Hybrid Control of the Pendubot

Mingjun Zhang, *Student Member, IEEE*, and Tzyh-Jong Tarn, *Fellow, IEEE*

***Abstract*—Swing up and balance control are two interesting control problems for the Pendubot. Many studies have been conducted for swing up control of the Pendubot. A few results have been reported for feedback stabilization of the Pendubot. In this paper, we will apply a new hybrid controller for feedback stabilization of the Pendubot. To the best of the authors' knowledge, this is the first implementation of a hybrid controller for feedback stabilization of the Pendubot. Furthermore, it is well-known that it is impossible to use smooth feedback to stabilize a class of underactuated mechanical systems around their equilibra, even locally. Various nonsmooth controllers have been presented for feedback stabilization of this type of system. However, most of the studies are either based on theoretical proofs or simulations. There is a strong need for experimental study. The Pendubot arises as a special test bed for this purpose. This experimental study has particular interest for feedback stabilization of underactuated mechanical systems that are not feedback stabilizable using smooth control.**

***Index Terms*—Hybrid control, nonholonomic systems, the Pendubot, underactuated mechanical systems.**

I. Introduction

A PENDUBOT [1] is a two-link (two-degree-of-freedom) planar robot, whose first link (shoulder) is actuated and second link (elbow) is not actuated. It is a simple underactuated mechanical system (see Fig. 1).

The position shown in the figure [1] is an unstable inverted equilibrium, which is the most difficult case for feedback stabilization among all the equilibria. In order to feedback stabilize the Pendubot to this position, swing up control is usually used for moving the Pendubot close to the equilibrium manifold; then switch to a balance controller. Many studies have been conducted for swing up control of the Pendubot [2], [3]. This is not the purpose of this paper. We are interested in the balance control of the Pendubot, particularly, feedback stabilization of the Pendubot around an inverted equlibrium.

Spong and Block [4] used a linear quadratic regulator (LQR) and pole placement for the balancing and stabilizing controller. Fantoni, Lozano and Spong [3] discussed some results using energy based control by simulations. As well addressed in [3], these are the only solutions existed in the literature for balance control. No hybrid controller has been reported in the literature for feedback stabilization of the Pendubot. This is the purpose of this paper, which is the first contribution of this paper. The Pendubot also possesses some unique features and challenges for control research not found in other underactuated mechanical systems, such as control of a cart [2], control of an Acrobot [5] and position control of the underactuated robot [6].

Manuscript received September 18, 2000 revised February 28, 2001. Recommended by Technical Editor R. Isermann. This work was supported by the National Science Foundation under Grant BES-9800829.

The authors are with the Department of Systems Science and Mathematics, Washington University in St. Louis, St. Louis, MO 63130 USA (e-mail: mjz@zach.wustl.edu).

Publisher Item Identifier S 1083-4435(02)01672-1.

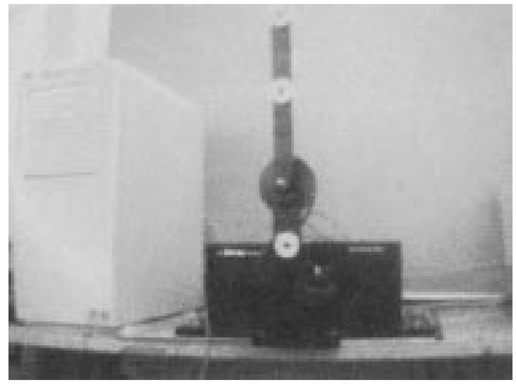


Fig. 1. Picture of the Pendubot.

For example, the Pendubot exhibits second-order nonholonomic properties, which means the dynamics of the Pendubot are subject to second-order nonintegrable differential constraints.

The systems that subject to nonintegrable differential constraints are called nonholonomic systems. It has also been shown that a class of underactuated mechanical systems can be regarded as second-order nonholonomic systems [7], [8]. Control of nonholonomic systems has been one of the most active research areas in the last few years. The difficulty is that for a class of nonholonomic systems, it is impossible to use smooth feedback to stabilize the system around an equilibrium even locally. Hybrid control has been considered as a good choice.

The hybrid controller presented in this paper is developed based on a general dynamic model of underactuated mechanical systems by extended application of the new stability theories for hybrid dynamical systems [9], [10]. The particular interest of the hybrid controller is for feedback stabilization of nonholonomic systems. As reported in [7], [8], [11], and [12], many theoretical studies have been performed for control of nonholonomic systems. However, few results have been implemented. There is a strong need for the experimental study of control of nonholonomic systems. The Pendubot arises as a special test bed for this purpose. As a result, this experimental study has special interests for feedback stabilization of nonholonomic systems. This is the second contribution of this paper.

In this paper, we will first present the dynamic model and control properties of the Pendubot. Then, we will discuss the new hybrid controller and its implementation for the Pendubot. Finally, experimental results of the hybrid controller are compared with the controller supplied by the manufacturer.

II. Dynamic Model and Control Properties of the Pendubot

The general dynamic model of underactuated mechanical systems with m actuated joints from a total of n joints can be expressed as follows [13]:

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + e(q) &= \tau \\ M(q) &= \begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix} \\ C(q,\dot{q}) &= \begin{bmatrix} C_{11}(q,\dot{q}) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q,\dot{q}) \end{bmatrix} \\ e(q) &= \begin{bmatrix} e_1(q) \\ e_2(q) \end{bmatrix} \\ \tau &= \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix} \end{aligned} \tag{1}$$

where $q = [q_1, q_2]^t \in R^n$ is the vector of joint variables. Here, $q_1 \in R^m$ represents the vector of the control links and q_2 represents the vector of the underactuated links. $M(q)$ is the $n \times n$ inertia matrix, $C(q,\dot{q})\dot{q}$ is the vector of Coriolis and centripetal torques, $e(q)$ is the gravitational term and τ_1 is the vector of control torque.

