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Brief Paper

Synchronized multiple spacecraft rotations ☆

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Abstract

The objective of this paper is to present formation control laws for maintaining attitude alignment among a group of spacecraft in either deep space or earth orbit. The paper presents two control strategies based on emergent behavior approaches. Each control strategy considers the desired formation behaviors of convergence to the final formation goal, formation keeping, and the desire to rotate the spacecraft about fixed axes. The first approach uses velocity feedback and the second approach used passivity-based damping. In addition, we prove analytically that our approach guarantees formation keeping throughout the maneuver. Simulation results demonstrate the effectiveness of our approach. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Formation flying; Spacecraft; Attitude control

1. Introduction

In recent years there has been considerable interest in spacecraft formation flying (Das, Cobb, & Stallard, 1998; DeCou, 1991; How, Twiggs, Weidow, Hartman, & Bauer, 1998; Lau, Lichten, Young, & Haines, 1996). Most of the reported results have focused on the problem of relative positioning between the spacecraft. For example, leader-following approaches to maintain satellite formations in earth orbit are reported in de Queiroz, Kapila, and Qiguo Yan (2000), Folta and Quinn (1998) and Kapila, Sparks, Buffington, and Qiguo Yan (2000). Maintaining relative positions during formation maneuvers in deep space have been considered in Beard, Lawton, and Hadaegh (2001) and Mesbahi and Hadaegh (2000). The problem of maintaining relative orientation during maneuvers has been addressed in Beard, Lawton, and Hadaegh (2001) and Wang and Hadaegh (1996).

In some applications, e.g. interferometry, it is important that spacecraft maintain relative alignment during formation maneuvers. This requires that the spacecraft reorient about the same axis. In essence, we would like spacecraft to execute eigenaxis rotations where the same eigenaxis is used for each spacecraft. In Lawton (2000) and Lawton and Beard (2001) we presented a model-independent feedback scheme that executes eigenaxis rotations for rigid bodies. In this paper, we will build upon that result to show how to execute attitude formation maneuvers where the spacecraft are synchronized to rotate about a given eigenaxis.

The salient features of our approach are as follows: (1) the control strategies are model independent and are therefore robust to variations in the inertia of the spacecraft; (2) the formation control strategies are distributed, and only require low-bandwidth communication between neighboring spacecraft. In fact, if a relative attitude sensor is available, then no communication is required; (3) formation keeping is guaranteed asymptotically.

The formation keeping strategy described in the paper can be categorized as a behavior-based strategy (Balch & Arkin, 1998). Behavioral strategies "blend" several behaviors together, usually in a linear way (Anderson & Robbins, 1998). The advantage of a behavioral approach is that they are distributed and have been found to be very robust (Balch & Arkin, 1998). The major disadvantage with behavioral approaches is that it is very difficult to guarantee analytical results, such as stability and convergence. One of the major contributions of this paper is to show how to guarantee convergence of formation error for a nearest-neighbor

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behavioral approach that blends formation keeping and goal-seeking behaviors.

2. Problem definition

In this section, we introduce our notation and formally define the attitude formation control problem in the context of elementary formation maneuvers. Throughout this paper, the attitude of each spacecraft will be represented by a unit quaternion (Wertz, 1978), where the set of unit quaternions is given by $\mathcal{Q} = \{\mathbf{q} \in \mathbb{R}^4 | \mathbf{q}^T\mathbf{q} = 1\}$. It will be convenient to separate the quaternion into a vector and scalar part as $\mathbf{q} = [\mathcal{V}(\mathbf{q})^T, \mathcal{S}(\mathbf{q})]^T$, where $\mathcal{V}(\mathbf{q})$ is the vector part and $\mathcal{S}(\mathbf{q})$ is the scalar part. Quaternion multiplication is defined by the following non-commutative operation:

$$\mathbf{qp} = \begin{bmatrix} \mathcal{S}(\mathbf{q})\mathcal{V}(\mathbf{p}) + \mathcal{S}(\mathbf{p})\mathcal{V}(\mathbf{q}) + \mathcal{V}(\mathbf{q}) \times \mathcal{V}(\mathbf{p}) \\ \mathcal{S}(\mathbf{q})\mathcal{S}(\mathbf{p}) - \mathcal{V}(\mathbf{q})^T\mathcal{V}(\mathbf{p}) \end{bmatrix}, \quad (1)$$

where $\times$ is the vector cross product operator. The inverse or conjugate of a quaternion is defined by (Wertz, 1978) $\mathbf{q}^* = (-\mathcal{V}(\mathbf{q})^T, \mathcal{S}(\mathbf{q}))^T$, with the quaternion identity given by $\mathbf{1} = (0,0,0,1)^T$.

