma_i(x)$ is the perceptive position of the formation based on the sensor information. For example, given any state x_0, let $x_d(s_0)$ be the orthogonal projection from x_0 to $x_d(s)$. If we define $\gamma(x_0) = s_0$, then it satisfies Eq. (4). However, an orthogonal projection is not the only way to define γ. It is shown in Sec. II that changing the projection transformation γ fundamentally changes the way subsystems coordinated with each other. This is a very important step in the coordination controller design. Several examples of the reference projection are discussed in Secs. II.C and III.B.

Step 4 involves the construction of non-time-based control laws. The feedbacks that are used to control the system are achieved by substitutions,

$$\alpha_i(x) = \alpha_i\left\{x, v^{-1}[\gamma_i(x)]\right\} \tag{5}$$

In this formula, $v^{-1}[\gamma_i(x)]$ serves as the perceptive time based on the perceptive position $s = \gamma_i(x)$. The closed-loop system with a non-time-based feedback is

$$\dot{x}_i = f_i[x, \alpha_i(x)] \tag{6}$$

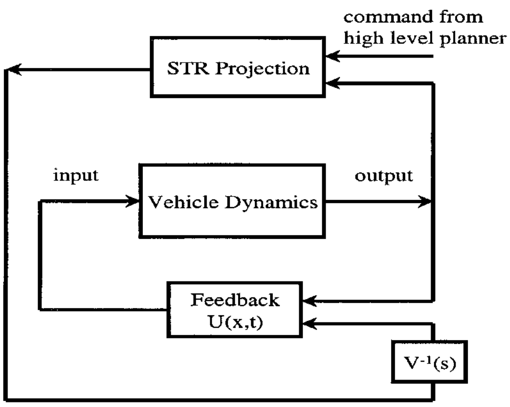

Fig. 1 Controller architecture for a subsystem.

In path tracking problems, it is preferred that the feedback be non-time based. For the purpose of coordination, it is better for all subsystem controllers to share the same value of action reference s. The value of the action reference is determined by $s = \gamma_i(x)$, which reflects the status of the formation at any given time. Different mission tasks require the designer to adopt different reference projections. Figure 1 shows the control architecture of a subsystem in a formation. The feedback $u(x, t)$ requires two pieces of information, the state and the time. The value of the time variable is filled by the perceptive time $t = v^{-1}(s)$, in which s is the perceptive position of the formation computed by the reference projection. In Fig. 1, the input of the reference projection box depends on the information required by the projection function. This is determined by the coordination or reconfiguration requirement. Some typical reference projections for satellite formations are explained in the next section.

B. Stability

In the design procedure, the time-varying feedbacks are designed so that the subsystems asymptotically approach the desired trajectory. However, the final control feedbacks are autonomous, that is, $u(x)$ is independent of the time variable. The next question to be answered is whether or not the non-time-based feedbacks drive the subsystems asymptotically to their desired trajectories.

The stability of the closed-loop system under the non-time-based feedback is proven in Refs. 13 and 14. The result is sketched in this section without proof. It is assumed that the time-varying feedback $u(x, t)$ drives the system exponentially to the desired trajectory. Assume that the derivative of the virtual time $v^{-1}[\gamma(x)]$ exponentially approaches 1. It is also assumed that all of the functions and their derivatives in this paper are bounded. The following theorem is proven in Refs. 13 and 14.

Theorem 1: There exists a neighborhood of the desired trajectory $x_d(s)$ such that $x(0)$ in the neighborhood implies that the trajectory $x(t)$ under the non-time-based feedback $u(x)$ exponentially approaches the desired trajectory.

C. Example

In the following, the idea of coordination control using perceptive frame is illustrated by an example of in-plane formation. The in-plane formation consists of a group of satellites occupying the same orbital plane and separated by a mean anomaly. Figure 2 shows an in-plane formation of four satellites in a circular orbit. The desired trajectory of each satellite is the circle

$$x_d = r_0 \cos(\omega t), \qquad y_d = r_0 \sin(\omega t) \tag{7}$$

where r_0 is the radius of the orbit and ω is the angular velocity. It is known that $\omega = \sqrt{(\mu/r_0^3)}$ and $\mu = 3.986 \times 10^{14}$ m³/s².

The coordinate system has inertially fixed x, y, and z directions. The origin of the coordinate system is the center of the Earth. The desired orbit is in the x–y plane. Following the design algorithm in Sec. II.A, the first step is to find feedbacks for each satellite to track

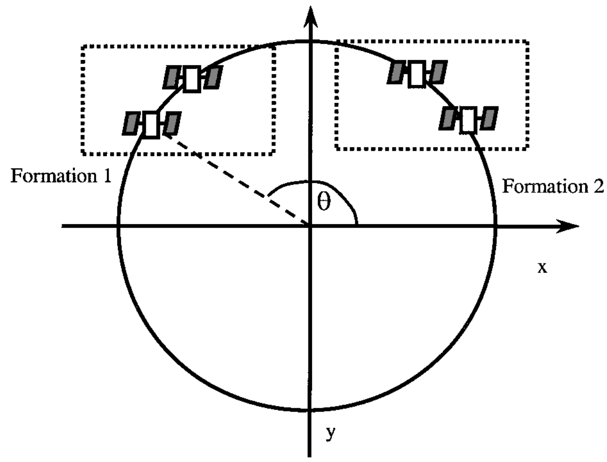

Fig. 2 In-plane formations in a circular orbit.

