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A Control Scheme For Improving Multi-Vehicle Formation Maneuvers

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Abstract

In this paper we present a control scheme for improving multiple vehicles formation maneuvering. The advantage of this control scheme is that it makes it easy to prescribe formation maneuvers, has stability guarantees and adds a type of robustness to maneuvering formations through the use of group feedback. For moving multiple vehicles in formation the added advantages of this control scheme are explored through simulation and hardware results using wheeled mobile robots.

1 Introduction

Maneuvering multiple vehicles in formation has become an active area of research in recent years. Applications for maneuvering formations range from synthesizing a space-based interferometer [1] to searching a grid using multiple mobile robots. There are several important issues involved with maneuvering multiple vehicles in formation. In this paper we examine these issues through examples and analysis of multiple wheeled mobile robots.

Currently there are two common approaches for robots to coordinate their control strategies to move in formation. The first is leader-following [2], [3], [4]. In leader-following, one robot is designated as the leader and the remaining robots follow the leader's motion offset by a distance. Leader-following essentially reduces to a tracking problem where stability of the tracking error is shown through standard control theoretic techniques. The advantage of leader-following is that maneuvers can be specified in terms of the leader's motion. One disadvantage of leader-following is that the leader's motion is independant of the followers. Hence, if the following robots are unable to maintain a small tracking error, the leader does not slow down and formation is broken. To compensate for this apparent lack of robustness, group feedback must be added from the followers to the leader. We hereafter refer to this specific form of feedback as *formation feedback*.

The second common approach to formation keeping is the so called behavior-based approach [5], [6], [7]. Behavior-based approaches place weightings on certain actions for each robot and the group dynamics emerge. The advantage of behavior-based schemes is that the group dynamics implicitly contain formation feedback by coupling the weightings of the actions. The disadvantage of using behavior-based schemes is that the emergent group dynamics are difficult to describe–making it difficult to prescribe formation maneuvers and show stability.

In examining the two most common approaches for formation keeping, we have seen that there is a need for a control scheme which easily prescribes formation maneuvers, allows for stability guarantees, and incorporates formation feedback. A recently introduced scheme which possesses the needed attributes is the virtual structure approach [8],[9]. In this approach, the formation is treated conceptually as a virtual structure with place-holders that represent the desired position for each robot. As the virtual structure (or virtual rigid body) evolves in time, the place-holders trace out trajectories for each robot to track. In [8] a virtual structure approach for spacecraft interferometry is presented which prescribes formation maneuvers and has stability guarantees, but does not include formation feedback. In [9] a virtual structure scheme is present which includes formation feedback, but cannot guarantee a formation converges to a final configuration. The contribution of our work is to develop a virtual structure scheme which easily prescribes formation maneuvers, makes stability guarantees and includes formation feedback.

To facilitate the development of our virtual structure scheme we use the architecture shown in Figure 1. The system $\mathcal{R}_i$ represents the i^{th} robot, with control input vector $\mathbf{u}_i$, and the output vector $\mathbf{y}_i$ represents the measurable outputs of the robot. The system $\mathcal{K}_i$ represents the local controller for the i^{th} robot. The inputs to $\mathcal{K}_i$ are the output of the i^{th} robot $\mathbf{y}_i$ and the output of the coordination mechanism block $\boldsymbol{\xi}$. We will refer to $\boldsymbol{\xi}$ as the coordination variable. The outputs of $\mathcal{K}_i$ are the control vector $\mathbf{u}_i$, and the performance variable $\mathbf{z}_i$. The system $\mathcal{F}$ represents the primary coordination mechanism in our scheme. The formation control block outputs the coordination variable which is broadcast to each robot. In addition, the formation control block outputs $\mathbf{z}_F$, which is used to calculate performance of the formation. The inputs to $\mathcal{F}$ are the performance variables from each robot $\mathbf{z}_i$, and the output of the supervisor ξ_G. The system $\mathcal{G}$ is a discrete event supervisor which prescribes various formation maneuvers for the robots through $\boldsymbol{\xi}_G$. In [8] it was shown that leader-following, behavior-based, and virtual structure schemes are subsumed by this architecture. In the case of a virtual structure approach, the coordination mechanism is accomplished through the virtual structure.