The equations of motion of the ith spacecraft are given by

$$\dot{\mathbf{q}}_i = \tfrac{1}{2}\mathbf{q}_i\bar{\boldsymbol{\omega}}_i, \qquad J_i\dot{\boldsymbol{\omega}}_i = -\boldsymbol{\omega}_i \times J_i\boldsymbol{\omega}_i + \boldsymbol{\tau}_i, \quad (2)$$

where $\mathbf{q}_i$ is the attitude of the ith spacecraft with respect to its desired orientation, $\boldsymbol{\tau}_i \in \mathbb{R}^3$ is the control torque, $\bar{\boldsymbol{\omega}}_i = (\boldsymbol{\omega}_i^T, 0)$, where $\boldsymbol{\omega}_i$ is the angular velocity, and $J_i \in \mathbb{R}^{3\times 3}$ is the moment of inertia expressed in the body frame.

There are three objectives in synchronized rotation. The first objective is to rotate each individual spacecraft to zero attitude error. Mathematically, we would like

$$E_G = \sum_{i=1}^{N} \|\mathbf{q}_i - \mathbf{1}\|^2 \to 0$$

asymptotically. The second objective is to maintain formation throughout the maneuver, i.e., we desire

$$E_F = \sum_{i=1}^{N} \|\mathbf{q}_i - \mathbf{q}_{i+1}\|^2 \to 0$$

asymptotically, where the index of summation is defined modulo N, i.e., $\mathbf{q}_{N+1} = \mathbf{q}_1$ and $\mathbf{q}_0 = \mathbf{q}_N$. The final objective is to rotate the spacecraft about a defined axis of rotation. Let $\hat{\mathbf{u}}$ be the desired eigenaxis and consider the quaternion factorization $\mathbf{q}_i = \mathbf{q}_{iu}\mathbf{q}_{iR}$, where $\mathcal{V}(\mathbf{q}_{iu}) = \sin(\theta_i/2)\hat{\mathbf{u}}$, $\mathcal{S}(\mathbf{q}_{iu}) = \cos(\theta_i/2)$, and $\theta_i = 2\operatorname{a}\tan 2(\hat{\mathbf{u}}^T\mathcal{V}(\mathbf{q}_i), \mathcal{S}(\mathbf{q}_i))$. $\mathbf{q}_{iu}$ represents a rotation about $\hat{\mathbf{u}}$ where $\mathbf{q}_{iR}$ represents a rotation about an axis perpendicular to $\hat{\mathbf{u}}$. As shown in Lawton (2000) and Lawton and Beard (2001), an eigenaxis rotation is ensured if $\mathbf{q}_{iR} = \mathbf{1}$. Therefore, we would like

$$E_e = \sum_{i=}^{N} \|\mathbf{q}_{iR} - \mathbf{1}\|^2 \to 0$$

asymptotically. Letting $E = k_G E_G + k_F E_F + k_e E_e$, the attitude formation problem is to drive $E \to 0$ asymptotically.