the ideal orbit (7). The feedback is designed based on the following satellite model in the cylindrical coordinates:

$$\ddot{r} = r\dot{\theta}^2 - \mu/r^2 + v_1, \qquad \ddot{\theta} = -(2/r)\dot{r}\dot{\theta} + (1/r)v_2$$
$$\ddot{z} = -(\mu/r^3)z + v_3 \tag{8}$$

where (r, θ, z) are the cylindrical coordinates. The variables v_1, v_2, and v_3 are thrust control inputs. The model was used in Ref. 5. The feedbacks are designed using the method of feedback decoupling and LQR. Because the perceptive frame method has no restriction on the lower-level feedbacks, it can be used with most existing control feedbacks of satellites. The following simple feedback is only used in this example for illustrative purposes. To design a practical feedback of satellite control, more restrictions such as fuel efficiency have to be considered in addition to the stability. The ΔV required for a more complicated reconfiguration in Sec. III is discussed in the last part of Sec. III.B. In the present example, we adopt the following feedback to linearize the dynamics of a satellite:

$$v_1 = -r\dot{\theta}^2 + \mu/r^2 + v_1', \qquad v_2 = r[(2/r)\dot{r}\dot{\theta} + v_2']$$
$$v_3 = (\mu/r^3)z + v_3' \tag{9}$$

The new control inputs v_1', v_2', and v_3' are given by

$$v_1' = a_1(r - r_0) + a_2\dot{r}, \qquad v_2 = b_1(\theta - \omega t) + b_2(\dot{\theta} - \omega)$$
$$v_3 = c_1 z + c_2 \dot{z} \tag{10}$$

The coefficients a_1 and a_2 are determined using the LQR method. More specifically, v_1' is the optimal feedback for the linear system $\dot{x} = Ax + Bv_1'$ with the cost function

$$J = \int_0^\infty \left([r - r_0 \quad \dot{r}]Q[r - r_0 \quad \dot{r}]^T + Rv_1^2\right) \mathrm{d}t \tag{11}$$

where $Q \geq 0$, $R > 0$, (A, B) is in the controller form of dimension two. In the simulations, Q is the 2×2 identity matrix. The coefficients b_i and c_i are determined in the same way.

To achieve the coordination between satellites, the action reference is defined. All feedbacks in the formation share the same reference information. The action reference for the circular orbit is defined to be $s = \theta$, the angle between the radius of a satellite and the x axis. On the desired path, $\theta = v(t) = \omega t$. If the satellite is not on the desired trajectory, it is necessary to calculate the reference θ based on the sensor information. The calculation uses a reference projection $\theta = \gamma_i(x, y)$. The definition of γ_i defines the relationship between the satellites. It determines the way that subsystems coordinate with each other. In the following, several reference projections are introduced. The coordination strategy associated with each projection formula is discussed. Simulations are carried out that clearly show the advantages of the controller design based on a perceptive frame.

Consider the in-plane formation of four satellites shown in Fig. 2. There are two formations in Fig. 2, and each formation consists of two satellites. The satellites are supposed to keep a constant distance from each other. The value of θ_i, $1 \le i \le 4$, is defined to be the angle between the radius of the ith satellite and the x-axis, that is,

$$\theta_i = \tan^{-1}(y_i/x_i)$$

The desired angle between the two satellites in a formation is θ_0. The feedback defined by Eqs. (9) and (10) is denoted by $u(x_i, y_i, \dot{x}_i, \dot{y}_i, t)$. The reference projection of each satellite is

$$\gamma_1 = \theta_1, \qquad \gamma_2 = \theta_1 - \theta_0, \qquad \gamma_3 = \theta_3, \qquad \gamma_4 = \theta_3 - \theta_0 \quad (12)$$

The final controller, which is time invariant, is defined by

$$\alpha_i(x_i, y_i, \dot{x}_i, \dot{y}_i) = u[x_i, y_i, \dot{x}_i, \dot{y}_i, (1/\omega)\gamma_i], \qquad i = 1, 2, 3, 4$$

Because γ_1 is a function of x_1 and y_1, the variation of x_i and y_i for $i \ge 2$ does not change the value of γ_1. Therefore, the performance of other satellites does not have any influence on the first satellite. Thus, the first satellite is a formation leader. For sat2, γ_2 is a function of θ_1. Therefore, the second satellite follows the first one. The constant angle θ_0 defines the distance between the satellites. Similarly, Eq. (12) also implies that sat3 is the team leader of the second formation in Fig. 2. Sat4 follows sat3. In this design, the leader does not fight against tracking error because its position θ_1 defines the perceptive position θ for the formation. The perceptive position defines the perceptive time $t = (1/\omega)\gamma_2$ for sat2. Only the error of sat2 relative to sat1 is corrected. Changing the leadership from sat1 to sat2 is simple. The following new reference projections assign sat2 as a leader:

$$\gamma_2 = \theta_2, \qquad \gamma_1 = \theta_2 + d_0$$

Our design method is different from the traditional decentralized design (master–slave design). In a master–slave controller, the slave has no information about the desired trajectory of the master. The feedback control is solely based on the current state of the master, no matter if the state is on the desired trajectory or not. In our design, the feedbacks of all subsystems, including both the leaders and the followers, are designed to follow the desired path of the formation. The final control force is calculated based on both the perceptive position (determined by the leader position) and the desired trajectory (including the position and the velocity determined by the mission).