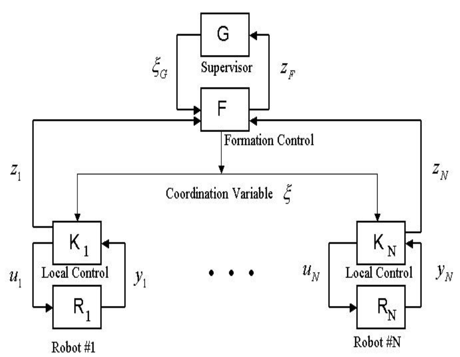

Figure 1: Architecture for Robot Maneuvers

The remainder of the paper is organized as follows. In Section 2 we describe the equations of motion for each robot. In Section 3 we describe the equations of motion for the virtual structure. In Section 4 we present stability guarantees of a control scheme which prescribes formation maneuvers for a virtual structure and includes formation feedback. Section 4 constitutes the main results of our paper. In Section 5 we present simulation and hardware results of the proposed coordination scheme. We analyze and discuss the important role formation feedback plays in maneuvering formations.

2 Robot Equations of Motion

The equations of motion for differentially driven wheeled mobile robots may be decomposed into the kinematic and dynamic equations. The kinematic equations are:

$$\dot{x} = v\cos(\theta), \tag{1}$$
$$\dot{y} = vsin(\theta), \tag{2}$$
$$\dot{\theta} = \omega, \tag{3}$$

where (x, y) is the location of the robot center, θ is its angle in the intertial frame, v is the velocity of the robot in the direction of θ and ω is the angular velocity of the robot about the axis center. The dynamic equations are :

$$m\dot{v} = F, \tag{4}$$
$$J\dot{\omega} = \tau, \tag{5}$$

where m is the mass, J is the inertia, F is force, and τ is the torque.

Two types of controls for mobile robots can be derived based upon these the equations of motion. A kinematic robot model assumes the kinematic equations describe a robot's motion with v and ω as control inputs. Many mobile robot control schemes and platforms presented in the literature assume this model. A more realistic robot model uses the kinematic and dynamic equations of motion with F and τ as the inputs.

In section 4 we present a formation control scheme for both a kinematic and a dynamic model. The kinematic equations of motion contain a nonholonomic constraint that makes even regulation difficult [10]. Some results for nonholonomic tracking tracking have been obtained, but the trajectories must satisfy certain constraints [11]. A common technique used to simplify the dynamics is feedback linearization of a point off the center of the wheel axis which we denote as $\mathbf{z}_i \triangleq (x_{hi}, y_{hi})^T$ [12],[4]. Fortunately, the zero dynamics are stable for regulation tasks, and asymptotically stable for tracking tasks if the reference trajcetory is well behaved. While feedback linearization is used in this paper to simplify the tracking laws for each robot, the virtual structure approach is general enough to allow other nonholonomic tracking controllers to be used. An off-axis point on the robot can be described by

$$\mathbf{z}_i = \begin{pmatrix} x_{hi} \\ y_{hi} \end{pmatrix} = \begin{pmatrix} x_i + L\cos(\theta_i) \\ y_i + L\sin(\theta_i) \end{pmatrix}. \tag{6}$$

Differentiating twice, we get:

$$\ddot{\mathbf{z}}_i = \begin{pmatrix} \ddot{x}_{hi} \\ \ddot{y}_{hi} \end{pmatrix} = R(\theta_i)\begin{bmatrix} \frac{F_i - F_s \text{sign}(v)}{M_i} - L\omega^2 \\ \frac{\tau L_i - rF_s \text{sign}(\omega)}{J_i} + v_i\omega_i \end{bmatrix}. \tag{7}$$

Setting F_i, and τ_i to:

$$\begin{pmatrix} \frac{F_i}{M_i} \\ \frac{\tau L_i}{J_i} \end{pmatrix} = \begin{pmatrix} L_i\omega^2 + \frac{F_s}{M_i}\text{sign}(v) \\ -v_i\omega + \frac{rF_s}{M_i}\text{sign}(\omega) \end{pmatrix} + R(-\theta_i)\begin{pmatrix} u_x \\ u_y \end{pmatrix}, \tag{8}$$

Equation (7) reduces to:

$$\ddot{\mathbf{z}}_i = \begin{pmatrix} \ddot{x}_{hi} \\ \ddot{y}_{hi} \end{pmatrix} = \begin{pmatrix} u_{xi} \\ u_{yi} \end{pmatrix} = \mathbf{u}_{zi}, \tag{9}$$

where $\mathbf{u}_{zi} \triangleq (u_{xi}, u_{yi})^T$. In this case feedback linearization reduces the equations for block $\mathcal{R}_i$ to double integrator dynamics.