3. Synchronized attitude maneuvers

In this section, we introduce a behavior-based strategy for synchronized attitude formation maneuvers. We propose the following control strategy for each spacecraft:

$$\begin{aligned}\boldsymbol{\tau}_i = &-k_G\mathcal{V}(\mathbf{q}_i) - d_G\boldsymbol{\omega}_i \\ &- k_F\mathcal{V}(\mathbf{q}_{i+1}^*\mathbf{q}_i) - d_F(\boldsymbol{\omega}_i - \boldsymbol{\omega}_{i+1}) \\ &- k_F\mathcal{V}(\mathbf{q}_{i-1}^*\mathbf{q}_i) - d_F(\boldsymbol{\omega}_i - \boldsymbol{\omega}_{i-1}) \\ &- k_e\mathcal{V}(\mathbf{q}_{iR}),\end{aligned} \quad (3)$$

where the first two terms are intended to drive $E_G \to 0$, the third and fourth terms are intended to maintain formation between the ith and $(i+1)$th spacecraft, the fifth and sixth terms are intended to maintain formation between the ith and $(i-1)$th spacecraft, and the last term is intended to maintain the rotation about the desired eigenaxis. Implementation of this control law requires that each spacecraft has knowledge of the orientation and angular velocity of itself and its neighbors. This information may be communicated or measured directly. Therefore, the control strategy is distributed and admits a potentially large number of spacecraft in the formation. Note that information flows in a bidirectional ring structure. In general, these types of information flow patterns are extremely difficult to analyze. One of the main contributions of this paper is to derive sufficient conditions, such that this type of information flow pattern leads to asymptotically stable attitude formation maneuvers.

One of the technical difficulties in working with unit quaternions is that both $\mathbf{q}$ and $-\mathbf{q}$ represent the same attitude. There exist some extreme initial conditions where this ambiguity may cause the formation, subject to the control law (3) to converge to a local equilibrium not equal to zero. A sufficient condition to remove this ambiguity is to require that $\mathcal{S}(\mathbf{q}_i) \geqslant 0$. We need a condition that ensures that if $\mathcal{S}(\mathbf{q}_i(0)) \geqslant 0$, then $\mathcal{S}(\mathbf{q}_i(t)) \geqslant 0$, for all $t \geqslant 0$. Towards this end, we have the following lemma.

Lemma 1. *Let*

$$\underline{E} = \min_j \min_{\mathcal{B}_j} k_G E_G + k_F E_F, \quad (4)$$

where

$$\mathcal{B}_j = \{(\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_N) \,|\, \mathcal{S}(\mathbf{q}_j) = 0, \mathcal{S}(\mathbf{q}_i) \geqslant 0,\ i \neq j\}.$$

If $E(t) < \underline{E}$ for all $t \geqslant 0$, then for each $1 \leqslant i \leqslant N$,

$$\mathcal{S}(\mathbf{q}_i(0)) > 0 \Rightarrow \mathcal{S}(\mathbf{q}_i(t)) > 0.$$

Proof. Suppose that the implication is not true, and let $t_i > 0$ represent the first time that $\mathscr{S}(\mathbf{q}_i)$ is equal to zero for the ith spacecraft, and let $k = \arg\min t_k$. Then at time t_k we have that $\mathscr{S}(\mathbf{q}_k(t_k)) = 0$, and $\mathscr{S}(\mathbf{q}_j(t_k)) \geqslant 0$, for all $j \neq k$. Therefore, by the definition of $\underline{E}$ we have that $k_G E_G(t_k) + k_F E_F(t_k) \geqslant \underline{E}$, and we have reached a contradiction. □

The following theorem is the main result of the paper.

Theorem 1. *If the formation given by Eq.* (2) *is subject to the distributed control strategy given in Eq.* (3), *and if*

(1) $k_G, d_G, k_e > 0$, $k_F, d_F \geqslant 0$,
(2) $k_G E_G(0) + k_F E_F(0) < \underline{E}$,
(3) $\mathscr{S}(\mathbf{q}_i(0)) > 0$,
(4) $\boldsymbol{\omega}_i(0) = 0$

then $E(t) \to 0$. *Furthermore, the eigenaxis error satisfies*

$$\sum_{i=1}^{N} \|\mathbf{q}_{iR}(t) - \mathbf{1}\|^2 \leqslant \frac{1}{k_e} E(0) \tag{5}$$

and the formation keeping error satisfies

$$\sum_{i=1}^{N} \|\mathbf{q}_i - \mathbf{q}_{i+1}\| \leqslant \frac{1}{k_F} E(0). \tag{6}$$

