Architecture and Algorithms for Constellation Control

Randal Beard
Electrical and Computer Engineering
Brigham Young University
Provo, Utah 84602

March 16, 1998

Final Report

Abstract

The objective of this research is to develop an architecture and algorithms for the control of a constellation of microspacecraft. It is envisioned that a constellation of spacecraft will be used by NASA for exosolar planet detection and astronomical observations. The basic architecture used in this work is that of a constellation template, i.e., a virtual rigid body that defines the motion of the constellation. Using this architecture, algorithms are developed for constellation initialization, constellation rotation, constellation expansion and contraction, and constellation reorientation.

Contents

Executive Summary 3

1 Introduction 5

2 Constellation Reorientation 9

3 Constellation Rotation 17

4 Constellation Expansion/Contraction 21

5 Constellation Initialization 25

6 Conclusions and Extensions 37

References 41

Acknowledgments 43

Architecture and Algorithms for Constellation Control

Randal Beard
Electrical and Computer Engineering
Brigham Young University
Provo, Utah 84602

March 16, 1998

Final Report

This report was prepared for the Jet Propulsion Laboratory, California Institute of Technology and was sponsored by the National Aeronautics and Space Administration.

Executive Summary

This report describes an architecture and algorithms that have been developed for constellation coordination and control. Algorithms for four basic constellation maneuvers are derived: constellation reorientation, constellation rotation, constellation expansion/contraction and constellation initialization.

A constellation reorientation maneuver requires the entire constellation to change orientation while the individual spacecraft in the constellation maintain their relative position and orientation with respect to each other. In other words, the constellation moves as a rigid body. The motivation for this type of maneuver is the metrology sensing system proposed for DS3 which requires that precise distances between spacecraft be maintained throughout maneuvers. The algorithms developed in this report accomplish constellation reorientation and consider the problem of picking a point of rotation *a priori*, such that the overall fuel is minimized and that the fuel usage is distributed equally among the spacecraft.

A constellation rotation maneuver is similar to a reorientation maneuver, except the constellation is rotated at a constant rate about an axis in the constellation. The motivation is to create mappings of the u-v plane for interferometry applications. In this report we develop a simple feedback control law for effecting a constellation rotation with on/off thrusters. We assume that thrusters can be pointed in an arbitrary direction.

A constellation expansion or contraction maneuver requires the volume of the convex hull of the spacecraft to uniformly increase over some time interval. The motivation is, again, u-v plane mapping. A simple algorithm is derived for expansion and contraction maneuvers.

Constellation initialization is required after launch of the spacecraft. The difficulty with initialization is that the spacecraft may have limited inter-spacecraft sensing and communication ability before they form into the desired constellation. In this report we apply the theory of satisficing games and epistemic utility theory to derive an efficient near-optimal initialization strategy for the case where the spacecraft have limited sensing and no communication capabilities.

1 Introduction

NASA is currently considering several missions where a constellation of small, inexpensive spacecraft will coordinate their actions to effect mission objectives that currently are addressed by more expensive spacecraft or that are currently too costly to achieve with a single spacecraft. An example is the proposed DS3 mission where three spacecraft will fly in precise formations to synthesize a space-based interferometer, enabling exosolar planetary observations. To realize these missions, architectures and algorithms that coordinate and control a constellation of spacecraft need to be developed. The objective of this research is to develop algorithms for:

- Constellation reorientation
- Constellation rotation
- Constellation expansion/contraction
- Constellation initialization.

Each of these problems will be considered in a separate section of the report. All algorithms will be developed under the assumption that the constellation is in free space, i.e., gravity is not a factor. One plan for DS3 is that the spacecraft will reside in an earth-depart sun orbit making the gravity gradient across the constellation negligible.

The remainder of this section defines the notation that will be used throughout the report. Figure 1 depicts four coordinate frames. Frame **O** is the inertial coordinate system. **C** is called the constellation frame and is a coordinate system that is fixed with respect to the constellation. **C** may be collocated with one of the spacecraft in the constellation, or it may be the baracenter or another convenient point of reference local to the constellation. Constellation rotation and reorientation maneuvers will require that we rotate the constellation about some point. **R** is the constellation rotation frame. Each spacecraft has a local coordinate frame that is represented by **i**. The unit attitude quaternion of frame B with respect to frame A is represented by ${}^B\mathbf{q}^A = \left(({}^B\eta^A)^T, {}^B\epsilon^A\right)^T$, where ${}^B\eta^A \in I\!R^3$ and ${}^B\epsilon^A \in I\!R$. The angular velocity of frame B with respect to frame A is represented by ${}^B\omega^A \in I\!R^3$. The position of frame B with respect to frame A is represented by ${}^B\mathbf{r}^A \in I\!R^3$. Similarly the velocity of frame B with respect to frame A is represented by ${}^B\mathbf{v}^A \in I\!R^3$. A subscript "$d$" will denote a desired quantity,

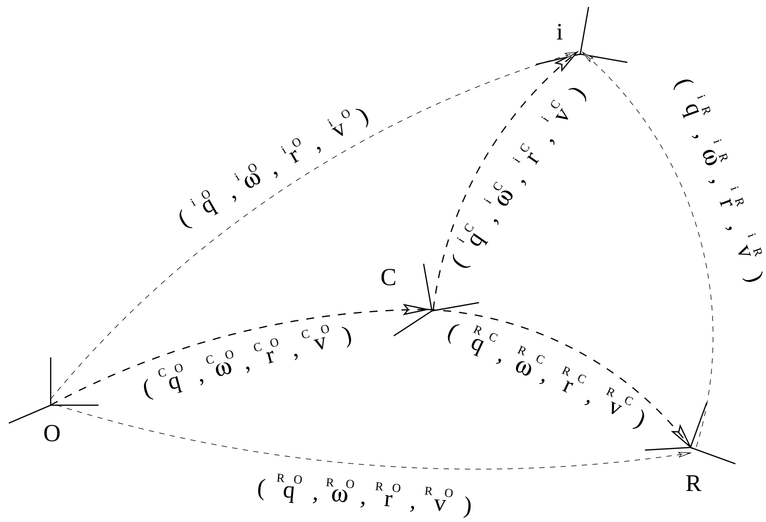

Figure 1: The geometry of formation flying.

i.e., ${}^i\mathbf{r}_d^O$ is the desired position of spacecraft i with respect to the inertial coordinate frame.

We will assume that each spacecraft is governed by the following rigid body dynamic equations [1, 2]:

Rotational Dynamics

$$
\begin{aligned}
\frac{d\,{}^i\eta^0}{dt} &= \frac{1}{2}({}^i\epsilon^0\,{}^i\omega^O - {}^i\omega^O \times {}^i\eta^0) \\
\frac{d\,{}^i\epsilon^0}{dt} &= -\frac{1}{2}({}^i\omega^O \cdot {}^i\eta^0) \\
I_i \frac{d\,{}^i\omega^O}{dt} + {}^i\omega^O \times (I_i\,{}^i\omega^O) &= \tau_c^i + \tau_e^i,
\end{aligned}
$$

where τ_c^i is the control torque and τ_e^i is the environmental disturbance torque,

Translational Dynamics

$$
\begin{aligned}
\frac{d\,{}^i\mathbf{r}^O}{dt} &= {}^i\mathbf{v}^O \\
M_i \frac{d\,{}^i\mathbf{v}^O}{dt} &= f_c^i + f_e^i
\end{aligned}
\tag{1}
$$

where f_c^i is the control force and f_e^i is the environmental disturbance force.

The algorithms developed in the next four sections will be for a general number of spacecraft and a general constellation. The task plan called for the algorithms to be demonstrated on a constellation comprising 18 spacecraft. While this is possible, the software tools that we are using (Matlab and Simulink) run extremely slow for this size of constellation on the computers that we are using. We have decided, therefore, to illustrate the algorithms on a constellation comprising only five spacecraft so that simulations can be demonstrated at reasonable speeds.