3 Virtual Structure Equations of Motion

A virtual structure can be thought of as a virtual rigid body. When describing a rigid body, we characterize its motion by the center of mass. Similarly for a virtual rigid body we can characterize its motion by a virtual center of mass denoted (x'_c, y'_c). Moving the virtual center of mass translates the position of the formation. In addition to translating the virtual rigid body, we can specify an orientation for the virtual rigid body through θ'_c. The virtual center of mass and its associated orientation θ'_c become the center for a non-inertial frame of reference which we denote $O' = (x'_c, y'_c, \theta'_c)$. As a matter of notation, variables in the non-inertial frame will be primed and variables in the inertial frame will be un-primed. The rest of the virtual rigid body is fixed in the primed coordinates to the center of the non-inertial frame O'. For robot formation maneuvers, each robot is assigned a position on the virtual rigid body. As the virtual rigid body evolves in time, the desired position traces out a desired trajectory for each robot to follow. These points are denoted (x'_{id}, y'_{id}) in Cartesian coordinates and (D'_{id}, θ'_{id}) in polar coordinates. Figure 2 shows a picture of the virtual rigid body. Thus, the virtual rigid body may be parameterized as

$$\zeta(t) = (x'_c(t), y'_c(t), \theta'_c(t), \mathbf{x}'_{\mathbf{d}}(t)^T, \mathbf{y}'_{\mathbf{d}}(t)^T)^T \quad (10)$$
$$= (x'_c(t), y'_c(t), \theta'_c(t), \mathbf{D}'_{\mathbf{d}}(t)^T, \Theta'_{\mathbf{d}}(t)^T)^T, \quad (11)$$

where $\mathbf{x}'_{\mathbf{d}}(t) = (x'_{1d}(t), \ldots, x'_{Nd}(t))^T$, $\mathbf{y}'_{\mathbf{d}}(t) = (y'_{1d}(t), \ldots, y'_{Nd}(t))^T$, $\mathbf{D}'_{\mathbf{d}}(t) = (D'_{1d}(t), \ldots, D'_{Nd}(t))^T$ and $\Theta'(t) = (\theta'_{1d}(t), \ldots, \theta'_{Nd}(t))^T$. Using this parameterization, we can relate the desired position for each robot from the non-inertial frame back to the inertial frame. This yields the following equations for the i^{th} desired position of each robot denoted as $\mathbf{z}_{id}$:

$$\mathbf{z}_{id} = \begin{pmatrix} x'_c(t) + D'_{id}(t)\cos(\theta'_c(t) + \theta'_{id}) \\ y'_c(t) + D'_{id}(t)\sin(\theta'_c(t) + \theta'_{id}) \end{pmatrix}. \quad (12)$$

Note that we have allowed all of the parameters of the virtual structure, $\zeta(t)$, to vary with time except θ'_i. This parameter can vary with time, however, we are concerned with formation maneuvers which preserve the overall formation shape. Thus, varying θ'_c rotates the formation, varying $x'_c(t)$ and $y'_c(t)$ translates the formation, and multiplying $\mathbf{D}'_{id}$ by some parameterization $\eta(t)$ expands or contracts the formation. Combinations of rotations, translations and expansions/contractions will constitute the formation maneuvers considered in the remainder of this paper. The virtual structure parameters act as the coordination variable ξ, in the architecture shown in Figure 1. Hereafter, we refer to the parameters of the virtual structure ζ as the coordination variable ξ.

4 Main Result

In this section we present a control scheme for performing formation maneuvers which is easy to describe, makes stability guarantees, and includes formation feedback. To prescribe a formation maneuver we must evolve ξ to ξ^d, the final desired values of the coordination variables. We also need to guarantee that $\mathbf{z}_i$ tracks $\mathbf{z}_{id}$. This motivates the following definition.

- **A formation maneuver is asymptotically achieved** if $\xi(t) \to \xi^d$ and $\mathbf{z}_i(t) \to \mathbf{z}_{id}(t)$ as $t \to \infty$.

The formation control block $\mathcal{F}$ in Figure 1 is responsible for forcing $\xi \to \xi_d$. The local control block $\mathcal{K}_i$ in Figure 1 is responsible for guaranteeing that $\mathbf{z}_i \to \mathbf{z}_{id}$.

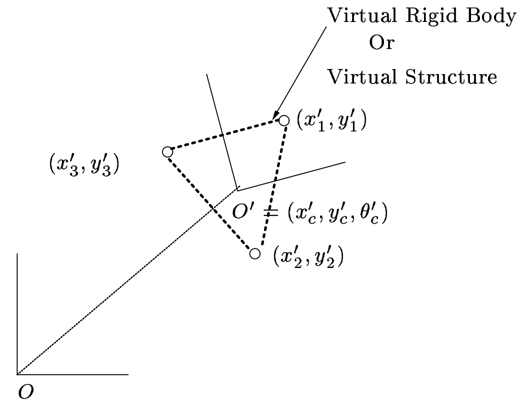

Figure 2: The virtual structure or virtual rigid body.