A. Dynamic Model of the Pendubot

For the dynamics of the Pendubot (see Fig. 2), define m_1 and m_2 as the mass of actuated link (called link one) and unactuated link (called link two), respectively, define q_1 and q_2 as the angles that link one and link two make with the horizontal lines, l_1 and l_2 the lengths of the two links, l_{c1} and l_{c2} the distances to the center of mass of link one and two, and I_1 and I_2 the moments of inertia of link one and link two about their centroids. It can be shown that the physical and geometrical characteristics of the plant can be described by seven dynamical parameters m_1, m_2, l_1, l_{c1}, l_{c2}, I_1, I_2. The seven dynamical parameters can be grouped into the following five new parameters for describing the dynamic of the Pendubot by a minimal set of parameters. This procedure is called reparameterization.

$$\begin{aligned} \theta_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1 \\ \theta_2 &= m_2 l_{c2}^2 + I_2 \\ \theta_3 &= m_2 l_1 l_{c2} \\ \theta_4 &= m_1 l_{c1} + m_2 l_1 \\ \theta_5 &= m_2 l_{c2}. \end{aligned}$$

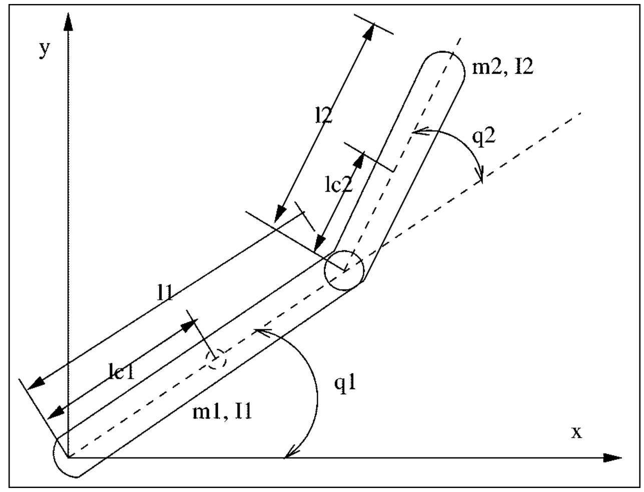


Fig. 2. Dynamics of the Pendubot.

Then, we obtain the following dynamic model of the Pendubot. See (2) at the bottom of the page.

B. Equilibrium Configuration

An equilibrium configuration is a particular value of the state and the control input for which the Pendubot is at rest, i.e, $\dot{q} = 0$ Examining the equations in (2) of the Pendubot, the equilibrium points are given by

$$\begin{aligned} \theta_4 g \cos(q_1) + \theta_5 g \cos(q_1 + q_2) &= \tau_{10} \\ \theta_5 g \cos(q_1 + q_2) &= 0. \end{aligned}$$

Suppose $\tau_{10}/\theta_4 g \leq 1$, then solving for the equilibrium configuration

$$\begin{aligned} q_1 &= \arccos\left(\frac{\tau_{10}}{\theta_4 g}\right) \\ q_2 &= n\frac{\pi}{2} - q_1; \text{ n=1,3,5,} \ldots \end{aligned}$$

which means the Pendubot will balance at a state $x = (q_1, q_2, 0, 0)$, if we apply a constant torque τ_{10}. The last two elements of the state are velocities.

We are interested in the natural equilibria of the Pendubot when $\tau_{10} = 0$. Examining the above solutions, we have the following four equilibrium configurations.

- $q_1 = -\pi/2$, $q_2 = 0$, (both link 1 and link 2 are in their lower positions).

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + e(q) &= \tau \\ M(q) &= \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3\cos(q_2) & \theta_2 + \theta_3\cos(q_2) \\ \theta_2 + \theta_3\cos(q_2) & \theta_2 \end{bmatrix} \\ C(q,\dot{q}) &= \begin{bmatrix} \theta_6 - \theta_3\sin(q_2)\dot{q}_2 & -\theta_3\sin(q_2)(\dot{q}_2 + \dot{q}_1) \\ \theta_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \\ e(q) &= \begin{bmatrix} \theta_4 g\cos(q_1) + \theta_5 g\cos(q_1 + q_2) \\ \theta_5 g\cos(q_1 + q_2) \end{bmatrix} \\ \tau &= \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix} \end{aligned} \tag{2}$$

- $q_1 = -\pi/2$, $q_2 = \pi$, (link 1 is in its lower position and link 2 is in its upper position).
- $q_1 = \pi/2$, $q_2 = 0$, (both link 1 and link 2 are in their upper positions).
- $q_1 = \pi/2$, $q_2 = \pi$, (link 1 is in its upper position and link 2 is in its lower position).

Note that only the first equilibrium point of the above four equilibria is stable. The remaining three equilibrium points are unstable. An arbitrary small disturbance causes at least one of the links to fall and consequently a large motion is produced. Furthermore, the third equilibrium configuration $q_1 = \pi/2$, $q_2 = 0$ is the most difficult case for feedback stabilization, since very small disturbances will cause both links to fall.

C. Control Properties of the Pendubot

From (2), we see the dynamics of the Pendubot are subject to a second-order differential constraint as follows:

$$(\theta_2 + \theta_3 \cos(q_2))\ddot{q}_1 + \theta_2\ddot{q}_2 + \theta_3 \sin(q_2)\dot{q}_1^2 + \theta_5 g \cos(q_1 + q_2) = 0. \quad (3)$$

The integrability of dynamic constraints is an important property for many physical systems. Depending on the integrability of their dynamic constraints, dynamic systems can be classified as either holonomic or nonholonomic. Dynamic systems that subject to nonintegrable differential constraints are called nonholonomic systems. A class of underactuated mechanical systems, such as the Pendubot, are second-order nonholonomic systems. Controllability and stabilizability of underactuated mechanical systems are closely related to this integrability property. It is well known that it is difficult if not impossible to use smooth feedback to asymptotically stabilize a class of nonholonomic systems to the equilibrium state. In this case, nonsmooth feedback stabilization must be pursued or different control objectives must be addressed [8].

In order to check whether a system is holonomic or nonholonomic, integrability of the differential constraint needs to be checked. However, many integrability conditions in the literature can not be used for this purpose. They are either coordinate dependent or have strong assumptions.

We have developed new integrability condition for classifying holonomic or nonholonomic systems using the Frobenius Theorem in differential forms [14]. The condition is coordinate independent and in general can be applied for any order of differential constraint. The condition states that a differential constraint is integrable if and only if the wedge product, $\wedge$, of the constraint and the exterior derivative of the constraint in differential form is vanishing.