2 Constellation Reorientation

2.1 Problem Description

The problem of constellation reorientation is shown graphically in Figure 2. Before a star can be imaged in an interferometry mission, the constellation

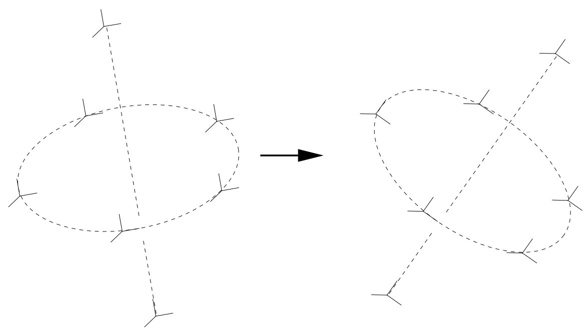

Figure 2: Constellation Reorientation.

must be reoriented such that the axis of the constellation points at the star. We will consider the case when the constellation is in a solar orbit far from the earth and, therefore, does not experience a serious gravitational gradient from one end of the constellation to another. Also, we will assume that the precise inertial position of the constellation is not important, i.e., only the orientation is important. This is justified since the distance to stars that will be imaged is so great that the required orientation of the spacecraft is not significantly altered as the inertial position of the constellation is changed. Since the precise inertial position of the constellation is not important, we can choose the point of rotation, labeled **R** in Figure 1, to satisfy other criteria such as minimizing fuel consumption or equalizing the fuel consumed across the constellation.

We have investigated two approaches to the fuel equalization problem, namely (1) *a priori* selection of the point of rotation such that there is a tradeoff between fuel minimization and fuel equalization, (2) dynamic adjustments to the point of rotation to try to keep the fuel equalized across the constellation. Since we are still investigating the second approach we will limit the discussion in this report to *a priori* selection of the the point of rotation.

2.2 An Algorithm for Constellation Reorientation

In this section, we will first develop an algorithm for constellation reorientation given a fixed point of rotation **R**. We will then explain how to choose **R** to tradeoff fuel minimization and fuel equalization.

2.2.1 Reorientation given R

We will assume that the fleet of spacecraft is currently configured into a desired constellation. The desired constellation is specified in terms of the position and orientation of each spacecraft with respect to the coordinate frame **C**, i.e., the set $\{({}^i\mathbf{q}_d^C, {}^i\mathbf{r}_d^C)\}_{i=1}^N$ is specified. We will assume that at the beginning of the maneuver ${}^i\mathbf{q}^C(0) = {}^i\mathbf{q}_d^C$ and ${}^i\mathbf{r}^C(0) = {}^i\mathbf{r}_d^C$. We divide the constellation reorientation problem into three steps.

Step 1: First, the template is treated as a rigid body, and a control law is derived for controlling the orientation of the template.

Step 2: Next, the motion of the template is used to define the desired position and attitude trajectories for each spacecraft.

Step 3: Finally, control laws are designed for each spacecraft that cause them to track these desired trajectories.

Each of these items will be discussed below.

Step 1. The desired constellation defines a virtual rigid body whose inertia about **R** is given by [3, pp. 43]

$$J = \sum_{i=1}^{N} M_i \left[\left\| {}^i\mathbf{r}_d^R \right\|^2 I - ({}^i\mathbf{r}_d^R)({}^i\mathbf{r}_d^R)^T \right],$$

where I is the the 3×3 identity matrix. The rotational dynamics of this virtual rigid body about ${}^R\mathbf{r}^O$ are given by [1, 2]:

$$\begin{aligned} \frac{d^R\eta^0}{dt} &= \frac{1}{2}({}^R\epsilon^0\, {}^R\omega^O - {}^R\omega^O \times {}^R\eta^0) \\ \frac{d^R\epsilon^0}{dt} &= -\frac{1}{2}({}^R\omega^O \cdot {}^R\eta^0) \\ J\frac{d^R\omega^O}{dt} + {}^R\omega^O \times (J^R\omega^O) &= \tau_R. \end{aligned} \tag{2}$$

A virtual control torque τ_R that asymptotically stabilizes this rigid body is given by [1, 4]

$$\tau_R = K_1 \left[{}^C\epsilon_d^0 ({}^C\eta_d^0 - {}^R\eta^0) - ({}^C\epsilon_d^0 - {}^R\epsilon^0)^C\eta_d^0 + {}^C\eta_d^0 \times {}^R\eta^0 \right] - K_2 J^R\omega^O, \quad (3)$$

where K_1 and K_2 are positive constants and where we have assumed that the attitude of $\mathbf{R}$ is identical to the attitude of $\mathbf{C}$, i.e., ${}^R\mathbf{q}_d^O = {}^C\mathbf{q}_d^O$.

This control is essentially PD control where K_1 is the proportional gain and K_2 is the derivative gain. Other control laws could also be used to improve the performance of the system. For example, a control law that attempts to minimize the overall fuel consumption of the constellation could be used. The overall control architecture dictated by the constellation template approach is independent of the specific control law used to rotate the virtual rigid body constellation.

Step 2. The objective of this step is to translate the virtual motion of the template into desired rotation and translation trajectories for the spacecraft. Given the control law in equation (3), the virtual dynamics of the constellation, given in equation (2), can be integrated to produce trajectories for the variables ${}^R\mathbf{q}_d^O(t)$ and ${}^R\omega_d^O(t)$. The objective is to derive expressions for $\left\{({}^i\mathbf{q}_d^O(t), {}^i\omega_d^O(t), {}^i\mathbf{r}_d^O(t), {}^i\mathbf{v}_d^O(t), {}^i\mathbf{a}_d^O(t))\right\}_{i=1}^N$ as a function of the known quantities ${}^R\mathbf{q}_d^O(t)$, ${}^R\omega_d^O(t)$, ${}^R\mathbf{r}^C$, ${}^C\mathbf{r}^O$ and $\left\{({}^i\mathbf{q}_d^C, {}^i\mathbf{r}_d^C)\right\}_{i=1}^N$, where ${}^i\mathbf{a}_d^O(t)$ is the desired acceleration of spacecraft i with respect to the inertial frame. The expressions are given below:

$$\begin{aligned}
{}^i\mathbf{q}_d^O(t) &= \left({}^R\mathbf{q}_d^O(t)\right)\left({}^i\mathbf{q}_d^C\right), \\
{}^i\omega_d^O(t) &= -\,{}^R\omega_d^O(t), \\
{}^i\mathbf{r}_d^O(t) &= {}^C\mathbf{r}^O + {}^R\mathbf{r}^C + \left({}^R\mathbf{q}_d^O(t)\right)\left({}^i\mathbf{r}_d^C - {}^R\mathbf{r}^C\right)\left({}^R\mathbf{q}_d^O(t)\right)^*, \\
{}^i\mathbf{v}_d^O(t) &= {}^R\omega_d^O(t) \times \left({}^R\mathbf{q}_d^O(t)\right)\left({}^i\mathbf{r}_d^C - {}^R\mathbf{r}^C\right)\left({}^R\mathbf{q}_d^O(t)\right)^*, \\
{}^i\mathbf{a}_d^O(t) &= {}^R\omega_d^O(t) \times {}^R\omega_d^O(t) \times \left({}^R\mathbf{q}_d^O(t)\right)\left({}^i\mathbf{r}_d^C - {}^R\mathbf{r}^C\right)\left({}^R\mathbf{q}_d^O(t)\right)^* \\
&\quad + \frac{d^R\omega_d^O(t)}{dt} \times \left({}^R\mathbf{q}_d^O(t)\right)\left({}^i\mathbf{r}_d^C - {}^R\mathbf{r}^C\right)\left({}^R\mathbf{q}_d^O(t)\right)^*.
\end{aligned}$$

Step 3. The third step is to design tracking controllers that cause each spacecraft to track the desired trajectories. As a first pass at this problem, PD tracking compensators were designed for both the rotational and translational motion. More sophisticated control laws will be investigated as this research progresses.

