Motion Planning using Potential Fields

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1 Introduction

The objective of these notes is to describe a reactive approach to motion planning for mobile robots. In a reactive approach, trajectories are not planned explicitly. Rather, robot interactions are defined explicitly and the robot motion is said to "emerge." The drawback of reactive methods is that it is sometimes difficult to get the robot to do exactly what you want. The reactive approach that will be described in these notes is called the "virtual fields" method and is commonly used in robotics.

The basic idea is to set up a virtual potential or force field which is defined by the location of objects, walls, other robots, the robot of interest, etc.. The robot then responds locally to the field. Accordingly, there are two key elements:

- Definition of the force or potential field.
- Specification of the robot response to the field.

In Section 2 we will discuss several common potential fields and how they might be used in robot soccer. In addition we will discuss a possible implementation using a global state variable. In Section 3 we discuss possible robot response to potential fields.

2 Potential Fields

In this section we will describe several possible potential fields.

A potential field will be described a function $E : \mathbb{R}^2 \to \mathbb{R}$. For example the function

$$E(\mathbf{x}) = c \tag{1}$$

where c is a constant defines a constant potential field. A constant potential field is not very useful, since there is no gradient to follow. A linear potential field is can be defined by the function

$$E(\mathbf{x}) = \mathbf{a}^T\mathbf{x} + c, \tag{2}$$

where $\mathbf{a}$ is a constant vector, and c is a constant. The gradient of this potential field is given by

$$\frac{\partial E}{\partial \mathbf{x}}(\mathbf{x}) = \mathbf{a}.$$

Therefore the gradient is constant and points in the direction of $\mathbf{a}$. If the robot is programmed to follow the negative gradient of E, then a linear potential field would cause the robot to move in the direction of $\mathbf{a}$.

Another common potential field is a quadratic potential:

$$E(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{c})^T(\mathbf{x} - \mathbf{c}), \tag{3}$$

where $\mathbf{c}$ is a constant vector. A plot of constant potential lines for (3) is shown in Figure 1. The gradient of the potential field defined in (3) is

$$\frac{\partial E}{\partial \mathbf{x}}(\mathbf{x}) = \mathbf{x} - \mathbf{c},$$

which always points away from $\mathbf{c}$. If the robot is programmed to follow the negative gradient of E, the quadratic potential will cause the robot to

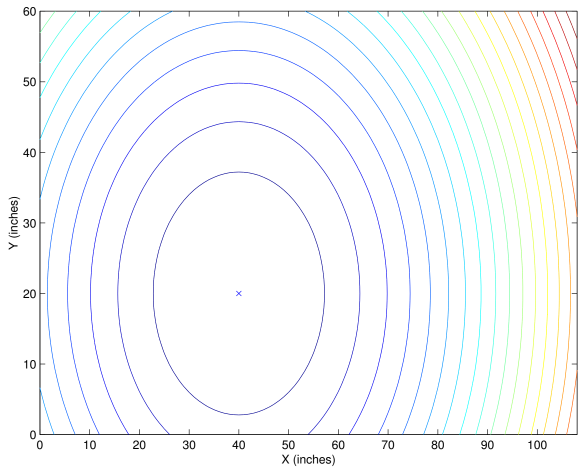

Figure 1: Constant potential lines for quadratic potential field.

move in the direction of $\mathbf{c}$. Equation (3) defines an attractive potential for $\mathbf{c}$. Accordingly, $\mathbf{c}$ is called an attractor. Alternatively, if E is defined as

$$E(\mathbf{x}) = -\frac{1}{2}(\mathbf{x}-\mathbf{c})^T(\mathbf{x}-\mathbf{c}), \tag{4}$$

then $\mathbf{c}$ is a repulsor, since following the negative gradient of E will cause the robot to move away from $\mathbf{c}$.