To interpret the above differential form condition, we introduce the following notations and definitions.

A function $f(x,y,z)$ can be considered as a 0-form. Its exterior derivative $df(x,y,z) = f_x dx + f_y dy + f_z dz$ is called 1-form. Further exterior derivative of 1-form, such as $(f_{zy} - f_{yz})dydz + (f_{xz} - f_{zx})dzdx + (f_{yx} - f_{xy})dxdy$ is called 2-form and so on. A differential form is a k-form for some k, where k is a positive integer or zero. The Frobenius Theorem in differential forms gives necessary and sufficient condition for integrability of differential constraints and the condition is coordinate independent. Interested readers many refer [14] for detailed information. The $\wedge$-sign represents wedge product, which is an alternating multi-linear functional. The wedge product takes N-form and M-form to create an $(N+M)$-form. It is the only outer product possible given the change of sign that incurs when differentials are passed over one another. For example, the wedge product of zdx and $dx + dy$ is $(zdx) \wedge (dx + dy) = zdxdy$. The wedge product may be considered as set intersection. For example, surfaces of constant $f(x,y,z)$ and surface of constant $g(x,y,z)$ intersects along the lines given by $df \wedge dg$. The notion of interpreting the wedge product as set intersection is appealing from a topological standpoint.

Consider the second-order differential constraint (3). After simple transformation, we obtain the following differential forms

$$\begin{aligned}\omega =& (\theta_2 + \theta_3 \cos(q_2))d\dot{q}_1 + \theta_2 d\dot{q}_2 \\ &+ [\theta_3 \sin(q_2)\dot{q}_1^2 + \theta_5 g \cos(q_1 + q_2)]dt \\ d\omega =& d(\theta_2 + \theta_3 \cos(q_2)) \wedge d\dot{q}_1 \\ &+ d[\theta_3 \sin(q_2)\dot{q}_1^2 + \theta_5 g \cos(q_1 + q_2)] \wedge dt\end{aligned}$$

where ω is a differential form obtained from the original differential constraints and $d\omega$ is the exterior derivative of ω.

It is easy to check that $\omega \wedge d\omega \neq 0$. We conclude that the Pendubot is a second-order nonholonomic system.

Oriolo and Nakamura [15] have shown that the dynamic constraint of an underactuated two-link robot is holonomic if the gravity term vanishes and only the second link is controlled. If the first link is actuated, it is a second-order nonholonomic system. For the Pendubot, not only is the first link actuated, but also the gravity term is not zero. Thus, it is a second-order nonholonomic system. This observation is consistent with our conclusion using new integrability conditions in differential forms.

III. Hybrid Control for the Pendubot

In order to feedback stabilize the Pendubot around the equilibrium $q_1 = \pi/2$ and $q_2 = 0$ (both links are in their upper position), we need to move the Pendubot from its stable downward position (both links in their lower positions) to an unstable equilibrium manifold close to the inverted position. Our strategy is to use swing up control first to move the Pendubot close to the equilibrium manifold, then switch to the hybrid controller for feedback stabilization. For swing up control, we use the same technique as the manufacturer. However, the manufacturer's controller will switch to LQR for the balance control. We will first introduce the swing up controller, then the hybrid controller.

A. Swing Up Control

Moving the Pendubot from its downward position to a neighborhood of its equilibrium manifold is called swing up control. Swing up has been well studied in the literature. A good choice for swing up for the Pendubot is partial feedback linearization [1], [5].

It has been shown that the Pendubot dynamics are not feedback linearizable with static state feedback and nonlinear coordinate transformation [5], [16]. However, we may achieve a linear response from either link, but not both, by suitable nonlinear partial feedback linearization.

Same as in [1], [4], expanding (1) for one control input of total two degrees of freedom, then

$$M_{11}(q)\ddot{q}_1 + M_{12}(q)\ddot{q}_2 + C_{11}(q,\dot{q})\dot{q}_1 + C_{12}(q,\dot{q})\dot{q}_2 + e_1(q) = \tau_1 \quad (4)$$

$$M_{21}(q)\ddot{q}_1 + M_{22}(q)\ddot{q}_2 + C_{21}(q,\dot{q})\dot{q}_1 + C_{22}(q,\dot{q})\dot{q}_2 + e_2(q) = 0. \quad (5)$$

From (5), one gets

$$\ddot{q}_2 = -M_{22}^{-1}(q)\big[M_{21}(q)\ddot{q}_1 + C_{21}(q,\dot{q})\dot{q}_1 + C_{22}(q,\dot{q})\dot{q}_2 + e_2(q)\big]. \quad (6)$$

Substitute (6) into (4), one obtains

$$\bar{M}(q)\ddot{q}_1 + \bar{C}(q,\dot{q}) + \bar{e}(q) = \tau_1 \quad (7)$$

where

$$\begin{aligned}\bar{M}(q) =& M_{11}(q) - M_{12}(q)M_{22}^{-1}(q)M_{21}(q)\\ \bar{C}(q,\dot{q}) =& C_{11}(q,\dot{q})\dot{q}_1 + C_{12}(q,\dot{q})\dot{q}_2\\ &- M_{12}(q)M_{22}^{-1}(q)\left[C_{21}(q,\dot{q})\dot{q}_1 - C_{22}(q,\dot{q})\dot{q}_2\right]\\ \bar{e}(q) =& e_1(q) - M_{12}(q)M_{22}^{-1}(q)e_2(q).\end{aligned}$$

Choose the swing up control as

$$\tau_1 = \bar{M}(q)u + \bar{C}(q,\dot{q}) + \bar{e}(q)$$

then

$$\ddot{q}_1 = u. \quad (8)$$

Substitute (8) into (5), the result is

$$\ddot{q}_2 = \hat{M}(q)u + \hat{C}(q,\dot{q}) + \hat{e}(q) \quad (9)$$

where

$$\begin{aligned}\hat{M}(q) =& -M_{22}^{-1}(q)M_{21}(q)\\ \hat{C}(q,\dot{q}) =& -M_{22}^{-1}(q)\left[C_{21}(q,\dot{q})\dot{q}_1 + C_{22}(q,\dot{q})\dot{q}_2\right]\\ &- M_{22}^{-1}(q)\left[M_{21}(q)k_3\dot{q}_1 + M_{21}(q)k_4\dot{q}_2\right]\\ \hat{e}(q) =& -M_{22}^{-1}(q)\left[e_2(q) + M_{21}(q)k_1(q_1 - q_{e1})\right]\\ &- M_{22}^{-1}(q)M_{21}(q)k_2(q_2 - q_{e2}).\end{aligned}$$

The system in (1) is partially feedback linearized and u is an additional (outer loop) control input to be designed. Similar control technique is also used in [5].