The control law for attitude tracking control is given by [1, 4]

$$\tau_{ci} = K_{i1}\left[{}^i\epsilon_d^C\left({}^i\eta_d^C - {}^i\eta^C\right) - \left({}^i\epsilon_d^C - {}^i\epsilon^C\right) - {}^i\eta_d^C \times {}^i\eta^C\right] \\ - \frac{1}{2}{}^i\omega_d^C \times \left(\tilde{I}_i\left({}^i\omega_d^C - {}^i\omega^C\right)\right) + K_{2i}\tilde{I}_i\left({}^i\omega_d^C - {}^i\omega^C\right).$$

The translational motion is controlled via

$$f_{ci} = \tilde{M}_i\left[{}^i\mathbf{a}_d^C + K_{vi}\left({}^i\mathbf{v}_d^C - {}^i\mathbf{v}^C\right) + K_{pi}\left({}^i\mathbf{r}_d^C - {}^i\mathbf{r}^C\right)\right],$$

where K_{vi} and K_{pi} are positive constants. This control law, together with the translational dynamics of the spacecraft given in equation (1) result in second order error dynamics given by

$$\ddot{e} + K_{vi}\dot{e} + K_{pi}e = f_e,$$

where $e = {}^i\mathbf{r}^C - {}^i\mathbf{r}_d^C$.

2.2.2 Choice of R

In this section we will describe how to pick the vector ${}^R\mathbf{r}^C$ *a priori*, such that there is a quantitative tradeoff between minimizing the fuel used by the sum of the spacecraft and equalizing the fuel used by each spacecraft. The objective is to avoid starving any particular spacecraft.

The key idea is to derive an expression, as a function of ${}^R\mathbf{r}^C$, for the amount of fuel used by each spacecraft during a minimum-time, minimum-fuel maneuver about the constellation eigen-axis and then to minimize an expression of the form

$$\sum_{i=1}^{N} f_i + \mu \sum_{i=1}^{N} \frac{f_i}{\sum_{j=1}^{N} f_j} \log \frac{f_i}{\sum_{j=1}^{N} f_j}, \tag{4}$$

where f_i is the expected amount of fuel expended by the i^{th}spacecraft during the maneuver. The first term attempts to minimize the total fuel expended in the maneuver. The second term is minimized when $f_1 = f_2 = \cdots = f_N$.

The problem is decomposed into the following steps.

Step 1. Find the eigen-axis and the Euler angle.

Step 2. Assign the peak thrust for each spacecraft according to its distance from the rotation axis.

Step 3. Determine the thrust profile for each spacecraft according to a minimum-time, minimum-fuel tradeoff.

Step 4. Determine the amount of fuel used by each spacecraft during a maneuver, and the total amount of fuel used by the constellation.

Step 5. Write equation (4) as a function of ${}^R\mathbf{r}^C$ and minimize.

Step 1. The first step is to determine the eigen-axis $\mathbf{k}$ and Euler angle $\hat{\phi}$, from the current constellation orientation given by ${}^C\mathbf{q}^O$ and the desired orientation given by ${}^C\mathbf{q}_d^O$. The appropriate equations are [2]

$$\mathbf{q}_e = \left({}^C\mathbf{q}^O\right)^* \left({}^C\mathbf{q}_d^O\right) \triangleq \begin{pmatrix} \mathbf{k}\sin(\frac{\hat{\phi}}{2}) \\ \cos(\frac{\hat{\phi}}{2}) \end{pmatrix}$$

$$\hat{\phi} = 2\cos^{-1}(\epsilon_e)$$

$$\mathbf{k} = \begin{cases} \eta_e \sin(\frac{\hat{\phi}}{2}) & \text{if } \hat{\phi} \neq 0, \\ \mathbf{0} & \text{if } \hat{\phi} = 0. \end{cases}$$

Step 2. Assume that the rotation will be about the eigen-axis $\mathbf{k}$, and that all spacecraft rotate synchronously about $\mathbf{R}$ and that the maximum thrust available to each spacecraft is $\hat{A}$. Assuming that $\mathbf{k}$ emanates from $\mathbf{R}$, the distance of the i^{th}spacecraft from the eigen-axis is given by ${}^i\mathbf{r}^R - (\mathbf{k}^T{}^i\mathbf{r}^R)\mathbf{k} = (I - \mathbf{k}\mathbf{k}^T){}^i\mathbf{r}^R$. Since all spacecraft rotate about a single axis, the equation of motion of each spacecraft can be projected onto a single plane to obtain $M_i \left\|(I - \mathbf{k}\mathbf{k}^T){}^i\mathbf{r}^R\right\| \ddot{\theta} = \|A_i\|$ where M_i is the mass of the i^{th}spacecraft, A_i is the thrust of the i^{th}spacecraft, and $\ddot{\theta}$ is the generalized (combination of linear and centripetal) acceleration. Since

$$\ddot{\theta} = \frac{\|A_i\|}{M_i \left\|(I - \mathbf{k}\mathbf{k}^T){}^i\mathbf{r}^R\right\|}$$

is identical for each spacecraft we have that

$$\frac{\|A_1\|}{M_1 \left\|(I - \mathbf{k}\mathbf{k}^T){}^1\mathbf{r}_d^R\right\|} = \cdots = \frac{\|A_N\|}{M_N \left\|(I - \mathbf{k}\mathbf{k}^T){}^N\mathbf{r}_d^R\right\|}.$$

Define α as

$$\alpha = \arg\max_{1\leq i\leq N} \left\|(I - \mathbf{k}\mathbf{k}^T){}^i\mathbf{r}^R\right\|.$$

Then $\|A_\alpha\| = \hat{A}$ is the maximum thrust for spacecraft α and

$$\|A_i\| = \frac{\hat{A}\left\|(I - \mathbf{k}\mathbf{k}^T)\,{}^i\mathbf{r}^R\right\|}{M_\alpha\left\|(I - \mathbf{k}\mathbf{k}^T)\,{}^\alpha\mathbf{r}^R\right\|} = \frac{\hat{A}\left\|(I - \mathbf{k}\mathbf{k}^T)({}^i\mathbf{r}^C - {}^R\mathbf{r}^C)\right\|}{M_\alpha\left\|(I - \mathbf{k}\mathbf{k}^T)({}^\alpha\mathbf{r}^C - {}^R\mathbf{r}^C)\right\|} \tag{5}$$

is the maximum thrust for the i^{th}spacecraft when $i \neq \alpha$. Note that an upper bound on the maximum angular acceleration of the constellation is

$$A \triangleq \frac{\hat{A}}{M_\alpha\left\|(I - \mathbf{k}\mathbf{k}^T)({}^\alpha\mathbf{r}^C - {}^R\mathbf{r}^C)\right\|}. \tag{6}$$

Step 3. As shown in [5], a minimum-time, minimum-fuel trajectory for a double integrator plant with actuator saturation is given by a bang-off-bang control trajectory (i.e., a trust trajectory like the one shown in equation (7)). Therefore we obtain the following possible trajectories for ϕ, the angle about $\mathbf{k}$:

$$\ddot{\phi}(t) = \begin{cases} A; & 0 \le t \le \hat{T} \\ 0; & \hat{T} \le t \le T - \hat{T} \\ -A; & T - \hat{T} \le t \le T, \end{cases} \tag{7}$$

$$\dot{\phi}(t) = \begin{cases} At; & 0 \le t \le \hat{T} \\ A\hat{T}; & \hat{T} \le t \le T - \hat{T} \\ A(T - t); & T - \hat{T} \le t \le T, \end{cases}$$

$$\phi(t) = \begin{cases} \frac{1}{2}At^2; & 0 \le t \le \hat{T} \\ A\hat{T}t - \frac{1}{2}A\hat{T}^2; & \hat{T} \le t \le T - \hat{T} \\ A\hat{T}((T - \hat{T}) - \frac{1}{2}A(T - t)^2; & T - \hat{T} \le t \le T. \end{cases}$$