Figure 3 and the associated simulation explain the advantage of our design in formation reconfiguration. For example, if the mission requires a new formation in which the first three satellites are flying closely, the challenge is to smoothly shift sat3 closer to sat2, while keeping sat4 in its original desired trajectory (Fig. 3). This can be achieved easily in our control architecture by modifying the reference projection without changing the feedback law $u(x, y, \dot{x}, \dot{y}, t)$ in Eqs. (9) and (10). The third satellite is not the leader in the new formation. It follows the first satellite. Sat4 is the new leader of its self. For this purpose, the reference projection γ_4 is defined to be

$$\gamma_4 = \theta_4 \quad (13)$$

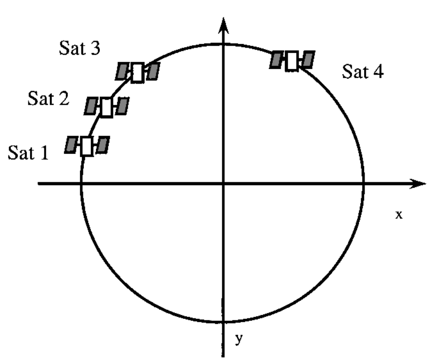

Fig. 3 New formations.

To make sat3 a member in the first formation, the new reference projection for the third satellite is

$$\gamma_3 = \theta_1 - 2\theta_0 \quad (14)$$

The term $-2\theta_0$ in the projection implies that sat3 and sat1 form an angle of $2\theta_0$. If the angle between sat2 and sat3 is important, another γ_3 can be used:

$$\gamma_3 = \theta_2 - \theta_0 \quad (15)$$

Under either Eq. (14) or (15), the satellites form the formation in Fig. 3. If Eq. (14) is used, sat3 follows sat1. The relative position between sat2 and sat3 does not have any influence on the performance. Under Eq. (15), sat3 follows sat2. The controller adjusts the position of sat3 relative to the second satellite, rather than the first one.

Two simulations are designed to show the advantage of the perceptive frame. In the first simulation, four satellites are flying in two formations as shown in Fig. 2. The reference projection is Eq. (12). At the time $t = 5$ (days), a new formation shown in Fig. 3 is to be achieved. The reference projections, γ_3 and γ_4, are switched to Eqs. (13) and (14). In Fig. 4, the relative angles between sat1 and the rest of the three satellites are shown. Thus, the curves in Fig. 4 represent $\theta_1 - \theta_2$, $\theta_1 - \theta_3$, and $\theta_1 - \theta_4$. When $t < 5$, the formation is kept by the feedback and the reference projection (12). At $t = 5$, the projections are changed to reconfigure the formation. Under the new projections (13) and (14), the third satellite is merged to the first formation, and sat4 stays in the second formation. During the transition, the leadership of the second formation is smoothly transferred from sat3 to sat4. In the simulation, $r_0 = 42{,}241$ km. In the LQR design, the constant R in Eq. (11) is $10^{17}(1/\omega)$. The closed-loop eigenvalues are $-0.3671(1 \pm \sqrt{-1}) \times 10^{-5}$. The distance between the two satellites in a formation is 1 km. Thus, $\theta_0 = 0.0013564$ deg. At the initial position, the distance between sat1 and sat3 is 5 km. The control inputs of sat3 are shown in Fig. 5. Because of the small closed-loop eigenvalues, the magnitude of the control input is limited within the range of 10^{-9} km/s^2.

In the second simulation, the four satellites fly in the formation shown in Fig. 2. The reference projection is Eq. (12). The reconfiguration of the formation started at $t = 5$ (days). The second formation in Fig. 2 is to be combined with the first formation. The new formation is shown in Fig. 6. For this purpose, the reference projection γ_3 is changed at $t = 5$. The new projection is

$$\gamma_3 = \theta_1 - 2\theta_0 \quad (16)$$

Because sat4 follows sat3, it is not necessary to change γ_4. In the transition, the leadership of sat3 is removed. It is assigned as a follower satellite. Figure 7 shows the relative angles of the satellites.

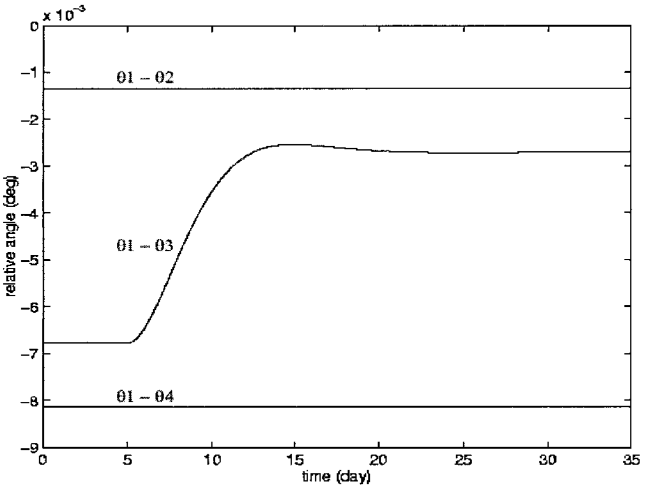

Fig. 4 Formation reconfiguration.

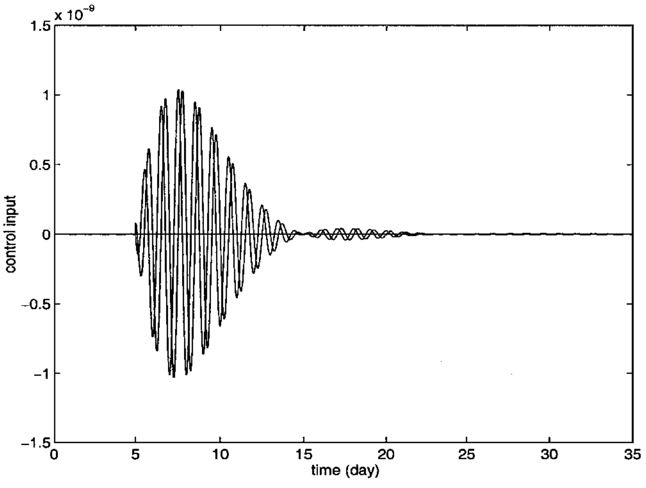

Fig. 5 Control input.