Proof. Consider the function

$$V = E + \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{\omega}_i^{\mathrm{T}} J_i \boldsymbol{\omega}_i.$$

After some manipulation, it can be shown that the time derivative of V is

$$\dot{V} = \sum_{i=1}^{N} \{-d_G \boldsymbol{\omega}_i^{\mathrm{T}} \boldsymbol{\omega}_i - d_F (\boldsymbol{\omega}_i - \boldsymbol{\omega}_{i+1})^{\mathrm{T}} (\boldsymbol{\omega}_i - \boldsymbol{\omega}_{i+1})\} \leqslant 0.$$

Therefore, V is a Lyapunov function. If follows directly that $E(t) \leqslant V(t) \leqslant V(0) = E(0) < \underline{E}$, which implies that $\mathscr{S}(\mathbf{q}_i(t)) > 0$ for all $t \geqslant 0$. The inequality $E(t) \leqslant E(0)$ implies that Eqs. (5) and (6) hold for all $t \geqslant 0$.

We will show convergence of E using an invariance argument. Let $\Omega = \{(\mathbf{q}_1, \ldots \mathbf{q}_N, \boldsymbol{\omega}_1, \ldots \boldsymbol{\omega}_N) | \dot{V} = 0\}$ and let $\bar{\Omega}$ be the largest invariant subset of Ω. On $\bar{\Omega}$

$$k_G \mathscr{V}(\mathbf{q}_i) + k_e \mathscr{V}(\mathbf{q}_{iR}) + k_F \mathscr{V}(\mathbf{q}_{i+1}^* \mathbf{q}_i) + k_F \mathscr{V}(\mathbf{q}_{i-1}^* \mathbf{q}_i) = 0 \tag{7}$$

for each i.

Note that since $\mathbf{q}_{iR} = \mathbf{q}_{iu}^* \mathbf{q}_i$, Eq. (7) can be written as

$$\mathscr{V}(\mathbf{p}_i^* \mathbf{q}_i) = 0, \tag{8}$$

where $\mathbf{p}_i = k_G \mathbf{1} + k_F \mathbf{q}_{i+1} + k_F \mathbf{q}_{i-1} + k_e \mathbf{q}_{ui}$. By the rules of quaternion multiplication Eq. (8) can be written as

$$-\mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{p}_i) + \mathscr{S}(\mathbf{p}_i)\mathscr{V}(\mathbf{q}_i) + \mathscr{V}(\mathbf{q}_i) \times \mathscr{V}(\mathbf{p}_i) = 0. \tag{9}$$

Multiplying Eq. (9) by $(\mathscr{V}(\mathbf{q}_i) \times \mathscr{V}(\mathbf{p}_i))^{\mathrm{T}}$ gives

$$\|\mathscr{V}(\mathbf{q}_i) \times \mathscr{V}(\mathbf{p}_i)\|^2 = 0. \tag{10}$$

Using this result in Eq. (9) we get

$$-\mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{p}_i) + \mathscr{S}(\mathbf{p}_i)\mathscr{V}(\mathbf{q}_i) = 0. \tag{11}$$

From Eq. (11) and the definition of $\mathbf{p}_i$ we can write

$$\{k_G + k_F \mathscr{S}(\mathbf{q}_{i+1}) + k_F \mathscr{S}(\mathbf{q}_{i-1})\}\mathscr{V}(\mathbf{q}_i) - k_F \mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{q}_{i+1}) - k_F \mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{q}_{i-1}) + k_e \mathscr{S}(\mathbf{q}_{ui})\mathscr{V}(\mathbf{q}_i) - k_e \mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{q}_{ui}) = 0. \tag{12}$$

From the definition of $\mathbf{q}_{ui}$, we get

$$\mathscr{V}(\mathbf{q}_{ui}) = \frac{\mathscr{S}(\mathbf{q}_{ui})}{\mathscr{S}(\mathbf{q}_i)} \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}} \mathscr{V}(\mathbf{q}_i),$$

therefore it follows that

$$k_e \mathscr{S}(\mathbf{q}_{ui})\mathscr{V}(\mathbf{q}_i) - k_e \mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{q}_{ui}) = k_e \mathscr{S}(\mathbf{q}_{ui})(I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}})\mathscr{V}(\mathbf{q}_i). \tag{13}$$