Therefore $\hat{\phi} = \phi(T) = A\hat{T}(T - \hat{T})$, which implies that

$$T = \frac{\hat{\phi}}{A\hat{T}} + \hat{T}$$

and

$$\hat{T} = \frac{T}{2} - \sqrt{\frac{T^2}{4} - \frac{\hat{\phi}}{A}}. \tag{8}$$

Assuming that the amount of fuel consumed by each spacecraft is linearly proportional to the amount of thrust, the amount of fuel consumed by the i^{th}spacecraft during the maneuver is

$$f_i = 2\gamma\hat{T}\left\|A_i\right\|. \tag{9}$$

Therefore the total amount of fuel consumed by the constellation is

$$F \triangleq \sum f_i = 2\gamma\hat{T}\sum\left\|A_i\right\|.$$

Substituting our previous expression for $\hat{T}$ gives

$$F = 2\gamma\sum\left\|A_i\right\|\left(\frac{T}{2} - \sqrt{\frac{T^2}{2} - \frac{\hat{\phi}}{A}}\right).$$

Solving for T we obtain

$$T = \frac{F}{2\gamma\sum\left\|A_i\right\|} + \frac{2\gamma\hat{\phi}\sum\left\|A_i\right\|}{AF}. \tag{10}$$

To trade off minimum-fuel and minimum-time we minimize the function

$$\min F + \lambda T = \min_F\left\{F + \lambda\left[\frac{F}{2\gamma\sum\left\|A_i\right\|} + \frac{2\gamma\hat{\phi}\sum\left\|A_i\right\|}{AF}\right]\right\},$$

for $\lambda > 0$. The minimum-time, minimum-fuel tradeoff is now explicitly in terms of λ. For $\lambda = 0$ we get the minimum-fuel (maximum-time, i.e. infinity) solution, as $\lambda \to \infty$ we get the minimum-time (maximum-fuel) solution. Setting the derivative of the above expression to zero and solving for F gives

$$F = 2\gamma\sum\left\|A_i\right\|\sqrt{\frac{\lambda\hat{\phi}}{2\gamma A\sum\left\|A_i\right\| + A\lambda}}. \tag{11}$$

Step 4. The fuel used by each spacecraft is given by equations (5–11). Note that since $\left\|A_i\right\|$ and A can be expressed in terms of ${}^R\mathbf{r}^C$, so can f_i, $\hat{T}$, T, and F.

Step 5. The final step is to minimize the function

$$H = F + \mu \frac{\sum_{i=1}^{N} f_i}{\sum_{j=1}^{N} f_j} \log \frac{f_i}{\sum_{j=1}^{N} f_j}. \tag{12}$$

Since F and f_i can be expressed as a function of ${}^{R}\mathbf{r}^{C}$ we set

$${}^{R}\mathbf{r}^{C} = \arg\min H.$$

H is a nonlinear functional that can be minimized effectively with a number of numerical tools, given a suitable initial condition for the algorithm. Most notably, we may use the Matlab function `fmins` to minimize the function. As an initial condition on the algorithm we use the center of fuel mass

$$\frac{\sum \frac{{}^{i}\mathbf{r}^{O}}{F_i(0)}}{\sum \frac{1}{F_i(0)}}, \tag{13}$$

where $F_i(0)$ is the amount of fuel possessed by the i^{th}spacecraft before the maneuver.

3 Constellation Rotation

3.1 Problem Description

The constellation rotation problem is shown graphically in Figure 3.

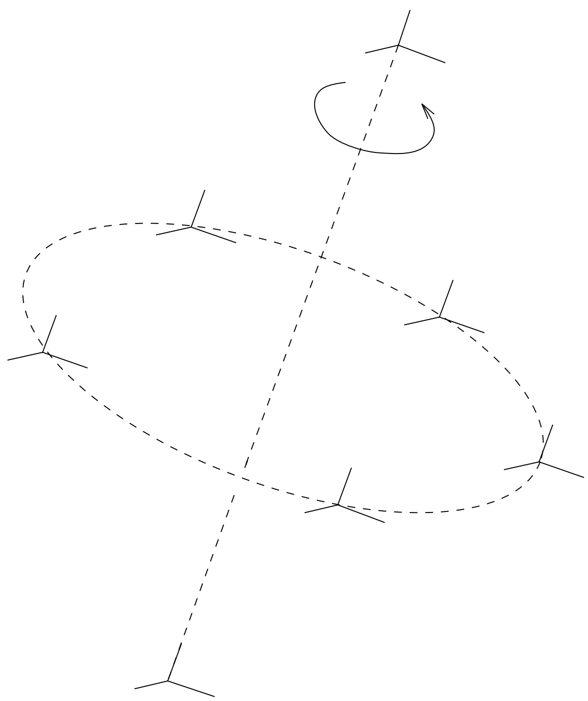

Figure 3: Constellation Rotation

The objective is to rotate the constellation about the axis joining the two combiner spacecraft. This problem has also been addressed by a similar approach in [6]. The difficulty with rotation maneuvers is that in order for the constellation to rotate as if it were a rigid body, the spacecraft are commanded to follow smooth arcs. To do so with on/off thrusters requires either continual firing of the thrusters, which is what would result from a pulse-width modulation approximation of the continuous control law. Frequent firing of the thrusters will drain the fuel and is, therefore, undesirable. For interferometry missions, we do not necessarily require that the constellation move precisely as a rigid body: there is an allowable tolerance on the relative distances between two spacecraft. Using these considerations, we can derive sub-optimal control laws that maintain the required tolerances while firing the thrusters a minimal amount of time.

3.2 An Algorithm for Constellation Rotation

Assuming continuous thrusters, the constellation rotation problem can be solved by the same algorithm developed for a constellation reorientation, with a few modification. First, fix the point $\mathbf{R}$ to be the center of the rotation. Presumably, $\mathbf{R}$ would be the baracenter of the two combiner spacecraft. Second, set

$$ {}^{C}\mathbf{q}^{O}{}_{d}(t) = \begin{pmatrix} \mathbf{k}\sin(\omega_0 t) \\ \cos(\omega_0 t) \end{pmatrix}, $$

where $\mathbf{k}$ is the axis of rotation and ω_0 is the desired rate of rotation about $\mathbf{k}$. The algorithm developed in Section 2 can now be applied directly to accomplish the continuous rotation about $\mathbf{k}$.

In the remainder of this section, we will consider the non-continuous thruster case. Suppose that for the purposes of the interferometry mission, we need to rotate the constellation at a fixed rate ω_0 while maintaining the distance between the $i^{\text{th}}$and j^{th} spacecraft to be $\|{}^{j}\mathbf{r}^{i}\| = d_{ij} \pm \epsilon_{ij}$. Let $\epsilon_i = \min_{j \neq i} \epsilon_{ij}$. We can guarantee that all of the distances are maintained within the required tolerances by ensuring that $\left\|{}^{i}\mathbf{r}^{R}(t) - {}^{i}\mathbf{r}^{R}_{d}(t)\right\| \leq \epsilon_i/2$. In addition, we want to maintain these tolerances while firing the thrusters as little as possible. Note that starting and stopping the rotation is not considered in this section. However, the ideas in this section extend easily to these tasks.

The idea that we will pursue is to only fire the i^{th}spacecraft's thrusters when it reaches its tolerance boundary. As shown in figures 4 and 5, the requirement that $\left\|{}^{i}\mathbf{r}^{R}(t) - {}^{i}\mathbf{r}^{R}_{d}(t)\right\| \leq \epsilon_i/2$ sweeps out a tube around the trajectory ${}^{i}\mathbf{r}^{R}_{d}(t)$. Assuming that the constellation is being commanded to rotate about $\mathbf{k}$, the motion of each spacecraft will be in a plane perpendicular to $\mathbf{k}$. By the conservation of momentum, we must apply an impulse thrust at the point that the spacecraft touches the boundary of the tolerance tube such that

$$ T_i = M_i {}^{i}\mathbf{v}^{O} - M_i {}^{i}\mathbf{v}^{O}{}_{new}, $$

where ${}^{i}\mathbf{v}^{O}{}_{new}$ is the velocity vector after the thrust is applied, and T_i is an impulsive thrust. Note that if an impulsive thrust is not used, then a firing sequence will need to be initiated before the spacecraft touches the boundary of the tolerance tube. Since the thruster will be limited to a magnitude of $\|A_i\|$, we apply a trust $\frac{\|A_i\| T_i}{\|T_i\|}$ for a period of $\|T_i\| \,/\, \|A_i\|$ seconds. Note that this will require that the tolerances be tightened slightly to allow momentum changes that are not instantaneous. It remains to find ${}^i\mathbf{v}^O{}_{new}$. We will consider two cases: when the spacecraft encounters an outside edge, and when it encounters an inside edge.