To motivate the dynamics for ξ we first note that our mobile robots have velocity constraints. The coordination variable has physical meaning since it is related to $\mathbf{z}_{id}$. To take into account the velocity constraints, the coordination variable needs to be bounded. One way to bound the coordination variable is to evolve it as a vector of first order systems with saturation. Such a vector of first order systems has its j^{th} component given by:

$$\dot{\xi}_j = -\mathrm{k}_1 \mathrm{K} \tanh\left(\frac{1}{\mathrm{K}}(\xi_j - \xi_j^d)\right), \quad (13)$$

where $\xi^d = (\xi_1^d, \ldots, \xi_M^d)^T$ is a constant vector. If we use Equation (13), the virtual structure evolves towards its final goal without formation feedback. In order to include formation feedback we augment Equation (13) by a nonlinear gain dependent on the quantity $\mathbf{z}_i - \mathbf{z}_{id}$. In designing this nonlinear gain we would like for the virtual structure to slow down and stop as the robots get out of formation. We would also like the virtual structure to move towards its final goal if the robots are maintaining formation. One such nonlinear gain, γ, is given by:

$$\gamma = \frac{1}{K_F \phi(\mathbf{z}_1, \ldots, \mathbf{z}_N) + \frac{1}{k_1}} = \frac{1}{K_F \phi(\tilde{\mathbf{Z}}) + \frac{1}{k_1}}, \quad (14)$$

where $\phi(\tilde{\mathbf{Z}})$ is a continuously differentiable function which measures how the robots maintain formation, K_F, k_1 are constants. To simplify the notation, we will use the variable $\mathbf{Z}$ to denote a vector of robot positions. This gives the quantities:

$$\mathbf{Z} = (\mathbf{z}_1^T, \ldots, \mathbf{z}_N^T)^T \quad (15)$$
$$\mathbf{Z}_d = (\mathbf{z}_{1d}^T, \ldots, \mathbf{z}_{Nd}^T)^T \quad (16)$$
$$\tilde{\mathbf{Z}} = \mathbf{Z} - \mathbf{Z}_d. \quad (17)$$

We may differentiate these quantities to get the velocity vectors $\dot{\mathbf{Z}}$, $\dot{\mathbf{Z}}_d$, $\dot{\tilde{\mathbf{Z}}}$ and the acceleration vectors $\ddot{\mathbf{Z}}$, $\ddot{\mathbf{Z}}_d$, $\ddot{\tilde{\mathbf{Z}}}$. Thus, $\phi(\tilde{\mathbf{Z}})$ is a function of all the robot positions and must be continuously differentiable so that our Lyapunov arguments will hold. We also require that if ϕ

is large then at least one of the $\tilde{\mathbf{z}}_i$ is large. The constant K_F weights the importance of our performance measure $\phi(\tilde{\mathbf{Z}})$. γ has the two desired properties for a nonlinear gain discussed above, i.e., $\phi \to \infty \Rightarrow \gamma \to 0$ and $\phi \to 0 \Rightarrow \gamma \to k_1$.

There are many possible candidates which measure the performance of the formation maneuver, $\phi(\tilde{\mathbf{Z}})$. One possible candidate for $\phi(\tilde{\mathbf{Z}})$ is the average tracking error of the robots given by:

$$\phi(\tilde{\mathbf{Z}}) = \frac{1}{N}\tilde{\mathbf{Z}}^T\tilde{\mathbf{Z}}. \tag{18}$$

Another possible candidate for $\phi(\tilde{\mathbf{Z}})$ is a squared error metric for formations given in [13] which may be stated mathematically as:

$$\begin{aligned}\mathrm{FE}(\tilde{\mathbf{Z}}(t), \xi(t)) = \\ \sum_{i=1}^{N}(\tilde{\mathbf{z}}_i(\xi) - \tilde{\mathbf{z}}_{i+1}(\xi))^T(\tilde{\mathbf{z}}_i(\xi) - \tilde{\mathbf{z}}_{i+1}(\xi)),\end{aligned} \tag{19}$$

where $\mathrm{FE}(\mathbf{Z}, \xi)$ is the formation error and where the indices are defined modulo N, i.e., $N+1 = 1$. Thus we may let $\phi(\tilde{\mathbf{Z}}) = FE(\tilde{\mathbf{Z}}(t), \xi(t))$.