Define the equilibrium point $q_e = [q_{e1}, q_{e2}]^t$, where q_{e1} is the actuated part and q_{e2} is the unactuated part. Choose the control u as

$$u = k_p(q_{e1} - q_1) - k_d\dot{q}_1. \quad (10)$$

It can be shown that if we choose $k_p > 0$ and $k_d > 0$ and suppose that the output q_1 identically tracks the equilibrium $q_{e1} = \pi/2$, then the linearized subsystem defines a globally attractive invariant manifold. The remaining nonlinear subsystem can be defined as the zero dynamics of the system with respect to the output q_1. The strategy for swing up control is to excite the zero dynamics sufficiently by the motion of link one so that the pendulum swings close to its unstable equilibrium manifold.

B. Feedback Stabilization

By extended application of the stability theory for hybrid dynamical systems [9], [10], we obtained the following hybrid control for feedback stabilization of underactuated mechanical systems in the general dynamic model as (1).

The system (1) is uniformly asymptotically stable to q_e, under the following hybrid control v, if there exists constants k_1, k_2, k_3, k_4, C, D and a positive real number $T \le 1$, such that A is nonsingular and H has all its eigenvalues within the unit circle. Here, T represents the switching time for the discrete control $u(k)$

$$\begin{aligned}\tau_1 =& \bar{M}(q)[k_1(q_1 - q_{e1}) + k_2(q_2 - q_{e2})\\ &+ k_3\dot{q}_1 + k_4\dot{q}_2 + u(k)]\\ &+ \bar{C}(q,\dot{q}) + \bar{e}(q)\\ u(k+1) =& Cu(k) + D\begin{bmatrix} q_1 - q_{e1}\\ q_2 - q_{e2}\\ \dot{q}_1\\ \dot{q}_2\end{bmatrix}\\ \bar{M}(q) =& M_{11}(q) - M_{12}(q)M_{22}^{-1}(q)M_{21}(q)\\ \bar{C}(q,\dot{q}) =& C_{11}(q,\dot{q})\dot{q}_1 + C_{12}(q,\dot{q})\dot{q}_2\\ &- M_{12}(q)M_{22}^{-1}(q)[C_{21}(q,\dot{q})\dot{q}_1\\ &- C_{22}(q,\dot{q})\dot{q}_2]\\ \bar{e}(q) =& e_1(q) - M_{12}(q)M_{22}^{-1}(q)e_2(q)\\ H =& \begin{bmatrix} e^{AT} & A^{-1}(e^{AT} - I)B\\ D & C\end{bmatrix}\\ A =& \begin{bmatrix} 0 & 0 & I_{m\times m} & 0\\ 0 & 0 & 0 & I_{(n-m)\times(n-m)}\\ k_1 & k_2 & k_3 & k_4\\ A_{41} & A_{42} & A_{43} & A_{44}\end{bmatrix}\\ B =& \begin{bmatrix} 0\\ 0\\ I_{m\times m}\\ -M_{22}^{-1}(q_e)M_{21}(q_e)\end{bmatrix}\\ A_{41} =& -M_{22}^{-1}(q_e)\left[M_{21}(q_e)k_1 + \frac{\partial e_2(q_1,q_2)}{\partial q_1}|_{q=q_e}\right]\\ A_{42} =& -M_{22}^{-1}(q_e)\left[M_{21}(q_e)k_2 + \frac{\partial e_2(q_1,q_2)}{\partial q_2}|_{q=q_e}\right]\\ A_{43} =& -M_{22}^{-1}(q_e)\left[M_{21}(q_e)k_3 + c_{21}(q_e)\right]\\ A_{44} =& -M_{22}^{-1}(q_e)\left[M_{21}(q_e)k_4 + c_{22}(q_e)\right].\end{aligned}$$

All above parameters are consistent with the model described in (1).

By investigating the above controller τ_1, we can easily conclude that it contains a continuous-time control part v_c and a discrete-time control part v_d as follows:

$$\begin{aligned}v_c =& \bar{M}(q)[k_1(q_1 - q_{e1}) + k_2(q_2 - q_{e2})\\ &+ k_3\dot{q}_1 + k_4\dot{q}_2] + \bar{C}(q,\dot{q}) + \bar{e}(q)\\ v_d =& \bar{M}(q)u(k).\end{aligned}$$

As shown in the block diagram Fig. 3, this is a hybrid controller. The basic idea of the two parts of this hybrid controller is that the continuous-time control part depends on continuous-time state information and the discrete-time control part changes values at fixed time interval. The values are determined by both

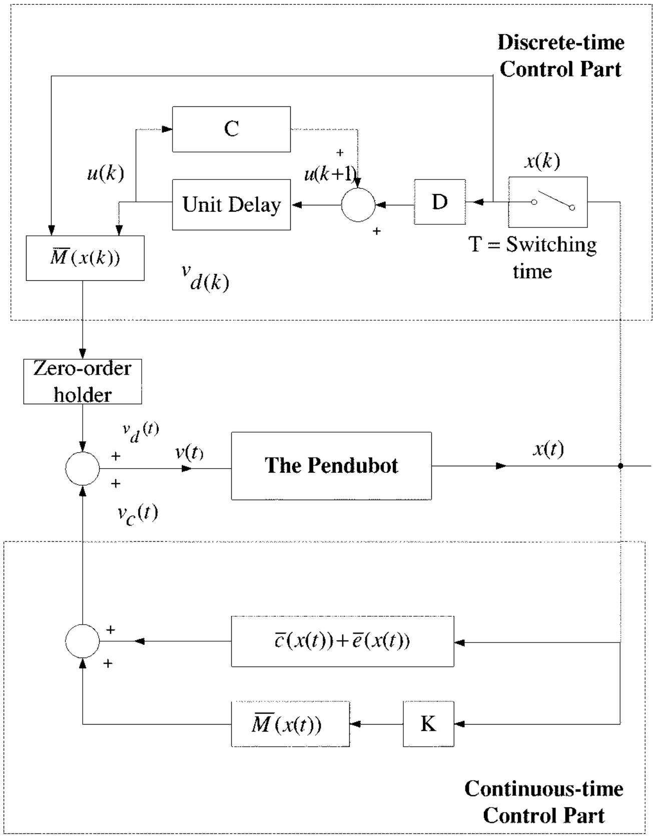


Fig. 3. Block diagram for the hybrid controller.