Case 1: Outside Edge. The geometry for this case is shown in Figure 4. From the figure we can see that

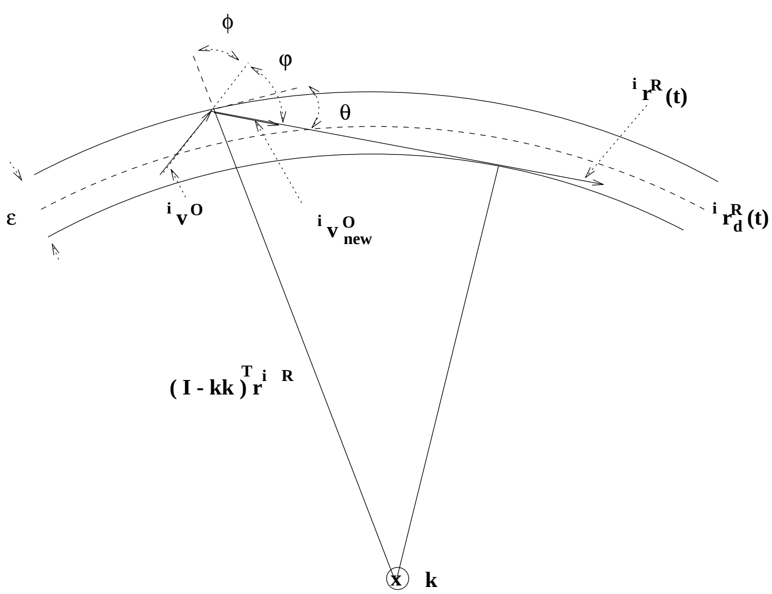

Figure 4: Constellation Rotations within a Specified Tolerance.

$$
\begin{aligned}
\phi &= \cos^{-1}\left[\frac{({}^i\mathbf{v}^O)^T(I-\mathbf{k}\mathbf{k}^T)\,{}^i\mathbf{r}^R}{\|{}^i\mathbf{r}^R\|\;\|(I-\mathbf{k}\mathbf{k}^T)\,{}^i\mathbf{r}^R\|}\right]\\
\theta &= \sin^{-1}\left[\frac{\|(I-\mathbf{k}\mathbf{k}^T)\,{}^i\mathbf{r}_d^R\|-\epsilon_i/2}{\|(I-\mathbf{k}\mathbf{k}^T)\,{}^i\mathbf{r}_d^R\|+\epsilon_i/2}\right]\\
\varphi &= \theta + (\pi/2-\phi).
\end{aligned}
$$

From the geometry we can see that

$$^{i}\mathbf{v}^{O}{}_{new} = \cos(\varphi)^{i}\mathbf{v}^{O} + \sin(\varphi)\mathbf{k} \times {}^{i}\mathbf{v}^{O}.$$

Case 2: Inside Edge. The geometry for this case is shown in Figure 5. From the figure we can see that

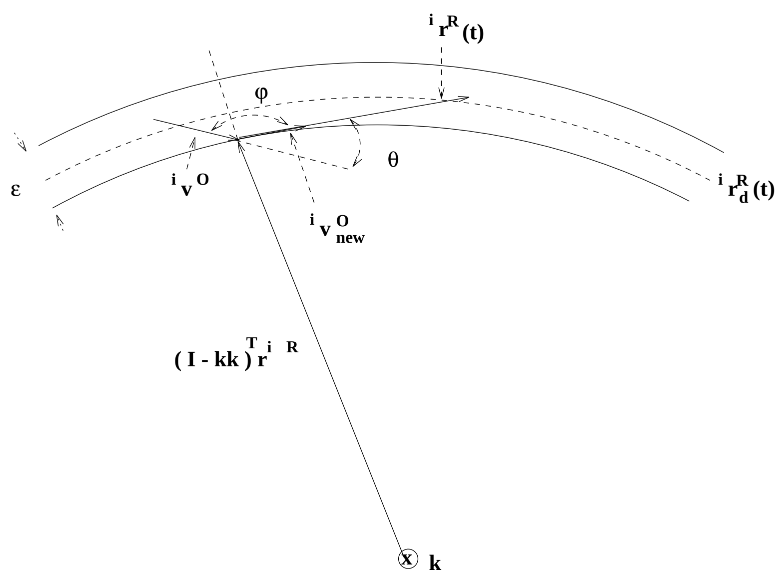

Figure 5: Constellation Rotations within a Specified Tolerance.

$$\theta = \cos^{-1}\left[\frac{-({}^{i}\mathbf{v}^{O})^{T}(I - \mathbf{k}\mathbf{k}^{T})^{i}\mathbf{r}^{R}}{\|{}^{i}\mathbf{v}^{O}\| \, \|(I - \mathbf{k}\mathbf{k}^{T})^{i}\mathbf{r}^{R}\|}\right]$$
$$\varphi = \pi - \theta.$$

From the geometry we can see that

$$^{i}\mathbf{v}^{O}{}_{new} = -\cos(\varphi)^{i}\mathbf{v}^{O} + \sin(\varphi)\mathbf{k} \times {}^{i}\mathbf{v}^{O}.$$

4 Constellation Expansion/Contraction

4.1 Problem Description

The problem of constellation expansion is illustrated in Figure 6.

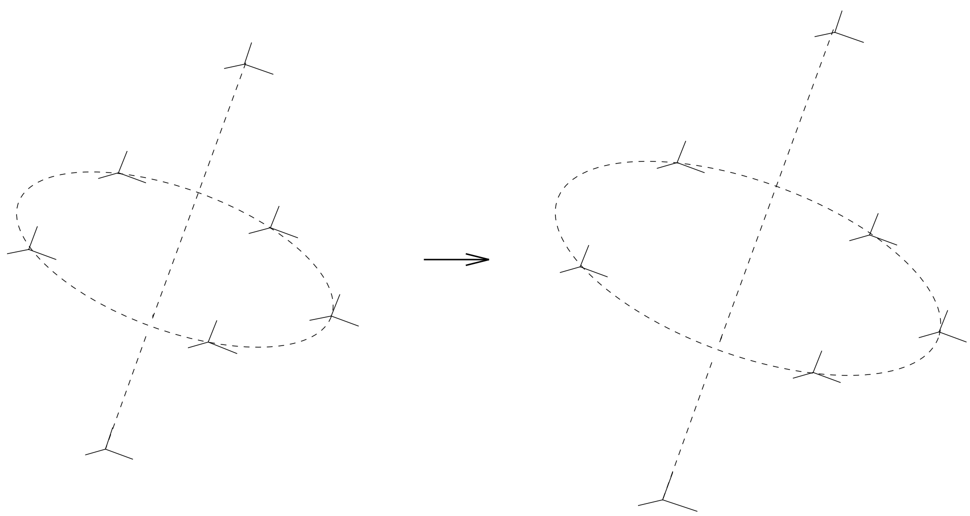

Figure 6: Constellation Expansion.

In this figure the constellation has been expanded by a factor of two with respect to the fixed point in space **R**. As before a wise placement of the point **R** will limit the distance that each spacecraft must travel and thus allow fuel to be conserved. The closer that **R** is chosen to a given spacecraft, the less distance that the spacecraft will have to move. Therefore, **R** may be placed close to a spacecraft that is low on fuel in order to conserve its limited resources.

In the expansion problem, rather than exert torque on the template, we will exert an expansion inducing force on the template. This force is analogous to the force that causes a balloon to expand.

4.2 An Algorithm for Constellation Expansion/Contraction

In this section we develop an algorithm for constellation expansion with respect to a fixed point **R**.