Substituting Eq. (13) into Eq. (12) we get that:

$$\{k_G + k_F \mathscr{S}(\mathbf{q}_{i+1}) + k_F \mathscr{S}(\mathbf{q}_{i-1})\}\mathscr{V}(\mathbf{q}_i) - k_F \mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{q}_{i+1}) - k_F \mathscr{S}(\mathbf{q}_i)\mathscr{V}(\mathbf{q}_{i-1}) + k_e \mathscr{S}(\mathbf{q}_{ui})(I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}})\mathscr{V}(\mathbf{q}_i) = 0. \tag{14}$$

If we write the system of equations given by Eq. (14) in matrix form we can write them as

$$\{(K \otimes I_3) + (D \otimes (I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}}))\}\mathscr{V}(\mathbf{q}) = 0, \tag{15}$$

where D is a diagonal matrix defined by $D_{ii} = k_e \mathscr{S}(\mathbf{q}_{ui})$, and K is defined by

$$K_{ij} = \begin{cases} k_G + k_F(\mathscr{S}(\mathbf{q}_{i-1}) + \mathscr{S}(\mathbf{q}_{i+1})) & \text{for } j = i \\ -k_F \mathscr{S}(\mathbf{q}_i) & \text{for } j = i \pm 1 \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Since K is strictly diagonally dominant when $\mathscr{S}(\mathbf{q}_i) > 0$, $K \otimes I_3$ is also strictly diagonally dominant. In order to show that $\mathscr{V}(\mathbf{q}) = 0$ on $\bar{\Omega}$ we must show that $K \otimes I_3 + D \otimes (I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}})$ is full rank. To do this we will show that all of the eigenvalues of $K \otimes I_3 + D \otimes (I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}})$ are positive. Consider the matrix $I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}}$. Let $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$ be any two linearly independent vectors that are both perpendicular to $\hat{\mathbf{u}}$. The vectors $\hat{\mathbf{u}}_1$, $\hat{\mathbf{u}}_2$ and $\hat{\mathbf{u}}$ are eigenvectors of $I_3 - \hat{\mathbf{u}}\,\hat{\mathbf{u}}^{\mathrm{T}}$ with corresponding eigenvalues $(1, 1, 0)$. Thus there exists a matrix, T such that

$$T^{-1}(I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}})T = \mathrm{diag}(1,1,0).$$

The eigenvalues of $K \otimes I_3 + D \otimes (I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}})$ are invariant under a similarity transformation. When we premultiply by $I_N \otimes T^{-1}$ and post multiply by $I_N \otimes T$ the eigenvalues are not changed. The resultant matrix is

$$\bar{K} = K \otimes I_3 + D \otimes \mathrm{diag}(1,1,0),$$

which will be strictly diagonally dominant provided that the elements of the diagonal matrix $D \otimes \mathrm{diag}(1,1,0)$ are non-negative. The non-zero elements of this matrix are given by $k_e \mathscr{S}(\mathbf{q}_{ui})$, where the scalar $k_e \geqslant 0$. Since $\mathbf{q}_{ui}$ is constructed using the a tan 2 function, $\mathscr{S}(\mathbf{q}_i) \geqslant 0$ implies that $\mathscr{S}(\mathbf{q}_{ui}) \geqslant 0$. Thus, $\bar{K}$ is strictly diagonally dominant which implies that it has positive eigenvalues. This in turn implies that $K \otimes I_3 + D \otimes (I_3 - \hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{T}})$ is full rank and $\mathscr{V}(\mathbf{q}(t)) \equiv 0$ on $\bar{\Omega}$. □