The nonlinear gain $\gamma(\tilde{\mathbf{Z}})$ can be incorporated into the evolution of the j^{th} component of ξ in the following manner.

$$\dot{\xi}_j = -\gamma(\tilde{\mathbf{Z}})\mathrm{K}\tanh\left(\frac{1}{\mathrm{K}}(\xi_j - \xi_j^d)\right), \tag{20}$$

where ξ^d is again a vector of constants. The j^{th} component of acceleration $\ddot{\xi}_j$ is given by:

$$\begin{aligned}\ddot{\xi}_j = -\gamma(\tilde{\mathbf{Z}})\left(\mathrm{sech}^2\left(\frac{1}{\mathrm{K}}(\tilde{\xi}_j)\right)\dot{\xi}_j\right) \\ +2\gamma(\tilde{\mathbf{Z}})^2\frac{k_F}{N}\left(\dot{\phi}(\tilde{\mathbf{Z}})\right)\mathrm{K}\tanh\left(\frac{1}{\mathrm{K}}(\tilde{\xi}_j)\right),\end{aligned} \tag{21}$$

which can be shown to be continuous as long as $\phi(\tilde{\mathbf{Z}})$ is continuously differentiable.

With the vector of first order systems given in Equation (20), the virtual structure evolves towards its final goal and includes formation feedback. The control law for the robots must allow them to track their desired trajectories traced out by the virtual structure. To accomplish this, a PD control law with a feed-forward term is used for $\mathbf{z}_i$, i.e.,

$$\mathbf{u}_{zi} = \ddot{\mathbf{z}}_i = \mathbf{z}_{id} - (\mathbf{k}_{pi})\tilde{\mathbf{z}}_i - (\mathbf{k}_{vi})\dot{\tilde{\mathbf{z}}}_i, \tag{22}$$

where $\mathbf{k}_{pi} = \mathrm{diag}(k_{px}, k_{py})$ and $\mathbf{k}_{vi} = \mathrm{diag}(k_{vx}, k_{vy})$. We may collect the controls for all the robots into a single vector given by

$$(\mathbf{u}_{z1}^T, \ldots, \mathbf{u}_{zN}^T)^T = \ddot{\mathbf{Z}} = \ddot{\mathbf{Z}}^d - \mathrm{A}\tilde{\mathbf{Z}} - \mathrm{B}\dot{\tilde{\mathbf{Z}}}, \tag{23}$$

where $\mathrm{A} = \mathrm{diag}(k_{px}, k_{py}, \ldots, k_{px}, k_{py})$, and $\mathrm{B} = \mathrm{diag}(k_{vx}, k_{vy}, \ldots, k_{vx}, k_{vy})$.

Using these tracking laws on each robot and the control law for the coordination variable, we can show that formation maneuvers can be asymptotically achieved. Before proceeding we prove the following lemma which will aid in obtaining our convergence results.

Lemma 1 *If the desired location for each robot is given by Equation* (12) *then*

(1) $\dot{\xi} = \mathbf{0} \Rightarrow \dot{\mathbf{Z}}_d = \mathbf{0}$.
(2) $\dot{\xi} = \mathbf{0}$ *and* $\ddot{\xi} = \mathbf{0} \Rightarrow \ddot{\mathbf{Z}}_d = \mathbf{0}$.

Proof: Computing $\dot{\mathbf{z}}_{id}$ which constitute the components of $\dot{\mathbf{Z}}$ we get:

$$\dot{\mathbf{z}}_{id} = \begin{pmatrix}\dot{x}'_c + \dot{D}'_{id}\cos(\theta'_c + \theta'_{id}) - D'_{id}\dot{\theta}'_c\sin(\theta'_c + \theta'_{id}) \\ \dot{y}'_c + \dot{D}'_{id}\sin(\theta'_c + \theta'_{id}) + D'_{id}\dot{\theta}'_c\cos(\theta'_c + \theta'_{id})\end{pmatrix}, \tag{24}$$

Computing $\ddot{\mathbf{z}}_{id}$ which constitute the components of $\ddot{\mathbf{Z}}$ we get:

$$\ddot{\mathbf{z}}_{id} = \begin{pmatrix}\ddot{x}'_c + \ddot{D}'_{id}\cos(\theta'_c + \theta'_{id}) - 2\dot{D}'_{id}\dot{\theta}'_c\sin(\theta'_c + \theta'_{id}) \\ -D'_{id}(\dot{\theta}'_c)^2\cos(\theta'_c + \theta'_{id}) - D'_{id}\ddot{\theta}'_c\sin(\theta'_c + \theta'_{id}) \\ \ddot{y}'_c + \ddot{D}'_{id}\sin(\theta'_c + \theta'_{id}) + 2\dot{D}'_{id}\dot{\theta}'_c\cos(\theta'_c + \theta'_{id}) \\ -D'_{id}(\dot{\theta}'_c)^2\sin(\theta'_c + \theta'_{id}) + D'_{id}\ddot{\theta}'_c\cos(\theta'_c + \theta'_{id})\end{pmatrix}. \tag{25}$$