Define a sizing factor $\nu(t)$ that measures the growth of the constellation. The template is defined by the desired trajectories of the of the individual spacecraft. The desired trajectory of the i^{th}spacecraft will obey

$$\begin{aligned} {}^i\mathbf{r}_d^R(t) &= \nu(t)\,{}^i\mathbf{r}^R(0), \\ {}^i\mathbf{v}_d^R(t) &= \dot{\nu(t)}\,{}^i\mathbf{r}^R(0), \\ {}^i\mathbf{a}_d^R(t) &= \ddot{\nu(t)}\,{}^i\mathbf{r}^R(0). \end{aligned} \tag{14}$$

Define the tracking error as

$$\begin{aligned} e_i &= {}^i\mathbf{r}^R - {}^i\mathbf{r}_d^R \\ \dot{e}_i &= {}^i\mathbf{v}^R - {}^i\mathbf{v}_d^R. \end{aligned}$$

We ensure that the initial tracking error is zero simply by setting the initial conditions on $\nu(t)$ to be

$$\begin{aligned} \nu(0) &= 1 \\ \dot{\nu}(0) &= 0. \end{aligned}$$

As before, we first develop a virtual control for the template, and then we will develop a control for the individual spacecraft.

Given a final desired size, ν_f, for the template, the template resizing force

$$F_{ti} = M_i(-k_1\,{}^i\mathbf{v}_d^R - k_2({}^i\mathbf{r}_d^R - \nu_f\,{}^i\mathbf{r}^R(0)) \tag{15}$$

will result in a closed loop template dynamics for ν of

$$\ddot{\nu} + k_1\dot{\nu} + k_2(\nu - \nu_f) = 0.$$

Equation (14) gives the desired trajectories for each member of the constellation. The i^{th}spacecraft may now track its desired trajectory using the control law

$$F_i = M_i({}^i\mathbf{a}_d^R - k_{1i}({}^i\mathbf{v}^R - {}^i\mathbf{v}_d^R) - k_{2i}({}^i\mathbf{r}^R - {}^i\mathbf{r}_d^R)), \tag{16}$$

which results in the closed loop dynamics

$$\ddot{e}_i + k_1\dot{e}_i + k_2 e_i = 0.$$

A careful selection of gains will allow exponential convergence of the tracking error. Moreover, if two neighboring spacecraft are within some tolerance of each other, they will remain within that tolerance.

A simple application of Lyapunov theory gives us the following result.

Theorem 4.1 (Constellation Expansion Convergence) *Given the sizing factor dynamics*

$$\ddot{\nu} + k_3\dot{\nu} + k_4(\nu - \nu_f) = 0,$$

and the spacecraft force

$$F_i = m({}^i\mathbf{a}_d^R - k_1{}^i\mathbf{v}^R - {}^i\mathbf{v}_d^R) - k_2({}^i\mathbf{r}^R - {}^i\mathbf{r}_d^R)), \tag{17}$$

then for positive gains, $\nu \to \nu_f$ and $e_i(t) \to 0$ as $t \to \infty$.

5 Constellation Initialization

5.1 Problem Description

This section considers the problem of initializing a constellation of spacecraft. There are several instances when a constellation must be initialized. For example, initialization must take place 1) immediately after the spacecraft are deployed from the launch vehicle, 2) when one spacecraft in the constellation experiences catastrophic failure, and 3) when the overall task of the community of spacecraft dramatically changes.

In this section we will use a radically different approach than in the three previous sections. The reason for doing so is that the constellation initialization problem is complicated by the following issues which are not easily addressed in the constellation template framework.

- The constellation may be a heterogeneous collection. For example, to build a space based interferometer, combiner and collector spacecraft are needed. The functions, and hence hardware, of these spacecraft will be radically different. Initializing the constellation may require that spacecraft of type A be located along the axis of the constellation, while spacecraft of type B be positioned in a concentric ring about the center of the axis. One spacecraft, of the correct type, must occupy each specified location, but individual spacecraft of the same type are not necessarily assigned to particular locations.

- The spacecraft must position themselves without colliding with other spacecraft.

- Each spacecraft may be equipped with sensors that detect the position of other spacecraft near its vicinity, but the sensors may be limited in range. Hence it may not be possible for any given spacecraft to know the relative or inertial positions of all the other spacecraft.

- Spacecraft may not initially be able to communicate with each other. Therefore, it may not be possible for a given spacecraft to know which positions in the constellation its nearest neighbors have decided to seek.

- Spacecraft life is limited by fuel. Therefore it is desirable that any initialization scheme keep the total distance traveled by all spacecraft to a minimum.

To be concrete, we will focus on the following scenario, which is designed to include all of the difficulties listed above. Consider a system of five spacecraft that are collectively instructed, via some form of ground communication, to initialize themselves into a specified configuration with a fixed inertial position and orientation. Each spacecraft must occupy exactly one location in the configuration and must effect its transition without colliding with others. Simultaneously, the total distance traveled by all spacecraft must be minimized. Each spacecraft is able to position itself in an inertial coordinate frame, but no centralized control is possible; decision making is distributed, with each entity deciding for itself which location to seek. The sensors on board each spacecraft permit it to determine the relative positions of only its two closest neighbors. Thus, it is not possible for any given spacecraft to know the relative positions of all other spacecraft when deciding which location to seek. In addition, no communication between spacecraft is possible.

The above scenarios may be viewed as instantiations of an N-agent coordination game, the classical solution of which is for all players to seek a coordination equilibrium that is globally optimal [7, 8]. Unfortunately, due to knowledge limitations, this is a game of asymmetric information, so the classical solution cannot be implemented.

We approach the problem by creating a hierarchical controller structured after the theory of intelligent machines developed in [9, 10, 11]. The control architecture consists of three levels: the Organization Level, the Coordination Level, and the Execution Level. For the scenario described above, the organization level dictates the desired configuration for the constellation. The execution level consists of simple control laws that move a spacecraft to a desired location and avoid collisions. The coordination level must decide, with limited information, which position a particular spacecraft should seek.

5.2 An Algorithm for Constellation Initialization

A block diagram of the system architecture is shown in Figure 7.

The spacecraft are at the lowest level, interacting directly with the environment through sensors and actuators. The execution level consists of two algorithms. A simple PD controller that moves each spacecraft toward the location specified by the Coordination Level, and a collision avoidance algorithm. The collision avoidance algorithm creates a control force that repels the spacecraft from any other spacecraft that is a distance D from it.

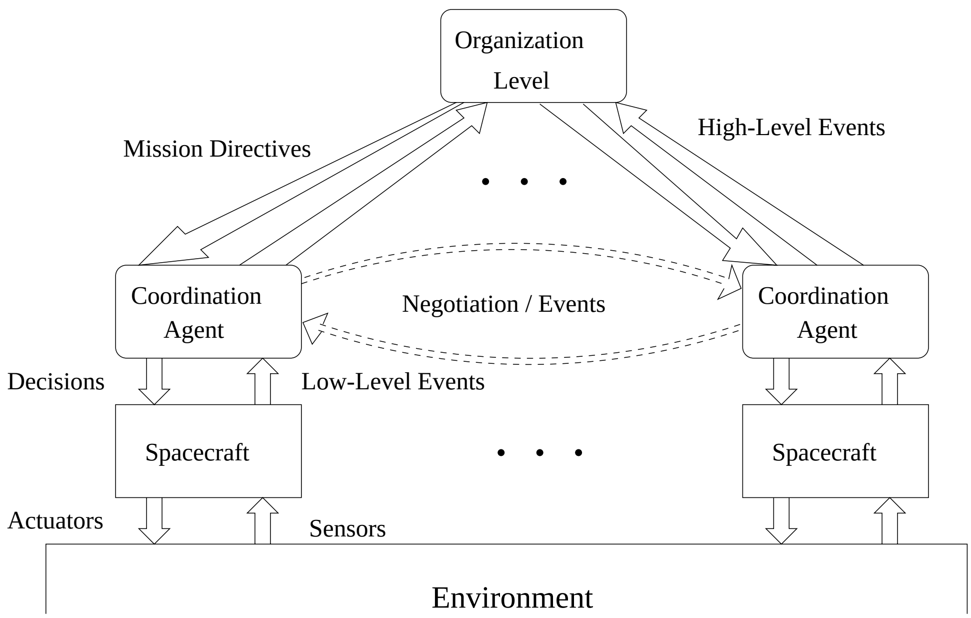

Figure 7: Block Diagram of the System Architecture.