4. Passivity-based formation maneuvers

The distributed control strategy of Theorem 1 requires that each spacecraft knows its own angular velocity and the angular velocity of its two neighbors. In this section we will remove this requirement by extending the results of Lizarralde and Wen (1996) to the distributed formation control case. The proposed dynamic control strategy is given by the following equations:

$$\dot{\boldsymbol{\alpha}}_i = A\boldsymbol{\alpha}_i + B\mathbf{z}_i, \tag{17}$$

$$\mathbf{y}_i = B^{\mathrm{T}}PA\boldsymbol{\alpha}_i + B^{\mathrm{T}}PB\mathbf{z}_i, \tag{18}$$

$$\mathbf{z}_i = d_G\mathbf{q}_i + d_F(\mathbf{q}_i - \mathbf{q}_{i+1}) + d_F(\mathbf{q}_i - \mathbf{q}_{i-1}), \tag{19}$$

$$\begin{aligned}\boldsymbol{\tau}_i = &-k_G\mathscr{V}(\mathbf{q}_i) - k_F\mathscr{V}(\mathbf{q}_{i+1}^*\mathbf{q}_i) - k_F\mathscr{V}(\mathbf{q}_{i-1}^*\mathbf{q}_i)\\ &- k_e\mathscr{V}(\mathbf{q}_{iR}) - \mathscr{V}(\mathbf{q}_i^*\mathbf{y}_i).\end{aligned} \tag{20}$$

Note that the damping forces in Eq. (3) have been replaced by the term $\mathscr{V}(\mathbf{q}_i^*\mathbf{y}_i)$. Eqs. (17)–(19) take the role of a filter whose output $\mathbf{y}_i$ provides damping to the formation.

Theorem 2. *If the formation given by Eq.* (2) *is subject to the dynamic distributed control strategy given by Eqs.* (17)–(20), *and if*

(1) $k_G, d_G > 0$, $k_F, d_F, k_e \geqslant 0$, *$A$ is Hurwitz, B is full rank*, $Q = Q^{\mathrm{T}} > 0$, $P = P^{\mathrm{T}} > 0$ *is the solution to* $PA + A^{\mathrm{T}}P = -Q$,
(2) $k_G E_G(0) + k_F E_F(0) < \underline{E}$,
(3) $\mathscr{S}(\mathbf{q}_i(0)) > 0$,
(4) $\boldsymbol{\omega}_i(0) = 0$,
(5) $\mathbf{y}_i(0) = 0$

then $E(t) \to 0$. Furthermore, the eigenaxis and formation error satisfy respectively,

$$\sum_{i=1}^{N} \|\mathbf{q}_{iR}(t) - \mathbf{1}\|^2 \leqslant \frac{1}{k_e} E(0),$$

$$\sum_{i=1}^{N} \|\mathbf{q}_i(t) - \mathbf{q}_{i+1}\|^2 \leqslant \frac{1}{k_F} E(0).$$

Proof. Letting $\bar{\boldsymbol{\alpha}} = (\dot{\boldsymbol{\alpha}}_1^{\mathrm{T}}, \ldots, \dot{\boldsymbol{\alpha}}_N^{\mathrm{T}})^{\mathrm{T}}$, we can write the filter equation as

$$\dot{\bar{\boldsymbol{\alpha}}} = (I_N \otimes A)\bar{\boldsymbol{\alpha}} + (D \otimes B)\mathbf{q},$$

$$\mathbf{y} = (I_N \otimes B^{\mathrm{T}}P)\bar{\boldsymbol{\alpha}},$$

where $\mathbf{q}$ and $\mathbf{y}$ are formed by stacking the $\mathbf{q}_i$'s and $\mathbf{y}_i$'s end on end, and I_N is an $N \times N$ identity matrix and the (i,j)th component of D is defined by

$$d_{ij} = \begin{cases} 1 & \text{for } j = i, \\ -1 & \text{for } j = i+1, \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

Consider the Lyapunov function candidate

$$V = E + \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{\omega}_i^{\mathrm{T}} J_i \boldsymbol{\omega}_i + \bar{\boldsymbol{\alpha}}^{\mathrm{T}}(D^{-1} \otimes P)\bar{\boldsymbol{\alpha}}$$

whose derivative is

$$\dot{V} = -\bar{\boldsymbol{\alpha}}^{\mathrm{T}}(D^{-1} \otimes Q)\bar{\boldsymbol{\alpha}} \leqslant 0.$$

The remainder of the proof follows arguments similar to the proof of Theorem 1. □

5. The domain of attraction

The domain of attraction for Theorems 1 and 2 are determined by the relationship $k_G E_G(0) + k_F E_F(0) < \underline{E}$, where $\underline{E}$ is given in Eq. (4). In this section, we will show that $\underline{E}$ can be determined by solving a convex optimization problem which scales linearly in the number of spacecraft in the formation.