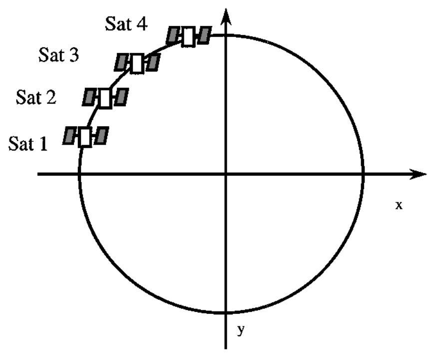

Fig. 6 New formation.

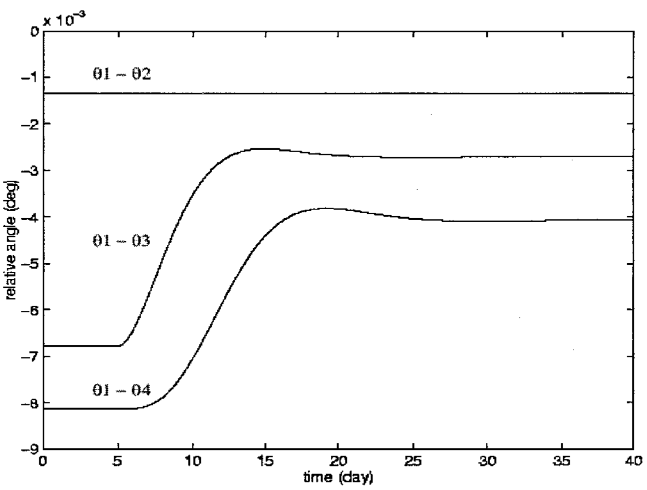

Fig. 7 Formation reconfiguration.

III. Coordination and Control of Circular Formation

In Ref. 2, satellite formations that are optimal for interferometric Earth imaging are discussed. In Ref. 15, four satellite formations are obtained for a variety of satellite applications. The desired trajectories are carefully designed natural orbits so that the energy cost to fly along these trajectories is minimized. Because of initial state errors and disturbances, local corrections in position and velocity are necessary. Feedback control is used to keep the relative positions accurate among the satellites in formation. In the following, feedbacks are designed for the circular formation (see Ref. 15).

A. Satellite Control in the Relative Frame

The circular formation is derived from Hill's equations (see, for instance, Ref. 15). The center of the formation is a satellite, called sat1, flying in a circular orbit (7). A moving frame is attached to sat1. Its x and y directions are shown in Fig. 8. The z direction is pointing upward orthogonal to both x and y axes. Under this coordinate system, the dynamic equations of a second satellite, called sat2, are

$$\ddot{x} - 2\omega\dot{y} - \omega^2(r_0 + x)\left\{1 - r_0^3\left[(r_0 + x)^2 + y^2 + z^2\right]^{-\frac{3}{2}}\right\} = v_1$$

$$\ddot{y} + 2\omega\dot{x} - \omega^2 y\left\{1 - r_0^3\left[(r_0 + x)^2 + y^2 + z^2\right]^{-\frac{3}{2}}\right\} = v_2$$

$$\ddot{z} + \omega^2 r_0^3 z\left[(r_0 + x)^2 + y^2 + z^2\right]^{-\frac{3}{2}} = v_3 \tag{17}$$

Under the moving frame, the desired trajectory of sat2 in the circular formation satisfies

$$x_d(t) = (\dot{x}_0/\omega)\sin(\omega t) + x_0\cos(\omega t)$$

$$y_d(t) = (2\dot{x}_0/\omega)\cos(\omega t) - 2x_0\sin(\omega t)$$

$$z_d(t) = \sqrt{3}(\dot{x}_0/\omega)\sin(\omega t) + \sqrt{3}x_0\cos(\omega t) \tag{18}$$

The desired path is a nonthrust trajectory for the linearized dynamics of Eq. (17) (see Ref. 15). The free variables, x_0 and $\dot{x}_0$, are the initial value of x and the initial derivative of x. In the inertially fixed frame, the path is an elliptical orbit. In the moving frame, the path determined by Eq. (18) is a circle centered at the origin (or sat1). The radius of the circle is

$$2\sqrt{\dot{x}_0^2/\omega^2 + x_0^2}$$

Furthermore, the circle is in the plane $z = \pm\sqrt{(3)}x$ (Ref. 15). A desired circular path in the plane $z = \sqrt{(3)}x$ is shown in Fig. 9. To stabilize the satellite system around the circle, a feedback is used. The first part of the feedback is the following linearization feedback:

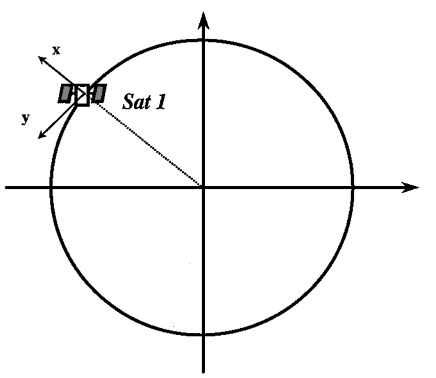

Fig. 8 Moving frame.