As an extension of (3) and (4) the following potential field might be defined:

$$E(\mathbf{x}) = \sum_{\mathbf{a}\in\mathcal{A}} \frac{1}{2}(\mathbf{x}-\mathbf{a})^T(\mathbf{x}-\mathbf{a}) - \sum_{\mathbf{r}\in\mathcal{R}} \frac{1}{2}(\mathbf{x}-\mathbf{r})^T(\mathbf{x}-\mathbf{r})$$

where $\mathcal{A}$ is a set of attractors and $\mathcal{R}$ is a set of repulsors. The gradient of E is given by

$$\begin{aligned}\frac{\partial E}{\partial \mathbf{x}}(\mathbf{x}) &= \sum_{\mathbf{a}\in\mathcal{A}}(\mathbf{x}-\mathbf{a}) - \sum_{\mathbf{r}\in\mathcal{R}}(\mathbf{x}-\mathbf{r}) \\ &= (|\mathcal{A}| - |\mathcal{R}|)\mathbf{x} - \left(\sum_{\mathbf{a}\in\mathcal{A}}\mathbf{a} - \sum_{\mathbf{r}\in\mathcal{R}}\mathbf{r}\right),\end{aligned}$$

where $|A|$ is the number of elements in $\mathcal{A}$. In other words, the robot will be attracted (or repulsed from) the center of mass of the attractors and repulsors. A potential field with five attractors and three repulsors is shown in Figure 2.

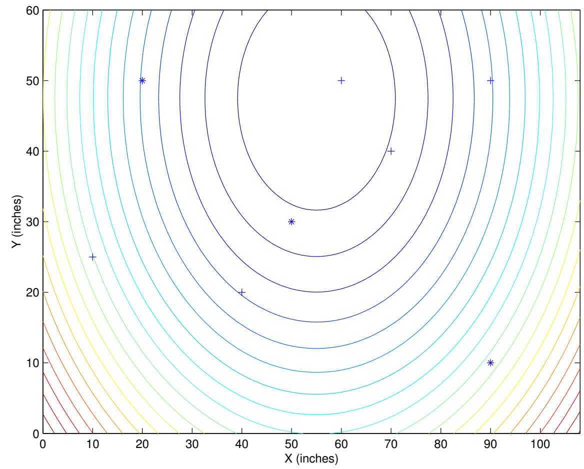

Figure 2: Quadratic potential field with five attractors (+) and three repulsors (*).

Suppose that we would like the robot to move toward near-by attractors and move away from near-by repulsors, then we could define the potential field as

$$E(\mathbf{x}) = -\sum_{\mathbf{a}\in\mathcal{A}} \alpha_a e^{-\frac{\gamma_a}{2}\|\mathbf{x}-\mathbf{a}\|^2} + \sum_{\mathbf{r}\in\mathcal{R}} \beta_r e^{-\frac{\gamma_r}{2}\|\mathbf{x}-\mathbf{r}\|^2}, \tag{5}$$

where the constants α_a, γ_a, β_r, and γ_r can be used to specify the strength of the attractor or repulsor. The gradient of (5) is given by

$$\frac{\partial E}{\partial \mathbf{x}}(\mathbf{x}) = +\sum_{\mathbf{a}\in\mathcal{A}} \alpha_a \gamma_a (\mathbf{x}-\mathbf{a}) e^{-\frac{\gamma_a}{2}\|\mathbf{x}-\mathbf{a}\|^2} - \sum_{\mathbf{r}\in\mathcal{R}} \beta_r \gamma_r (\mathbf{x}-\mathbf{r}) e^{-\frac{\gamma_r}{2}\|\mathbf{x}-\mathbf{r}\|^2}.$$

The potential field with the same attractors and repulsors as Figure 2, but with potential field (5) is shown in Figure 3

It may also be desirable to include a potential field that repulses the robot from the walls. A simple potential field that does the job is

$$E(\mathbf{r}) = \alpha e^{-\frac{\gamma}{2}(r_y - \mathrm{YMAX})^2} + \alpha e^{-\frac{\gamma}{2}r_y^2} + \alpha e^{-\frac{\gamma}{2}(r_x - \mathrm{XMAX})^2} + \alpha e^{-\frac{\gamma}{2}r_x^2}, \tag{6}$$

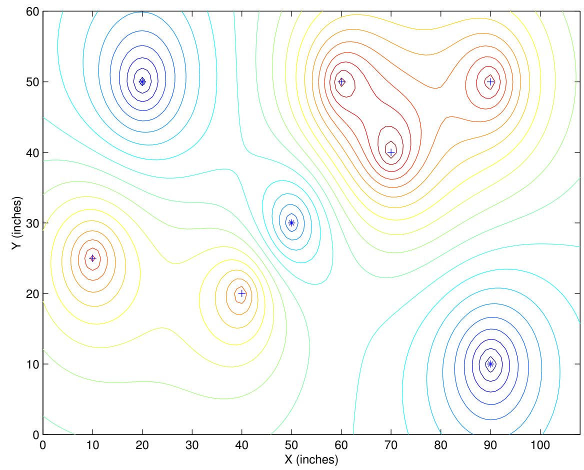

Figure 3: Exponential potential field with five attractors (+) and three repulsors (*).