Each spacecraft has an associated coordination agent at the Coordination Level. The coordination agent receives a high level directive from the Organization Level. For the formation initialization problem, the directive consists of a list of desired locations. The coordination agent uses this information, along with the current location of the other spacecraft in its field of view, to make a decision as to which location the spacecraft should move to. It then directs its associated spacecraft to move to the appropriate location. Decisions at the Coordination Level are only made when low-level events occur. For example, when a new spacecraft comes into the field of view of the current spacecraft or when a neighboring spacecraft has reached one of the specified locations. Since decisions are event driven, the data flow to the Coordination Level is significantly reduced. This is in accordance with the principle of "increasing precision with decreasing intelligence" [10]. If communication between spacecraft is allowed, then the coordination agents may negotiate their decisions with the other agents. In this paper we will not consider communication among coordinating agents.

The Organization Level shown in Figure 7 is, in our example, an entity (probably human) on earth that is communicating to the system the desired configuration. The high-level events passed back to the Organization Level are success/failure events.

5.2.1 Coordinated Decision Making

Making coordinated decisions is an extremely complex problem. For an agent to coordinate effectively with other agents without negotiations, it must possess a model of itself and all other individual agents, as well as a model of how each agent will interact with the other agents in the system. It must then use these models to make a decision that is appropriate, given the available knowledge. Viewed from this perspective, there are two main issues to be considered: the representation of knowledge, and the rationale for making decisions based on that knowledge. To facilitate discussion, we first describe our model of knowledge representation and decision making for a single agent, then generalize to the case of multiple agents.

Knowledge Representation. Knowledge representations in common use include dynamic models based on physical principles, models consisting of production rules, and probability models. Multiple-agent coupled dynamic models are difficult to specify and often involve unknown parameters whose uncertainty must be included in the model, as well as discrete events that may alter model structure. Production rules are well-suited for representing local, conditional behavior, but it has proven difficult in practice to anticipate every possible scenario and include an appropriate rule, especially in dynamic, uncertain environments. Probability models, however, are readily applicable for characterizing both global and local behavior. Probabilities are typically used as a measure of truth support for a set of events, with unconditional probabilities characterizing global truth support, and conditional probabilities representing local truth support, that is, truth support given specific circumstances. As indicated by Shafer, "probability is not really about numbers, it is about the *structure of reasoning* [italics added]." [12, page 15]. Probability networks such as those developed by Pearl, for example, have been shown to be powerful and convenient means for representing knowledge [12].

Epistemic utility theory employs **two** distinct probability models to represent knowledge; one to serve as a measure of *truth support,* and the other to serve as a measure of *informational value of rejection.* Probability as a measure of truth support uses the conventional semantics of probability theory but as a measure of informational value requires an entirely different semantics. The mathematics of probability has been so intertwined historically with a particular semantics that the two may appear to be indistinguishable, but the mathematics stands by itself and can easily be associated with different semantics. With the conventional semantics, there is a unit of truth support apportioned over a Boolean algebra of events in a finite probability space. Choosing a particular event implies a corresponding amount of truth support is committed to the agent. With the alternative semantics, a unit of informational value is apportioned over the same Boolean algebra. By *rejecting* a particular event, the corresponding amount of informational value is committed to the agent. These two probability functions thus encode knowledge pertaining to both belief and importance of the events, and do so with comparable units.

Although epistemic utility theory was developed as a model of cognitive decision making, it may be readily adapted to practical decision making by assigning new semantics to both of the probability functions. In the place of "truth," let us associate "accuracy," meaning *conformity to a standard.* In the cognitive context, the standard is factuality, but in a practical context, the standard is associated with the goals or aspirations of the agent. The corresponding probability function then becomes a measure of the degree to which accuracy is achieved by adopting a particular event. To distinguish this semantics from the traditional one, we will refer to this function as the *credibility* function and denote it p_C. Also, in the place of "informational value of rejection," let us associate the notion of *resource depletion,* and the corresponding probability function becomes a measure of the degree to which the adoption of a given event consumes the available resources. To distinguish this semantics, we shall refer to this probability function as the *rejectability* function and denote it p_R.

Decision-Making Rationale. Optimization is a mathematical sophistication of the common sense view that decision makers ought to do the best they can. The von Neumann-Morgenstern instantiation of this view requires that the set of options available to a decision maker be described by a single preference ordering. This ordering is provided by a utility function, which numerically weights the relative importance of the various attributes associated with the decision in question. An optimal option is then a maximal element with respect to this ordering, subject to whatever constraints are relevant. A decision maker who seeks an optimal solution is operating under the *superlative* paradigm. This paradigm is often challenged, however, since it is well known that people are poor optimizers, and strict adherence to the optimization paradigm inadequately describes how decisions are often made in naturalistic settings [13, 14, 15, 16, 17, 18, 19, 20, 21]. Furthermore, the superlative paradigm is very rigid. It requires that sufficient information be available to guarantee that the optimal solution can be identified; if the information is not sufficient, no decision can be reached.

Epistemic utility theory does not fit the superlative paradigm, since it employs two utility functions—credibility and rejectability—rather than the single utility function employed under the superlative paradigm. These two utility functions, being expressed in the same scale (i.e., each has a unit of its attribute to apportion), invite a binary *comparison* of these attributes for each option. A natural decision rule in this context is to reject all options for which the rejectability (the amount resources are depleted, i.e., the cost) exceeds the credibility (the degree to which the goal is achieved, i.e., the benefit). An example of a comparative approach is the familiar cost-benefit analysis of economics.

The *comparative* decision paradigm is distinctly different from the superlative paradigm of optimal decision theory. Whereas the superlative paradigm is designed to yield a single "best" option, the comparative paradigm is designed to yield a *set* of options (those for which credibility exceeds rejectability), each of which is "good enough," if not optimal. We shall refer to such non-rejected options as *satisficing*[1] options, and the set of satisficing options is termed the *satisficing set*. Any element of this set may be implemented; alternatively, a subjective tie-breaking mechanism may be invoked to make a unique selection.

Although this comparative procedure does not have the obvious normative power of a superlative procedure, it does serve as an apt description of the way people often behave and satisfies one of many possible definitions of common sense rationality. Furthermore, the two paradigms are consistent in the following sense: if the credibility and rejectability functions are designed according to the same criteria used to define an optimal performance index, then the optimal decision can be shown to be a member of the satisficing set.

[1] *Satisficing* = *satis*fy + suf*fice* is evidently a word of Scottish origin, and has been used by economists to describe decision rules that, though not optimal, achieve a performance commensurate with a given aspiration level.