the previous discrete-time control part and the state information. Whenever the discrete-time control part switches to a new value, the discrete-time control part will always keep part of the previous control information. The hybrid controller design easily calculates matrices to satisfy proper conditions. The continuous-time control part contributes for partial feedback linearization and the discrete-time control part can be regarded as cancellation of the drift terms. Once the Pendubot reaches the unstable equilibrium manifold via swing up control, the controller will switch to the new hybrid controller for asymptotically stabilizing the system to the equilibrium state.

IV. Implementation Results for the New Hybrid Control

The proposed swing up and hybrid controllers have been implemented for control of the Pendubot.

- For our implementation, the parameters of the Pendubot from the manufacturer's user manual are identified as follows [1]:

$$\begin{aligned}\theta_1 &= 0.0308 \text{ vs}^2\\ \theta_2 &= 0.0106 \text{ vs}^2\\ \theta_3 &= 0.0095 \text{ vs}^2\\ \theta_4 &= 0.2086 \text{ vs}^2/\text{m}\\ \theta_5 &= 0.0630 \text{ vs}^2/\text{m}.\end{aligned} \tag{11}$$

  For future discussion, we call this Model One. The units in the above parameters follow the International System for Units.

- For the purpose of robustness comparison, we also implemented the controllers for the following model with varied parameters (called Model Two).

$$\begin{aligned}\theta_1 &= 0.0260 \text{ vs}^2\\ \theta_2 &= 0.0119 \text{ vs}^2\\ \theta_3 &= 0.0098 \text{ vs}^2\\ \theta_4 &= 0.1673 \text{ vs}^2/\text{m}\\ \theta_5 &= 0.0643 \text{ vs}^2/\text{m}.\end{aligned} \tag{12}$$

  For implementing the swing up control of the Pendubot, we choose the following parameter values.
- For the Model One, we choose $k_p = 50/\text{s}^2$ and $k_d = 8.8/\text{s}$.
- For the Model Two, we choose $k_p = 53/\text{s}^2$ and $k_d = 8.62/\text{s}$.

Our particular interest is in the balance control. We compared the experimental results of the hybrid controller with the controller supplied by the manufacturer, which is the only controller in the literature implemented for feedback stabilization of the Pendubot.

We have implemented the hybrid control algorithm for both the Model One and the Model Two. One may use MAPLE or MATLAB to conduct the computation and design the hybrid control parameters for the balance control and asymptotically stabilizing the system to the equilibrium state.

- For implementing the balance control of the Model One, we choose

$$\begin{aligned}k_1 &= 2884.2, & k_2 &= 2192.0\\ k_3 &= 479.5, & k_4 &= 282.9\\ d_1 &= 0.2, & d_2 &= 0.3\\ d_3 &= 0.1, & d_4 &= 0.2\\ c &= 0.4, & T &= 0.1\end{aligned}$$

- For implementing the balance control of the Model Two, we choose

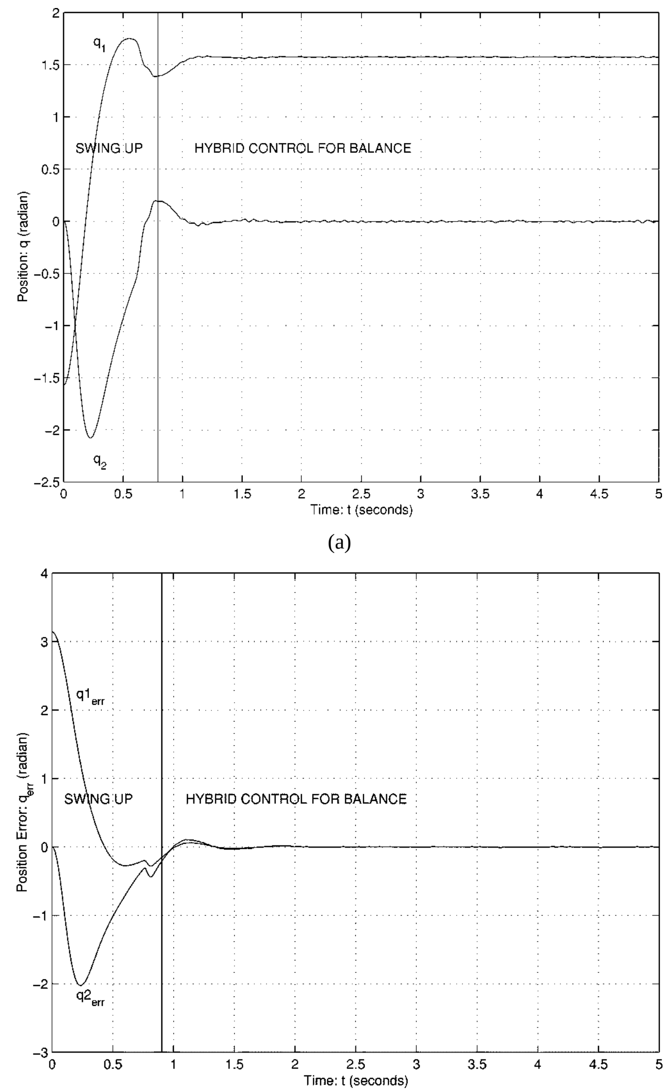
$$\begin{aligned}k_1 &= 3224.5, & k_2 &= 2463.0\\ k_2 &= 547.9, & k_4 &= 331.7\\ d_1 &= 0.2, & d_2 &= 0.3\\ d_3 &= 0.1, & d_4 &= 0.2\\ c &= 0.4, & T &= 0.1.\end{aligned}$$

For the purpose of comparison, some external disturbances were added randomly by lightly hitting the links using a metal stick to test the robustness of the algorithm. Trajectories for both link one and link two are given. The interesting fact is that the manufacturer's balance controller does not work for the model with variation. Our hybrid controller still works well. Please see the following various test cases for detailed information.