By hypothesis we have $\dot{x}'_c = 0$, $\ddot{x}'_c = 0$, $\dot{y}'_c = 0$, $\ddot{y}'_c = 0$, $\dot{\theta}'_c = 0$, $\ddot{\theta}'_c = 0$, $\dot{D}'_{id} = 0$, and $\ddot{D}'_{id} = 0$. Plugging these values into Equations (24) and (25) yields parts (1) and (2). ■

Part(1) of Lemma 1 states that if the velocity of the coordination variable is zero, then the desired velocity of each robot is zero. Part(2) of Lemma 1 states that if the coordination variable has zero velocity and acceleration, then the desired acceleration of each robot is also zero. The reason we require that both velocity and acceleration of the coordination variable be zero is that $\ddot{\mathbf{Z}}^{\mathbf{d}}$ is in the inertial frame may thus be a function of both ξ and $\dot{\xi}$ which are in the non-inertial frame. Lemma 1 will be useful in obtaining stability results.

Theorem 2 *Let* $\dot{\xi}$ *be given by Equation* (20) *and let the tracking laws for the robot be given by Equation* (23)*. Then the robot formation maneuvers are asymptotically achieved.*

Proof: Consider the Lyapunov function candidate:

$$V = \frac{1}{2}\tilde{\xi}^T\tilde{\xi} + \frac{1}{2}\tilde{\mathbf{Z}}^TA\tilde{\mathbf{Z}} + \frac{1}{2}\dot{\tilde{\mathbf{Z}}}^T\dot{\tilde{\mathbf{Z}}},$$

Differentiating we arrive at:

$$\dot{V} = -\gamma(\tilde{Z})\mathrm{K}\sum_{j=1}^{M}\tanh\left(\frac{1}{\mathrm{K}}(\xi_j - \xi_j^d)\right)(\xi_j - \xi_j^d) - \dot{\tilde{\mathbf{Z}}}^TB\dot{\tilde{\mathbf{Z}}}.$$

Let $\Sigma = \{\Omega | \dot{V} = 0\}$, where $\Omega = \mathbb{R}^M \times \mathbb{R}^{2N} \times \mathbb{R}^{2N}$ and $(\tilde{\xi}_{\mathbf{F}}^T, \tilde{\mathbf{Z}}^T, \dot{\tilde{\mathbf{Z}}}^T)^T \in \Omega$. Let $\bar{\Sigma}$ be the largest invariant set contained in Σ. On the set $\bar{\Sigma}$, $\dot{V} = 0$. When $\dot{V} = 0$ we must have $\dot{\mathbf{Z}} = \dot{\mathbf{Z}}^d$ and either $\xi = \xi^d$ or $\gamma(\tilde{\mathbf{Z}}) = 0$.

First we show that on $\bar{\Sigma}$, $\gamma(\tilde{\mathbf{Z}}) \neq 0$. Suppose that at time t_1 the robots enter $\bar{\Sigma}$ where $\gamma(\tilde{\mathbf{Z}}) = 0$. Then we have from Equations (20) and (21) that $\dot{\xi} = 0$ and