with gradient given by

$$\frac{\partial E}{\partial \mathbf{r}}(\mathbf{r}) = -\gamma\alpha(r_y - \text{YMAX})e^{-\frac{\gamma}{2}(r_y-\text{YMAX})^2} - \gamma\alpha r_y e^{-\frac{\gamma}{2}r_y^2} - \gamma\alpha(r_x - \text{XMAX})e^{-\frac{\gamma}{2}(r_x-\text{XMAX})^2} - \gamma\alpha r_x e^{-\frac{\gamma}{2}r_x^2}.$$

The potential field established by (6) is shown in Figure 4.

Combining the potential fields from (5) and (6) results in the potential field shown in Figure 5.

One possible implementation would be to define a list of attractors and repulsors in the global data structure as shown in myrobot.h, and then to write a skill that computes the gradient of the potential field at the current robot location, given the current positions of the attractors and repulsors. Interesting tactics could be constructed by "guiding" the robot around the field by dynamically changing the positions of the attractors and repulsors.

3 Robot Response

The second important aspect of the potential fields method is to define the response of the robot to the potential field. In the previous section we developed the potential fields under the assumption that the robot would be

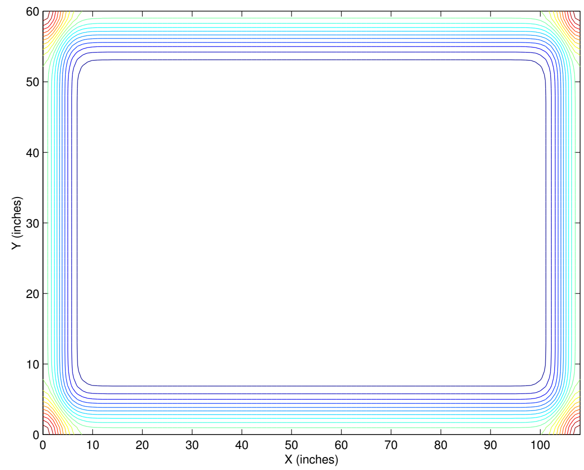

Figure 4: Potential field for avoiding the walls.

following the negative gradient of the field. Unfortunately, this is not possible to execute exactly due to the nonholonomic nature of the robot.

As an alternative one might attempt to orient the robot in the direction of the negative gradient, while simultaneously moving at velocity which is proportional to the projection of the velocity vector onto the negative gradient. The desired orientation of the robot is given by

$$\psi^d = \text{atan2}(-\frac{\partial E}{\partial r_y}, -\frac{\partial E}{\partial r_x}).$$

Therefore using a proportional control we set

$$\omega^d = -k_p(\psi - \psi^d).$$

The desired velocity is obtained by projecting the direction vector onto the negative gradient:

$$v^d = -\frac{\partial E}{\partial r_x}\cos(\psi) - \frac{\partial E}{\partial r_y}\sin(\psi).$$

If we have implemented a utility that moves the robot at linear speed v^d and angular speed ω^d, then we can invoke this utility to synthesize the desired motion.

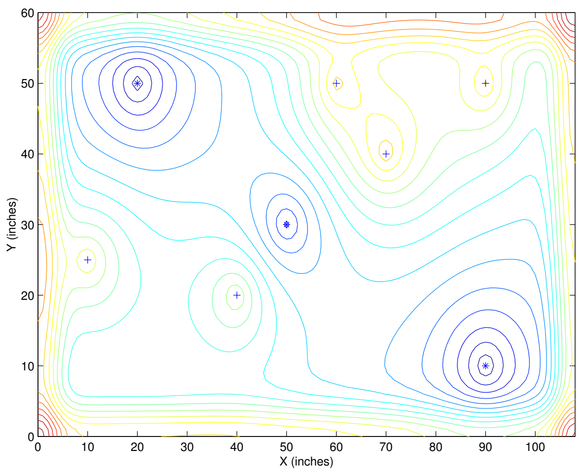

Figure 5: Potential field with attractors and repulsors and wall avoidance.