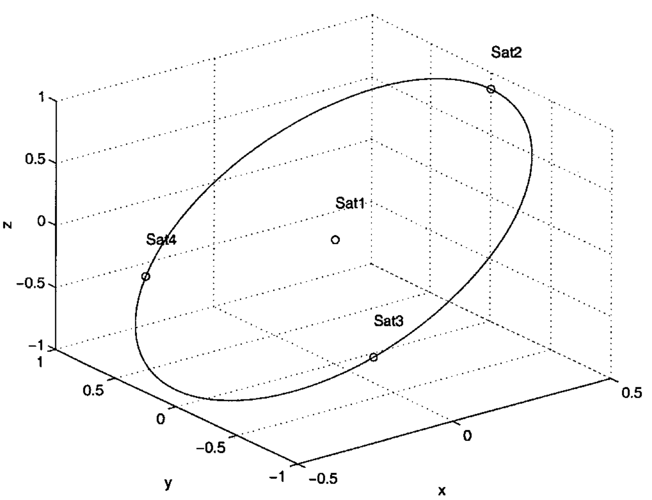

Fig. 9 Path of three satellites relative to sat1 in moving frame.

$$v_1 = -\omega^2(r_0+x)\left\{1 - r_0^3\left[(r_0+x)^2+y^2+z^2\right]^{-\frac{3}{2}}\right\} + 3\omega^2 x + v_1'$$

$$v_2 = -\omega^2 y\left\{1 - r_0^3\left[(r_0+x)^2+y^2+z^2\right]^{-\frac{3}{2}}\right\} + v_2'$$

$$v_3 = -\omega^2 z\left\{1 - r_0^3\left[(r_0+x)^2+y^2+z^2\right]^{-\frac{3}{2}}\right\} + v_3' \quad (19)$$

By substitution of it into Eq. (17), the resulting system is

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = v_1', \qquad \ddot{y} + 2\omega\dot{x} = v_2', \qquad \ddot{z} + \omega^2 z = v_3' \quad (20)$$

It is easy to check that this is the linearization of Eq. (17). It was proven in Ref. 15 that the circular path (18) satisfies Eq. (20) if $v_i' = 0$, for $i = 1, 2, 3$. The feedback for the new control variables v_i', $i = 1, 2, 3$, is a linear function

$$[v_1' \quad v_2' \quad v_3']^T = K[x - x_d \quad \dot{x} - \dot{x}_d \quad y - y_d \quad \dot{y} - \dot{y}_d \quad z - z_d \quad \dot{z} - \dot{z}_d]^T \quad (21)$$

where K is a 3×6 matrix. This matrix is calculated using LQR, which minimizes the following cost function:

$$\int_{t=0}^{\infty} \left[(x - x_d)^2 + (\dot{x} - \dot{x}_d)^2 + (y - y_d)^2 + (\dot{y} - \dot{y}_d)^2 + (z - z_d)^2 + (\dot{z} - \dot{z}_d)^2\right] + R\left(v_1'^2 + v_2'^2 + v_3'^2\right) \mathrm{d}t \quad (22)$$

where R is any constant. Larger values of R result in smaller control inputs. On the other hand, a feedback with a smaller value of R has larger feedback gain, and it takes less time to drive the system to its desired path. Because of the small control forces available on a microsatellite, the simulations are done for large values of R.

The final feedback law is the combination of Eqs. (19) and (21):

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\omega^2(r_0+x)\left\{1 - r_0^3\left[(r_0+x)^2+y^2+z^2\right]^{-\frac{3}{2}}\right\} + 3\omega^2 x \\ -\omega^2 y\left\{1 - r_0^3\left[(r_0+x)^2+y^2+z^2\right]^{-\frac{3}{2}}\right\} \\ -\omega^2 z\left\{1 - r_0^3\left[(r_0+x)^2+y^2+z^2\right]^{-\frac{3}{2}}\right\} \end{bmatrix} + K[x - x_d \quad \dot{x} - \dot{x}_d \quad y - y_d \quad \dot{y} - \dot{y}_d \quad z - z_d \quad \dot{z} - \dot{z}_d]^T \quad (23)$$

In this design, the feedback does not decouple the system. The first two equations in Eq. (20) depend on both x and y. Although the system can be decoupled using feedback, we prefer the form of Eq. (20) because the desired path is generated from Eq. (20). Therefore, the feedback Eq. (19) works with the desired path and the linearization of the dynamics, not against it.

In an inertially fixed coordinate system with the center of the Earth as the origin, the control feedback is

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{1}{r_0}\begin{bmatrix} X_0 & -Y_0 & 0 \\ Y_0 & X_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (24)$$

where (X_0, Y_0, Z_0) are the coordinates of the satellite in the inertially fixed frame. In Eq. (24), it is assumed that the orbit of sat1 is in the X–Y plane of the inertially fixed frame. The values of v_i are computed using Eq. (23).

B. Coordination and Simulation

The circular formation involves two types of feedback. The goal of the feedback for sat1, the center of the circular formation, is to keep sat1 on the circular orbit around the Earth. The feedbacks of the other satellites are designed to keep a constant distance between the follower satellites and sat1, while keeping the relative positions between sat2, sat3, and sat4 (Fig. 10) in a desired formation.

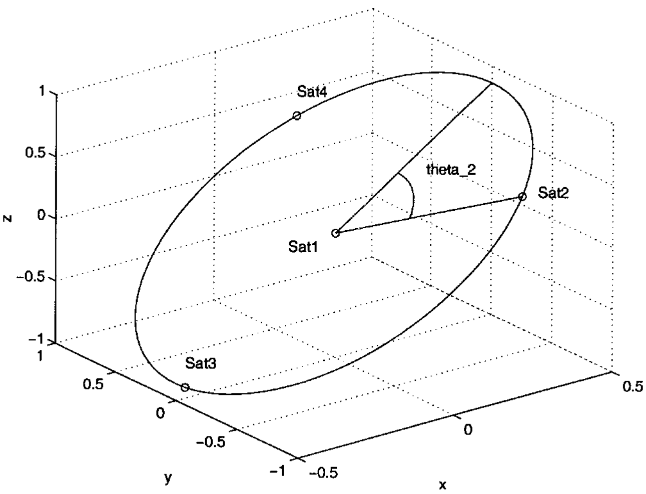

Fig. 10 Action reference θ_2.