$\ddot{\xi} = 0$. Applying Lemma 1 part(2) gives $\dot{\mathbf{Z}}^d = 0$ and $\ddot{\mathbf{Z}}^d = 0$. Since $\dot{\tilde{\mathbf{Z}}} = \mathbf{0}$ on $\bar{\Sigma}$, this implies $\dot{\mathbf{Z}} = \dot{\mathbf{Z}}^d = \mathbf{0}$. In addition $\gamma(\tilde{Z}) = 0$ implies, by our requirements on $\phi(\tilde{\mathbf{Z}})$, that at least one of the $\tilde{\mathbf{z}}_i$ be large. Writing $\ddot{\mathbf{Z}}_d = \mathbf{0}$ and $\dot{\tilde{\mathbf{Z}}} = \mathbf{0}$ in component form gives $\ddot{\mathbf{z}}_{id} = \mathbf{0}$ and $\dot{\tilde{\mathbf{z}}}_i = \mathbf{0}$. Plugging in these values and a large value for $\tilde{\mathbf{z}}_i$ into the control law in Equation (22) gives $\ddot{\mathbf{z}}_i = \mathbf{0} - \mathbf{k}_{pi}(\tilde{\mathbf{z}}_\mathbf{i}) - \mathbf{k}_{vi}(\mathbf{0})$. This means that one of the $\ddot{\mathbf{z}}_i$ is a large value, so it is immediately integrated to a non zero velocity, $\dot{\mathbf{z}}_i$. Thus, we have $\dot{\mathbf{z}}_i \neq \mathbf{0}$, $\dot{\mathbf{z}}_{id} = \mathbf{0} \Rightarrow \dot{\tilde{\mathbf{z}}}_i \neq \mathbf{0} \Rightarrow \dot{\tilde{\mathbf{Z}}} \neq \mathbf{0}$ which gives that on $\bar{\Sigma}$, $\gamma(\tilde{\mathbf{Z}}) \neq 0$.

On the set $\bar{\Sigma}$ we have established that $\dot{\mathbf{Z}} = \dot{\mathbf{Z}}^d$ and $\xi = \xi^d$. From Equation (20) $\xi = \xi^d \Rightarrow \dot{\xi} = \mathbf{0}$. Using $\dot{\xi} = \mathbf{0}$ in Equation (21) implies $\ddot{\xi} = \mathbf{0}$. Combining $\dot{\xi} = \mathbf{0}$ with Lemma 1 part(1) we have $\dot{\mathbf{Z}}^d = \mathbf{0} \Rightarrow \dot{\mathbf{Z}} = \dot{\mathbf{Z}}^d = \mathbf{0}$. Additionally combining $\dot{\xi} = \mathbf{0}$ and $\ddot{\xi} = \mathbf{0}$ with Lemma 1 part(2) we have that $\ddot{\mathbf{Z}}^d = \mathbf{0}$. Plugging in $\ddot{\mathbf{Z}}^\mathbf{d} = \mathbf{0}$ and $\dot{\tilde{\mathbf{Z}}} = \mathbf{0}$ into the control law in Equation (23), we must have $\mathbf{Z} = \mathbf{Z}^d$. Therefore, by Lasalle's invariance principle $\xi \to \xi^d$ and $\mathbf{Z} \to \mathbf{Z}^d$ asymptotically as $t \to \infty$. ■

5 Simulation and Hardware Results

We present simulation and hardware results which were obtained using three robots in BYU's MAGICC lab[1]. The control scheme implemented is the one outlined in Theorem 2 with $\phi(\tilde{\mathbf{Z}})$ given by Equation (18). The control gains on the robots were chosen to be $k_p = 10$, $k_v = 16$. The simulation and hardware results illustrate some practical advantages of including formation feedback.

A sequence of formation maneuvers were run on three robots. Using a combination of elementary translations and rotations the robots were able to cover a grid. Several different gains were chosen for the coordination variable and the values for K_F were chosen to be $K_F = 0$, $K_F = 5$ and K_F=10. Simulation and hardware plots of $\mathbf{z}_i$ and $\mathbf{z}_{id}$ are shown in Figures 3, 4, 5, 6, 7 and 8 which illustrate the importance of formation feedback. The gains on the coordination variable shown in these plots were chosen to be $k_1 = 2.3$ and $K = 0.2$. In Figures 3 and 4, K_F was chosen to be zero which corresponds to no formation feedback. Both plots show that the virtual structure moves around the grid, ignoring the robots. The virtual structure does not slow down for the robots who break formation and miss portions of the grid. It took 45(sec) to perform the maneuver sequence with $K_F = 0$.

Contrast the case of no formation feedback to the case of formation feedback with $K_F = 5$ and $K_F = 10$ shown in Figures 5, 6, 7 and 8. Significantly, the virtual structure slows down as the following robots are unable to track their desired position. The robots maintain formation and cover the entire grid. It took 70(sec) to complete the sequence of maneuvers for $K_F = 5$ and it took 80(sec) to complete the sequence of maneuvers for $K_F = 10$. Although it took longer to complete the maneuver, formation feedback allowed the robots to maintain formation.

The same sequence of maneuvers was also run with slower gains on the coordination variable for the previous three values of K_F. With lower gains on the coordination variable, the robots were able to maintain formation for all three values of K_F. However, it is hard to know a priori the gains to use for ξ so that the robots maintain formation. For example in the case of no formation feedback shown in Figures 3 and 4, the robots stay in formation for the first leg of the maneuver. Thereafter, the tracking error increases and the robots break formation. To overcome this problem, gain scheduling could be used, but that would require large amounts of memory to accommodate every possible maneuver sequence for three robots. Clearly using formation feedback does not require large amounts of memory and makes the gains for ξ more robust with respect to maintaining formation.