A. Control Based on Model One

The following cases have been performed using hybrid control for Model One(as supplied by the manufacturer).

- Without disturbances: Fig. 4(a) and (b) show the positions and position errors of the two links.
- With randomly added quick disturbances: Fig. 5 shows the positions of the two links.



(a)

(b)

Fig. 4. Without disturbances.

- With randomly added slow disturbances for visually estimating the region of attraction: Fig. 6 shows the positions of the two links.

B. Control Based on Model Two

Fig. 7(a) and (b) show the positions and position errors of the two links implementing hybrid control for Model Two.

V. Implementation Results for the Controller Supplied by the Manufacturer

The manufacturer [1] supplied controllers uses LQR and pole placement for balance control of the Pendubot at the open loop unstable equilibrium $q_1 = \pi/2$ and $q_2 = 0$. Since the effect of friction in the motor brushes and bearings at the first joint and in the bearings at the second joint generally result in limit cycle behavior, the controller supplied by the manufacturer also includes a small dither signal that reduces the amplitude of the limit cycle. This reduction is called the friction compensation technique. However, for our hybrid controller, we simply neglect friction without using any compensation technique and it still works better than the manufacturer's controller.

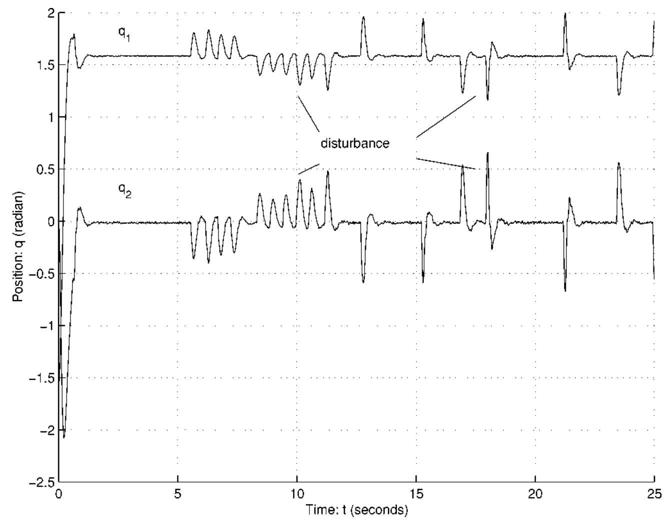


Fig. 5. Randomly added quick disturbances.

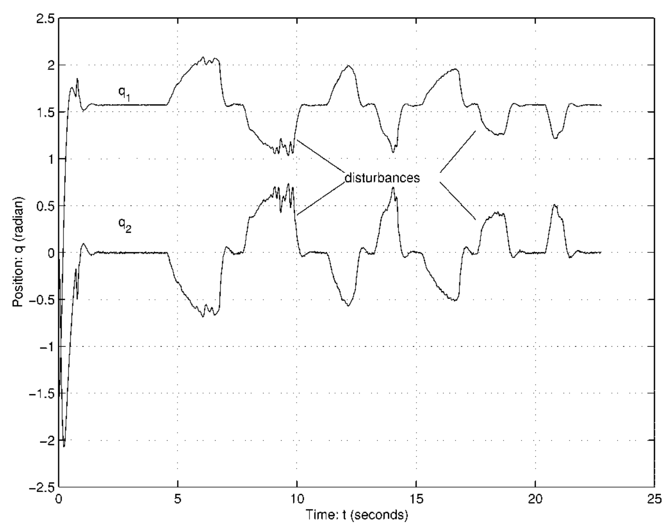


Fig. 6. Randomly added slow disturbances.

Fig. 8(a) and (b) show positions and position errors of link one and link two.

VI. Analysis and Comparison of the Experimental Results

From studying the above experimental results, we have come to the following conclusions.

- Implementation results show that the hybrid controller works very well. It shows a small transient shortly after the system is switched to hybrid control.
- Fig. 5 shows that the hybrid controller responds very quickly and robustly, even to large uncertain disturbances.
- Fig. 6 shows that the region of attraction is quite large for the hybrid controller. Slow uncertain disturbances were applied for estimating the region of attraction, which was found to be 72 degrees centered around the equilibrium state. However, we observed that the region of attraction for the controller supplied by the manufacturer is very small, which was about 38 degrees centered around the equilibrium.