The action reference for sat1 is defined to be θ_1, the angle between the radius of sat1 and the x axis in the inertially fixed frame. This is the same as the reference used for the in-plane formation. However, the action references (r_2, θ_2), (r_3, θ_3), and (r_4, θ_4) are defined using the moving frame. The distance from sat1 to the ith satellite is r_i. The radius is chosen as part of the action reference because the reconfiguration in this section involves transitions between different radius to change the baseline. The circular orbit in the moving frame (Fig. 10) lies in the plane $z = \sqrt{(3)}x$. The vector

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 0 \\ \sqrt{3} \end{bmatrix}$$

is in that plane, where the coordinates of $\boldsymbol{v}$ are relative to the moving frame in Fig. 8. The vector from sat1 to sat2 is $\boldsymbol{v}_2$. The action reference θ_2 is defined to be the angle between $\boldsymbol{v}$ and $\boldsymbol{v}_2$. Suppose the coordinate of sat2 in the moving frame is $[x \; y \; z]^T$, then

$$\theta_2 = \cos^{-1}\left(\frac{x + \sqrt{3}z}{2\sqrt{x^2+y^2+z^2}}\right)$$

The reference θ_2 is shown in Fig. 10. Similarly, θ_i of sat3 and sat4 are defined to be the angles between the radius of the satellite (in the moving frame) and the vector $\boldsymbol{v}$.

As it was discussed in the first section, depending on the mission requirements, several parameters of a satellite formation need reconfiguration. These parameters include baseline, satellite distribution, number of satellites, etc. During the process of reconfiguration, a team leader may have to be reassigned too. To test the capability of reconfiguration in the perceptive frame, we demonstrate a complicated reconfiguration, which is to replace a satellite in the formation. The process involves the change of several parameters in a formation, including the radius, the relative position, and the team leader.

In the simulation, the orbit of sat1 is a circle around the Earth with radius 7178.1363 km. The path of sat2 and sat3 is a circle in the moving frame (operation orbit). Its radius is 1 km. A new satellite, sat4, stays in a parking orbit that is 1.5 km from sat1 (Fig. 11). The reconfiguration is to remove sat3 from the operation orbit, then replace it by sat4 in the parking orbit. During the reconfiguration, the reference projection of sat3 and sat4 has the following form:

$$\gamma_j = \theta_i + \theta_0, \qquad r = r_0 \quad (25)$$

for $j = 3, 4$, where the values of i, θ_0, and r_0 are determined by higher-level controllers and the sensor information. How to determine i, θ_0, and r_0 is explained in the following. The entire process has three stages. During all stages, the reference projections for sat1 and sat2 are the same,

$$\gamma_1 = \theta_1, \qquad \gamma_2 = \theta_1, \qquad r_2 = 1 \quad (26)$$

In the first stage, the reference projections of sat3 and sat4 are

$$\gamma_3 = \theta_2 + \pi, \qquad r_3 = 1, \qquad \gamma_4 = \theta_2 + \pi, \qquad r_4 = 1.5 \quad (27)$$

In this projection, sat2 follows sat1 and sat3 and sat4 follow sat2 with a constant angle $\theta_0 = \pi$. By doing this, sat4 and sat3 are aligned. The second stage is to move sat3 to the parking orbit. This requires reassigning the leader of sat3 from sat2 to sat4. The new reference projection for sat1 and sat2 is the same as before. The other projections are

$$\gamma_3 = \theta_4 - \pi/6, \qquad r_3 = 1.5, \qquad \gamma_4 = \theta_2 + \pi, \qquad r_4 = 1.5 \quad (28)$$

In this projection, the action references θ and r of sat3 follow the references of sat4. To avoid collision, sat3 follows sat4 with a constant angle $\pi/6$. In this stage, sat4 is the leader of sat3. After sat3 is stabilized on the parking orbit, the process goes into its last stage. Sat4 is moved to the operation orbit to replace sat3. In this stage, sat4 cannot be the leader of sat3 anymore. Sat3 will fly in the parking orbit by itself. The new reference projection is

$$\gamma_3 = \theta_3, \qquad r_3 = 1.5, \qquad \gamma_4 = \theta_2 + \pi, \qquad r_4 = 1.0 \quad (29)$$

In this projection, sat3 follows itself. Sat4 follows sat2 with a constant angle $\theta_0 = \pi$.

Because of the multistage process, a hybrid controller is inevitable. The control architecture is shown in Fig. 12. The planner is a rule-based controller. The output of the planner is the values i, θ_0, and r_0 in the reference projection,

$$Y_j = [i \quad \theta_0 \quad r_0], \qquad j = 2, 3, 4$$

where i and θ_0 means that sat$j$ follows sat$i$ with a constant angle θ_0 and r_0 is the radius of the orbit in the relative frame. For example,

$$Y_4 = [2 \quad \pi \quad 1]$$

represents the same projection as the second equation in Eq. (29). Based on the measured values of state variables of each satellite, the planner determines the reference projection. Then the state-to-reference (STR) projection box in Fig. 12 computes the value of action reference based on the output of the planner. The STR projection box represents the reference projection. Its input is $Y_j = [i \; \theta_0 \; r_0]$. The formula used in the projection is given by Eq. (25). It covers all of the projections specified in Eqs. (27–29). The value of action reference is the output of the STR projection box. It is fed into the low-level controllers with the sensor information of the state variables. The final control input value is computed based on both s and x.