A final advantage gained by formation feedback is that it made formation keeping more robust to synchronization issues, differences in timing among each robot, and the manufacture variability on each robot.

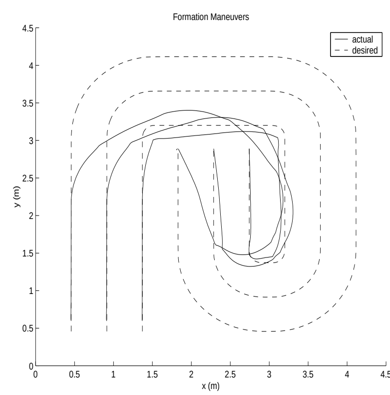

Figure 3: The (x, y) path taken in simulation by three robots with $K_F = 0$ and with no initial formation error.

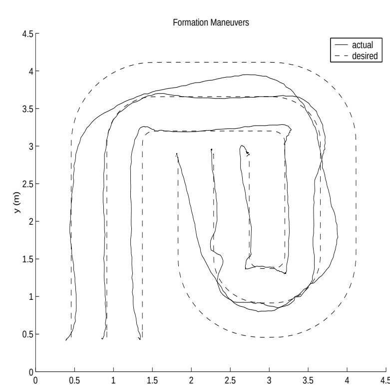

Figure 4: The (x, y) path taken in hardware by three robots with $K_F = 0$ and with no initial formation error.

[1]See [14] for a detailed description of the lab.

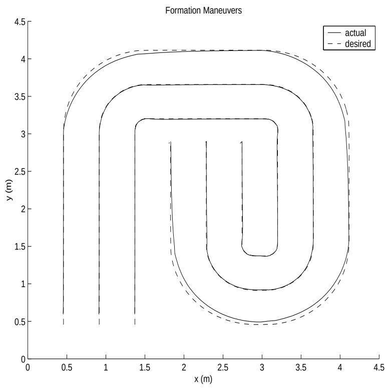

Figure 5: The (x, y) path taken in simulation by three robots with $K_F = 5$ and with no initial formation error.

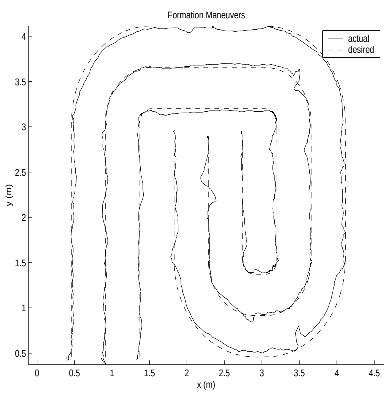

Figure 6: The (x, y) path taken in hardware by three robots with $K_F = 5$ and with no initial formation error.

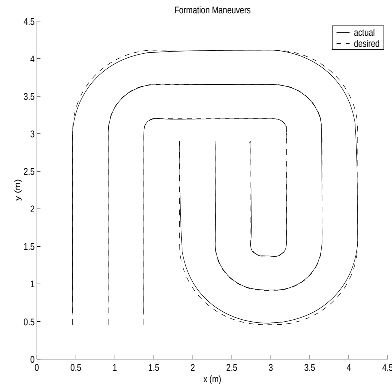

Figure 7: The (x, y) path taken in simulation by three robots with $K_F = 10$ and with no initial formation error.

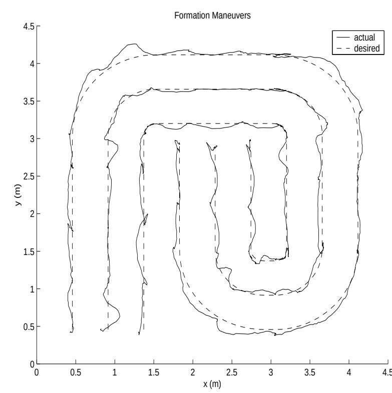

Figure 8: The (x, y) path taken in hardware by three robots with $K_F = 10$ and with no initial formation error.

6 Conclusions and Discussion

In this paper we have shown that traditional methods for keeping formation allow for some but not all of the following: easily prescribed formation maneuvers, stability guarantees, and formation feedback. We have developed a virtual structure control scheme for keeping formation which addresses all of these attributes. Simulation and hardware results have shown that formation feedback reduces formation error.

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