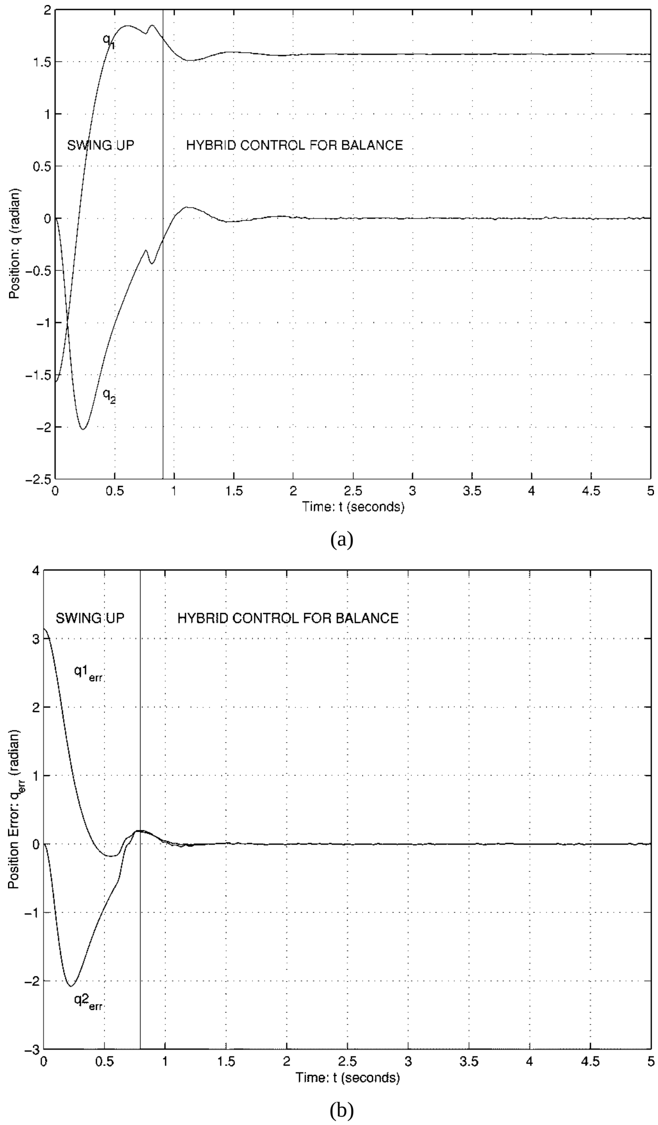


Fig. 7. Varied parameters model.

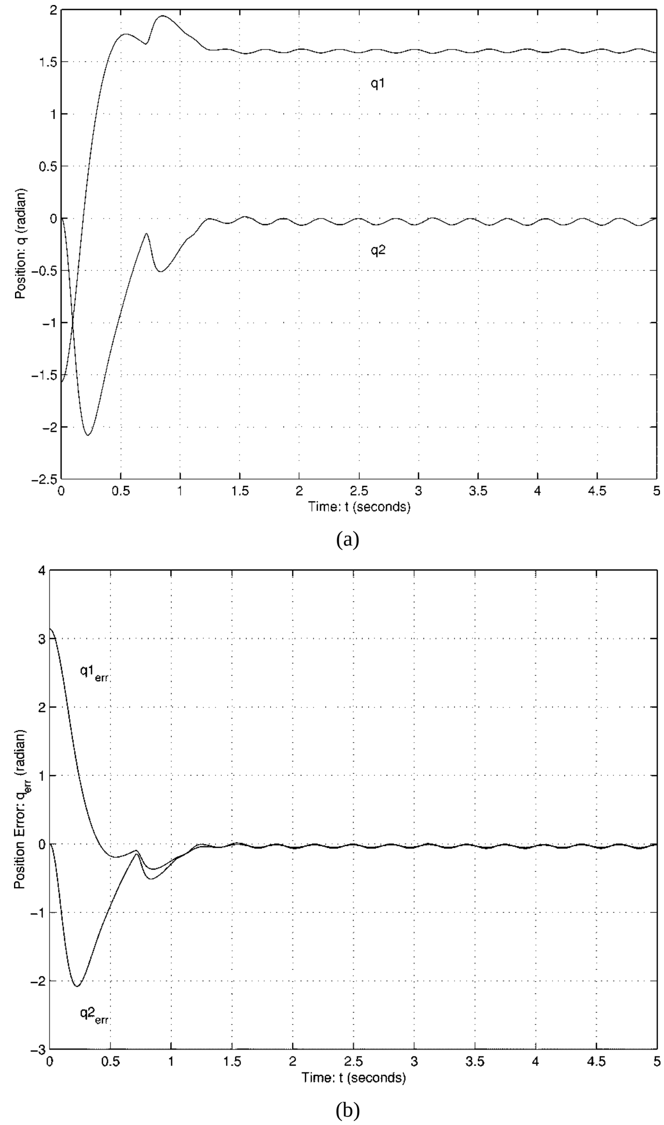


Fig. 8. Manufacturer's controller.

- The hybrid control is very robust with respect to the model variation. Fig. 7 shows a somewhat surprising result: the hybrid control still works very well for large variations of the dynamic model. However, the manufacturer's controller does not perform well under these conditions. For our hybrid controller, the swing up controller can be switched into it much earlier than the controller supplied by the manufacturer.
- By observation, we found in most cases the manufacturer's swing up controller can not be switched to the LQR for the model with variations. Since the region of attraction is very small for the linearized system, the manufacturer's controller works very well only when the swing up will move the Pendubot very close to the equalibrium states. However, in most cases it is very hard for the model with variation. The swing up either over shooting or under shooting the state under which the controller can be switched to the LQR. However, this is not the case for the hybrid controller, since the region of attraction is quite large, the swing up controller can always be easily switched to the hybrid controller for balance control.
- Comparison of Fig. 4 with Fig. 8 shows that the hybrid controller works better than the controller supplied by the manufacturer. For the hybrid controller, both position and velocity errors are significantly smaller than that of the controller supplied by the manufacturer. Also, the response time is much faster than that of the controller supplied by the manufacturer. Once our controller switches to hybrid control, it quickly reaches steady-state.
- The Pendubot remains a very special case among underactuated mechanical systems that can use Linear Quadratic Optimal Theory for achieving smooth control. For a class of underactuated mechanical systems, it is impossible to use smooth feedback to asymptotically stabilize the system around the equilibrium state. For this reason, we developed the hybrid control technique, which can be used for designing a hybrid controller for a class of underactuated mechanical systems, especially where smooth feedback cannot be used to asymptotically stabilize the equilibrium state.
- We should note that we did not consider any technique for friction compensation in our hybrid control. Even though

friction is large and asymmetric for the Pendubot, the hybrid controller still outperforms the controller supplied by the manufacturer.

VII. Conclusions

We have presented a new hybrid controller for feedback stabilization of the Pendubot. The hybrid controller was constructed based on a general form originally created for feedback stabilization of a class of underactuated mechanical systems. Various test cases have been presented and compared with the available controller from the manufacturer for balance control of the Pendubot. The experimental results show our hybrid control outperforms the existing algorithm and it is very robust. To the best of the authors' knowledge, this is the first hybrid controller proposed and implemented for feedback stabilization of the Pendubot.