The tracking feedback $u(x, t)$ of each satellite is designed in Sec. III.A. The reference projections are defined in Eqs. (27–29). The high-level planner is a rule-based controller using the following rules. They are derived from the three reconfiguration stages explained earlier.

1) The starting time for reconfiguration is $t_1 = 1$ (day).

2) If $t \le t_1$, the coordination strategy is

$$Y_3 = [2 \quad \pi \quad 1], \qquad Y_4 = [2 \quad \pi \quad 2]$$

3) If $t > t_1$, the coordination strategy is

$$Y_3 = [4 \quad -\pi/6 \quad 2], \qquad Y_4 = [2 \quad \pi \quad 2]$$

4) Sat4 does not leave the parking orbit until sat3 is stabilized around the parking orbit.

5) If sat3 is stabilized around the parking orbit, set

$$Y_3 = [3 \quad 0 \quad 2], \qquad Y_4 = [2 \quad \pi \quad 1]$$

Simulink is used to integrate the hybrid controllers with the satellite model. The simulation is done for $t = 4$ days. The plot of $r_3 - 1$ and $r_4 - 1$, where r_i is the radius of sat$i$ in the relative frame, is shown in Fig. 13. It shows that, when $t > t_1 = 1$, sat3 moves up to the parking orbit. Sat4 stays in the parking orbit as the leader of sat3. After sat3 is stabilized, sat4 starts to move down to the

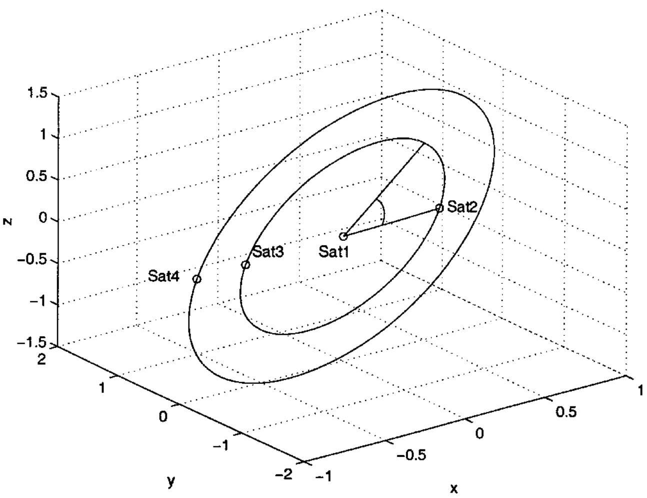

Fig. 11 Operation and parking orbits.

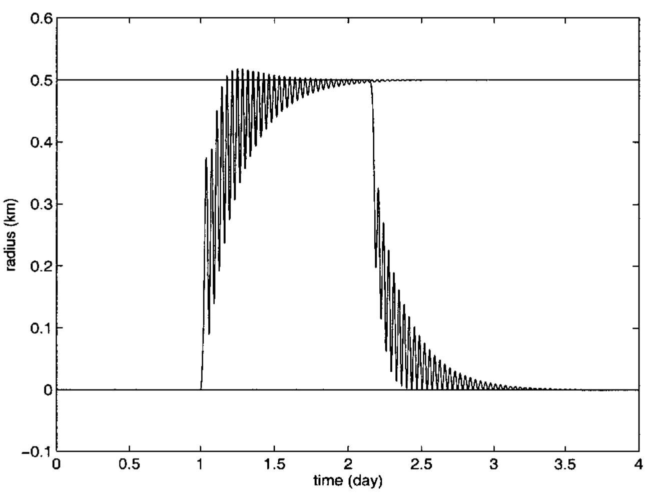

Fig. 13 Radius $r_3 - 1$ and $r_4 - 1$.

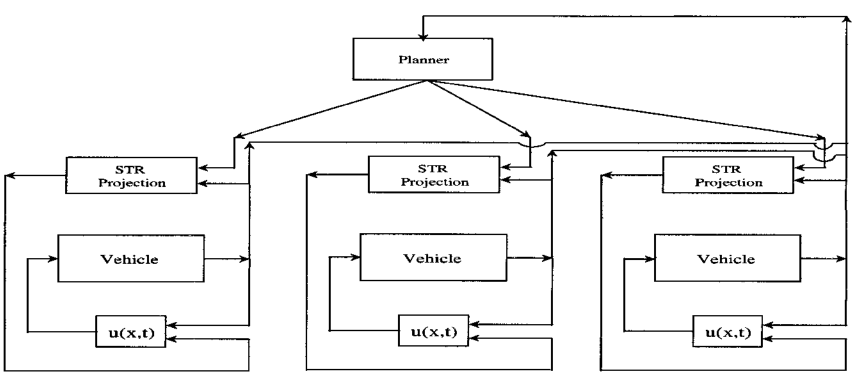

Fig. 12 Control architecture.

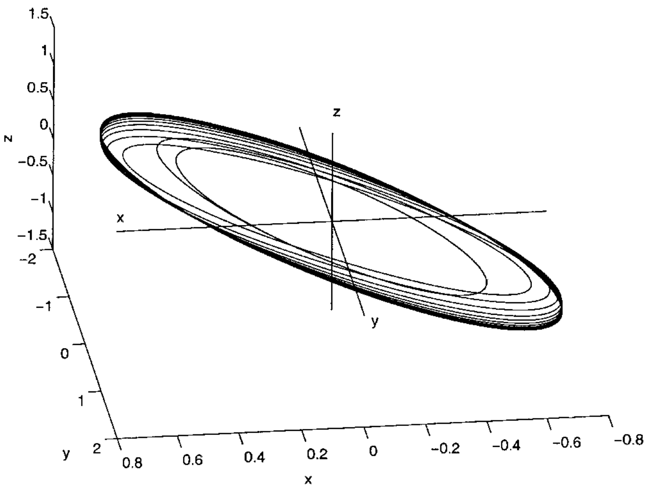

Fig. 14 Trajectory of sat3 in moving frame.