First note that E_G, E_F and B_j are symmetric with respect to the index of the spacecraft. Therefore Eq. (4) can be written as

$$\underline{E} = \min_{\mathscr{B}_1} k_G E_G + k_F E_F. \tag{22}$$

Also note that $(\mathbf{q}_1, \ldots, \mathbf{q}_N) \in \mathscr{B}_1$ implies that

$$\mathbf{q}_1 = \begin{pmatrix} \mathscr{V}(\mathbf{q}_1) \\ \mathscr{S}(\mathbf{q}_1) \end{pmatrix} = \begin{pmatrix} \sin(\theta_1/2)\hat{\mathbf{u}}_1 \\ \cos(\theta_1/2) \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{u}}_1 \\ 0 \end{pmatrix}.$$

If we assume that the spacecraft angles are referenced between $\pm\pi$, then it follows that $\theta_1 = \pi$. In addition,

$\mathscr{S}(\mathbf{q}_i)=\cos(\theta_i/2)\geqslant 0$, $i=2,\ldots,N$ implies that $-\pi\leqslant\theta_i\leqslant\pi$. Using the fact that $\mathbf{q}_i$ is a unit quaternion, after some manipulation we can write

$$E_G = 2Nk_G - 2k_G\sum_{i=1}^{N}\sqrt{1-\|\mathscr{V}(\mathbf{q}_i)\|^2},$$

$$E_F = 2Nk_F - 2k_F\sum_{i=1}^{N}\mathscr{V}(\mathbf{q}_i)^{\mathrm{T}}\mathscr{V}(\mathbf{q}_{i+1})$$

$$-2k_F\sum_{i=1}^{N}\sqrt{(1-\|\mathscr{V}(\mathbf{q}_i)\|^2)(1-\|\mathscr{V}(\mathbf{q}_{i+1})\|^2)}.$$

Letting $\mathscr{V}(\mathbf{q}_i)=\sin(\theta_i/2)\hat{\mathbf{u}}_i$ we get

$$E_G = 2Nk_G - 2k_G\sum_{i=1}^{N}\cos(\theta_i/2),$$

$$E_F = 2Nk_F - 2k_F\sum_{i=1}^{N}\hat{\mathbf{u}}_i^{\mathrm{T}}\hat{\mathbf{u}}_{i+1}\sin(\theta_i/2)\sin(\theta_{i+1}/2)$$

$$-2k_F\sum_{i=1}^{N}\cos(\theta_i/2)\cos(\theta_{i+1}/2).$$

Since E_F contains the only directional information, it can be seen that to minimize E_F, we should set $\hat{\mathbf{u}}_i=\hat{\mathbf{u}}_{i+1}$, which after simplification gives

$$E_F = 2Nk_F - 2k_F\sum_{i=1}^{N}\cos\left(\frac{\theta_i-\theta_{i+1}}{2}\right).$$

Therefore,

$$\underline{E} = 2N(k_G+k_F) - 2\max_{\Theta_1}\sum_{i=1}^{N}k_G\cos\left(\frac{\theta_i}{2}\right)$$
$$+k_F\cos\left(\frac{\theta_i-\theta_{i+1}}{2}\right),$$

where $\Theta_1=\{(\theta_1,\ldots,\theta_N):\theta_1=\pi,-\pi\leqslant\theta_i\leqslant\pi\}$. Note that Θ_1 is compact and convex, and that over Θ_1, the objective function is also convex. Therefore, there is a unique maximum on Θ_1, and $\underline{E}$ can be determined by solving a well-posed convex optimization problem with complexity scaling linearly in the number of spacecraft.