The Pendubot is a simple underactuated mechanical system that shows second-order nonholonomic properties. It is well-known that it is impossible to use smooth feedback to stabilize a class of nonholonomic systems even locally. Hybrid control has been considered as a good choice. However, few study has been implemented in real system. This experimental study has special meanings for control of nonholonomic systems. Gravity terms make the Pendubot a special case for second-order nonholonomic systems. If there were no gravity terms, it could not be feedback stabilized using smooth control and the controller supplied by the manufacturer could not be used. In such conditions, our hybrid control can be used. The case for the omission of gravity terms can be easily found for airplanes, space craft, underwater manipulators,and underwater robotic vehicles and vessels. However, a practical implementation remains to be demonstrated.

Acknowledgment

The authors would like to thank the anonymous reviewers and F. Celani for their helpful comments and suggestions.

References

[1] *The Pendubot user's manual*, Mechatronic Systems, Inc., Champaign, IL, 1997.
[2] K. J. Åström and K. Furuta, "Swing up a pendulum by energy control," *Automatica*, vol. 36, pp. 287–295, 2000.
[3] I. Fantoni, R. Lozano, and M. W. Spong, "Engergy based control of the Pendubot," *IEEE Trans. Automat. Contr.*, vol. 45, pp. 725–729, Apr. 2000.
[4] M. W. Spong and D. J. Block, "The Pendubot: A mechatronic system for control research and education," in *Proc. 34th IEEE Conf. Decision and Control*, 1995, pp. 555–556.
[5] M. W. Spong, "The swing up control problem for the acrobot," *IEEE Contr. Syst. Mag.*, vol. 15, pp. 49–55, 1995.
[6] M. Arai and S. Tachi, "Position control of a manipulator with passive joints using dynamic coupling," *IEEE Trans. Robot. Automat.*, vol. 7, pp. 528–534, 1995.
[7] M. Reyhanoglu, A. van der Schaft, N. H. McClamroch, and I. Kolmanovsky, "Dynamics and control of a class of under-actuated mechanical systems," *IEEE Trans. Automat. Contr.*, vol. 44, pp. 1663–1671, Sept. 1999.
[8] I. Kolmanovsky and N. H. McClamroch, "Developments in nonholonomic control problems," *IEEE Contr. Syst. Mag.*, vol. 15, pp. 20–36, 1995.
[9] A. N. Michel, "Recent trends in the stability analysis of hybrid dynamical systems," *IEEE Trans. Circuits Syst. I*, vol. 45, pp. 120–134, Jan. 1999.
[10] H. Ye, A. N. Michel, and L. Hou, "Stability theory for hybrid dynamical systems," *IEEE Trans. on Automat. Contr.*, vol. 43, pp. 461–472, Apr. 1998.
[11] J. P. Hespanha and A. S. Morse, "Stabilization of nonholonomic integrator via logic-based switching," *Automatica*, vol. 35, pp. 385–393, 1999.
[12] Z. P. Jiang, "Robust exponential regulation of nonholonomic systems with uncertainties," *Automatica*, vol. 36, pp. 189–209, 2000.
[13] M. W. Spong and M. Vidyasagar, *Robot dynamics and control*. New York: Wiley, 1989.
[14] H. Cartan, *Differential forms*. Paris, France: Hermann & Cie, 1970.
[15] G. Oriolo and Y. Nakamura, "Control of mechanical systems with second-order nonholonomic constraints: under-actuated manipulators," in *Proc. 30th IEEE Conf. Decision and Control*, 1991, pp. 306–310.
[16] M. W. Spong, "Engergy based control of a class of under-actuated mechanical systems," in *Proc. IFAC World Congress*, San Francisco, CA, 1996, pp. 431–435.

Mingjun Zhang (S'98) received the B.S. and M.S. degrees in mechanical engineering, and the Ph.D. degree in industrial automation, all from Zhejiang University, Hangzhou, China, in 1990, 1994, and 1997, respectively. He received the M.S. degree in computer science from the University of Missouri-Rolla, Rolla, and the Doctor of Science degree in systems, science, and mathematics from Washington University in St. Louis, St. Louis, MO, both in 2000.

He is currently working on developing automatic equipment for DNA analysis at the Bio-Research Solutions Unit in Agilent Technologies, Palo Alto, California. His research interests include control theory, differential games, image processing, biomedical engineering, robotics and industrial automation.

Tzyh-Jong Tarn (M'71–M'83–F'85) received the D.Sc. degree in control system engineering from Washington University, St. Louis, MO.

Currently, he is a Professor in the Department of Systems, Science, and Mathematics and the Director of the Center for Robotics and Automation at Washington University, St. Louis, MO. He served as the President of the IEEE Robotics and Automation Society from 1992 to 1993, the Director of the IEEE Division X (Systems and Control), from 1995 to 1996, and was a member of the IEEE Board of Directors, from 1995 to 1996. At present, he serves as the Chairman of the Management Committee of the IEEE/ASME Transactions on Mmechatronics and the Vice President for Publications of the IEEE Neural Network Council.

Dr. Tarn received the NASA Certificate of Recognition for the creative development of a technical innovation on Robot Arm Dynamic Control by Computer, in 1987, the Best Research Article Award, from the Japan Foundation for the Promotion of Advanced Automation Technology, in March 1994, the Best Paper Award at the IEEE/RSJ International Conference on Intelligent Robots and Systems, in 1995, and the Distinguished Member Award from the IEEE Control Systems Society, in 1996. He is the first recipient of both the Nakamura Prize (in recognition and appreciation of his contribution to the advancement of the technology on intelligent robots and systems over a decade) at the tenth anniversary of the international conference on Intelligent Robots and Systems IROS, in 1997, and the Ford Motor Company Best Paper Award at the Japan/USA Symposium on Flexible Automation, in 1998. In addition, he is the recipient of the prestigious Joseph F. Engelberger Award of the Robotic Industries Association for contributing to the advancement of the science of robotics, in 1999, and the Auto Soft Lifetime Achievement Award in recognition of his pioneering and outstanding contributions to the fields of Robotics and Automation, in 2000. He was featured in the Special Report on Engineering of the 1998 Best Graduate School issue of *US News and World Report* and his recent research accomplishments were reported in the *Washington Times*, Washington DC, the *Financial Times*, London, U.K., *Le Monde*, Paris, France, and the *Chicago Sun Times*, Chicago, IL.