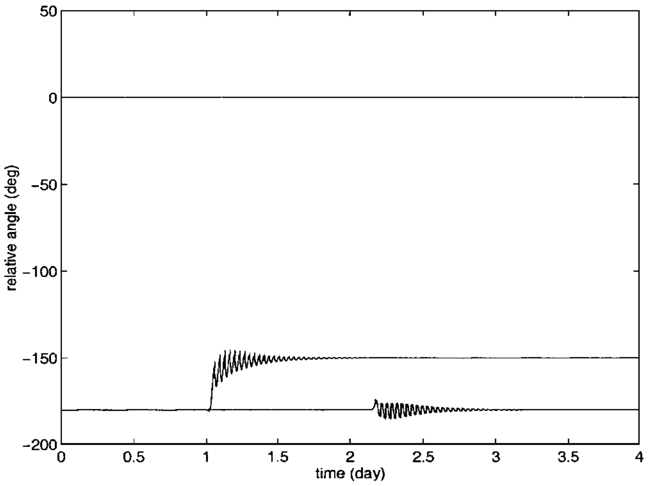

Fig. 15 Relative angles $\theta_2 - \theta_3$ and $\theta_2 - \theta_4$.

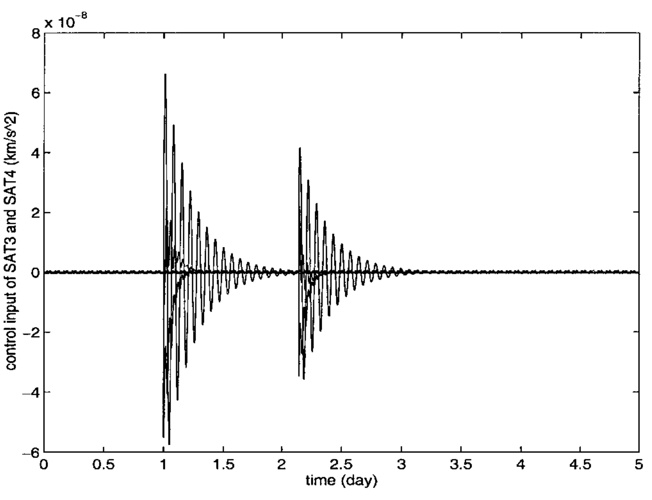

Fig. 16 Control inputs.

operation orbit. Sat3 stays on the parking orbit by itself. In Fig. 14, The transition of sat3 from the operation orbit to the parking orbit is shown in the moving frame fixed on sat1. The relative angles $\theta_2 - \theta_3$ and $\theta_2 - \theta_4$ are plotted in Fig. 15. When $t < t_1$, $\theta_3 = \theta_4$, but they have different r. When $t > t_1$, θ_3 follows θ_4 by $\theta_0 = \pi/6$. The control input of both sat3 and sat4 is shown in Fig. 16. The first peak is the control of sat3, and the second peak is the control of sat4.

The total ΔV is calculated for this simulation. The movement of sat3 costs 1.15617 m/s. The cost for sat4 is 0.74988 m/s. Considering that the reconfiguration performed in the simulation is a major one, the cost is acceptable. The perceptive frame is a higher-level coordination controller, the fuel cost is mainly due to the lower-level feedback. Because the reference projection works with different kinds of asymptotic tracking feedbacks, it gives the designer the flexibility of using different lower-level feedbacks, which is the most efficient way for the satellite to be controlled. Furthermore, the perceptive frame design has the capability of reassigning the leader in a formation online; it helps to coordinate the difference in the fuel cost among multiple satellites.

IV. Conclusions

The perceptive frame developed in this paper consists of a tracking feedback law for the satellites, the action reference, and the reference projection. The feedback law is used to determine the control input of each satellite. The action reference is chosen based on the mission. It serves as the online perceptive status of the formation for the entire system. This piece of information is shared among all subsystems, whereas the control inputs are computed separately in each satellite. The reference projection is a mapping that manages the sensor information to achieve a value for the action reference. In other words, the reference projection is the formula to compute the perceptive position of the formation. Different definitions of the projection define a different coordination strategy between the satellites in the formation. Control and formation reconfiguration is achieved by using a suitable reference projection. The reference projection is derived for reconfigurations related to leader–follower assignment, satellite distribution, formation baseline, and replacement of a satellite. Simulations are carried out to show the advantages of the design using a perceptive frame. There are many coordination strategies and reconfigurations that are not covered in this paper. However, the reference is an arbitrary function with a weak restriction. It can accommodate a large family of coordination strategies and reconfiguration requirements.

We also want to emphasize that the purpose of this paper is to demonstrate a general control architecture for the formation control of multiple vehicles. The architecture has to be used with meaningful, low-thrust formations and the fuel-efficient lower-level feedbacks. Feedback linearization in some of the examples may not be most fuel-efficient strategy.

For the future, research, control, and coordination of more complicated formations in the presence of disturbances is one of our research goals. Both continuous and discrete-time feedbacks for systems under J_2, drag force, and other disturbances will be developed. The integration of feedbacks designed under fuel or time efficiency restrictions in the perceptive frame will also be studied.

Acknowledgment

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