6. Simulations

In this section, we will consider a group of three spacecraft starting from rest with initial attitudes given by $\mathbf{q}_1(0)=(9,0,0,10)^{\mathrm{T}}/\sqrt{181}$, $\mathbf{q}_2(0)=(10,0,0,10)^{\mathrm{T}}/\sqrt{200}$, and $\mathbf{q}_3(0)=(11,0,0,10)^{\mathrm{T}}/\sqrt{221}$. The moment of inertia of each spacecraft is given by

$$J_i=\frac{m_i}{6}\begin{bmatrix}1 & 0.1 & 0.1\\ 0.1 & 1.1 & 0.1\\ 0.1 & 0.1 & 0.9\end{bmatrix},$$

where the mass of the three spacecraft are given by $m_1=250$ kg and $m_2=m_3=150$ kg. We assume that 10 mN m is the maximum torque exerted by each spacecraft. The control gains were chosen as $k_G=3.64$, $d_G=533$, $k_F=200k_G$, $d_F=100d_G$, and $k_e=100k_G$. For this choice of gains, $k_GE_G+k_FE_F=17.4$. Solving the optimization problem posed in the previous section, we get $\underline{E}=21.8$, therefore the above configuration is within the domain of attraction.

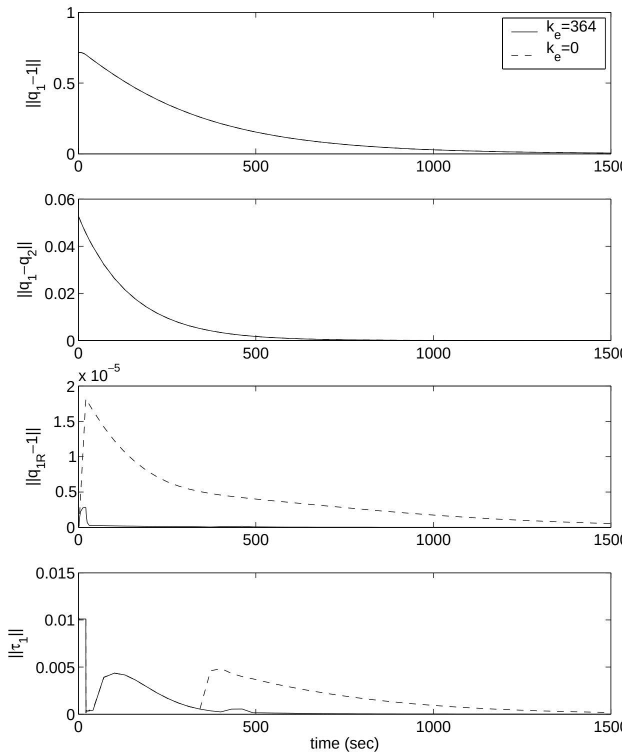

Fig. 1. Simulation for three spacecraft synchronized rotation.

Fig. 1 shows the performance of the control strategy given in Eq. (3) with (i.e., $k_e=100k_G$) and without (i.e., $k_e=0$) eigenaxis feedback. The solid line corresponds to eigenaxis feedback, and the dashed line corresponds to no eigenaxis feedback.

The first subplot shows the convergence of the first spacecraft to its goal. The plots of the other spacecraft look similar. The second subplot shows the formation error between the first and second spacecraft. Again, the formation error between the other spacecraft are qualitatively similar. The third subplot shows the eigenaxis error for the first spacecraft. From this plot it is clear that while eigenaxis feedback does not play a significant role in goal seeking or formation keeping, it does play an important role in maintaining synchronization between spacecraft. The fourth subplot shows the thrust trajectory of the first spacecraft.

7. Conclusions

In this paper, we have derived distributed formation control strategies for synchronized attitude maneuvers. The first strategy is given by Eq. (3) and requires angular velocity information. The second strategy is given by Eqs. (17)–(20) and does not require angular velocity information. Both strategies require that information is shared in a bi-directional ring topology. The proof of Theorem 1 demonstrates how distributed control strategies under this type of information flow structure can be analyzed. Also note that both control strategies are model independent in that they do not require knowledge of the moments of inertia. In Section 5, we showed that the domain of attraction for the control strategies can be found by solving a well-posed convex optimization problem.

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