Multiple Agent Coordination. Since epistemic utility theory involves probability functions, it may be extended to the case of multiple agents by defining multivariate probability functions and invoking a comparative decision rule that yields decision *vectors*—one component for each agent. Such a rule requires expressions for *joint credibility* and *joint rejectability*. To develop these expressions, we must consider the relationships that exist between agents. The credibility of a particular option for a given agent will, in general, be influenced by the credibility and rejectability that all other agents ascribe to their particular options. Let $\{X_1, \ldots, X_N\}$ denote a system of N agents, let U_i denote X_i's set of admissible options (assumed, for this discussion, to be finite), and consider the *joint* credibility/rejectability associated with X_i opting for $u_i \in U_i$ in the interest of credibility, and of opting for $v_i \in U_i$ in the interest of rejectability, $i = 1, \ldots, N$. We shall call this function the *interdependence function* and denote it $p_{C_1 \ldots C_N R_1 \ldots R_N}(u_1, \ldots, u_N, v_1, \ldots, v_N)$. The interdependence function is a $2N$-variate probability mass function that characterizes the joint credibility/rejectability associated with the joint credibility/rejectability state $(u_1, \ldots, u_N, v_1, \ldots, v_N)$, where $u_i \in U_i$ and $v_i \in U_i$, $i = 1, \ldots, N$. We interpret this function as follows: The joint credibility associated with the agents jointly adopting the options $(u_1, \ldots, u_N)$ is given by

$$p_{C_1 \cdots C_N}(u_1, \ldots, u_N) = \sum_{v_1 \in U_1} \cdots \sum_{v_N \in U_N} p_{C_1 \cdots C_N R_1 \cdots R_N}(u_1, \ldots, u_N, v_1, \ldots, v_N), \tag{18}$$

and the joint rejectability associated with the agents jointly adopting the options $(v_1, \ldots, v_N)$ is given by

$$p_{R_1 \cdots R_N}(v_1, \ldots, v_N) = \sum_{u_1 \in U_1} \cdots \sum_{u_N \in U_N} p_{C_1 \cdots C_N R_1 \cdots R_N}(u_1, \ldots, u_N, v_1, \ldots, v_N). \tag{19}$$

Once the interdependence function is specified, the joint credibility and rejectability functions may be obtained via (18) and (19), and the set of *jointly satisficing* options is

$$\mathbf{S}_b = \{(w_1, \ldots, w_N) \in U_1 \times \cdots \times U_N : p_{C_1 \cdots C_N}(w_1, \ldots, w_N) \geq b p_{R_1 \cdots R_N}(w_1, \ldots, w_N)\}, \tag{20}$$

that is, the jointly satisficing options are all joint options such that the joint credibility is not exceeded by b times the joint rejectability. The parameter,

b, is termed the rejectivity index, and is a measure of relative weight given credibility and rejectability. Nominally, we set $b = 1$, reflecting equal weight.

To apply this theory to the problem of spacecraft initialization, we must first settle on operational definitions for credibility and rejectability. Credibility will be defined to keep the distance a spacecraft must travel as small as possible. Thus, target locations close to a spacecraft will have higher credibility than distant target locations. Rejectability will reflect the undesirability of having more than one spacecraft at a given target. Thus, locations closer to other spacecraft and therefore likely to be occupied will have higher rejectability than locations more likely to be open. With these operational definitions in place, we may define the interdependence function for each coordination agent.

Let X_1 denote the subject agent, and let X_2 and X_3 denote the two spacecraft within X_1's field of view. We must define the interdependence function, $p_{C_1C_2C_3R_1R_2R_3}$, for the subject agent. By the product rule for probabilities, we may write the interdependence function as (suppressing arguments)

$$p_{C_1C_2C_3R_1R_2R_3} = p_{C_1|R_1C_2C_3R_1R_2R_3} p_{R_1C_2C_3R_2R_3}. \tag{21}$$

Since credibility deals only with the choice of the target location, it is reasonable to assume that X_1's credibility is independent of the rejectabilities associated with any of the agents, hence we may simplify (21) to become

$$p_{C_1C_2C_3R_1R_2R_3} = p_{C_1|C_2C_3} p_{R_1C_2C_3R_2R_3}. \tag{22}$$

Applying the product rule to the second term of (22) we obtain

$$p_{C_1C_2C_3R_1R_2R_3} = p_{C_1|C_2C_3} p_{R_1|C_2C_3R_2R_3} p_{C_2C_3R_2R_3}. \tag{23}$$

Now, assuming that rejectability can be determined independently by the three agents, we may simplify this expression to

$$p_{C_1C_2C_3R_1R_2R_3} = p_{C_1|C_2C_3} p_{R_1|C_2C_3} p_{C_2C_3R_2R_3}. \tag{24}$$

Further application of the product rule and applying the arguments above yields

$$p_{C_1C_2C_3R_1R_2R_3} = p_{C_1|C_2C_3} p_{R_1|C_2C_3} p_{C_2|C_3} p_{R_2|C_3} p_{C_3} p_{R_3}. \tag{25}$$

This factorization exercise illustrates one aspect of the great power of probability models—namely, conditional probability mass functions allow us to represent local information in a modular structure. For example, $p_{C_1|C_2C_3}(\alpha_1|\alpha_2, \alpha_3)$ represents the credibility that X_1 places on target location α_1, given that X_2 and X_3 place their entire units of credibility on target locations α_2 and α_3. Also, $p_{R_1|C_2C_3}(\beta_1|\alpha_2, \alpha_3)$ represents the rejectability that X_1 places on location β_1 given that X_2 and X_3 place their entire units of credibility on target locations α_2 and α_3. Simple analytical expressions for these conditional probabilities may easily be developed as functions of the target locations and the locations of the spacecraft. In this way, every possible scenario can be represented—specifying these conditional probability functions is tantamount to specifying a family of production rules to cover all possible circumstances.

As mentioned earlier, the complexity of this approach will grow rapidly with the number of agents, especially if the agents are permitted arbitrary interactions. However, it must be stressed that the coordination of multiple agents is intrinsically a very complex problem, and one would expect that models adequate to characterize complex relationships between agents must also be complex. It may be well to recall the pithy advice of Einstein: "Make things as simple as possible, but not simpler." In most problems, it will be desirable to reduce complexity by imposing reasonable assumptions, such as hierarchical relationships between coordination agents. The use of conditional probability mass functions is an efficient way to encode such simplifying structure.

With the conditional pmf's in place, the interdependence function for the agent is fully defined by (25). The next step is to form the joint credibility and rejectability functions and to compute the jointly satisficing sets for the collection of agents within the field of view. Each agent will then invoke a tie-breaker, if necessary, and implement its component of the selected joint option. The joint credibility is obtained as

$$\begin{aligned} p_{C_1C_2C_3}(\alpha_1, \alpha_2, \alpha_3) &= \sum_{\beta_1 \in U} \sum_{\beta_2 \in U} \sum_{\beta_3 \in U} p_{C_1C_2C_3R_1R_2R_3}(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3) && (26) \\ &= p_{C_1|C_2C_3}(\alpha_1|\alpha_2, \alpha_3) p_{C_2|C_3}(\alpha_2|\alpha_3) p_{C_3}(\alpha_3), && (27) \end{aligned}$$

and the joint rejectability is given by

$$p_{R_1R_2R_3}(\beta_1,\beta_2,\beta_3) = \sum_{\alpha_1\in U}\sum_{\alpha_2\in U}\sum_{\alpha_3\in U} p_{C_1C_2C_3R_1R_2R_3}(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\beta_3) \quad (28)$$

$$= p_{R_3}(\beta_3)\sum_{\alpha_2\in U}\sum_{\alpha_3\in U} p_{R_1|C_2C_3}(\beta_1|\alpha_2,\alpha_3)p_{C_2|C_3}(\alpha_2|\alpha_3)p_{R_2|C_3}(\beta_2|\alpha_3)p_{C_3}(\alpha_3) \quad (29)$$

The set of jointly satisficing options, from the point of view of X_1, is then

$$\mathbf{S}_b = \{(\alpha_1,\alpha_2,\alpha_3) \in U\times U\times U : p_{C_1C_2C_3}(\alpha_1,\alpha_2,\alpha_3) \geq b p_{R_1R_2R_3}(\alpha_1,\alpha_2,\alpha_3)\}. \quad (30)$$

We must now formally specify the myopic and joint credibility and rejectability functions. The myopic credibility function is computed by rewarding locations that are close to the agent, according to

$$p_{C_3}(\alpha_3) = \frac{\frac{1}{\|y_{d\alpha_3} - y_3\|}}{\sum_{\gamma\in U}\frac{1}{\|y_{d\gamma}-y_3\|}},$$

where U is the index set of the three closest desired locations, y_{dj} are the desired locations, and y_j are the current locations of the spacecraft. The rejectability function is computed by rejecting desired locations that have other spacecraft closer than the deciding agent, as described by

$$p_{R_3}(\beta_3) = \frac{\|y_{d\beta_3} - y_3\| - \rho(y_{d\alpha_3})}{\sum_{\gamma\in U}\left(\|y_{d\gamma} - y_3\| - \rho(y_{d\gamma})\right)},$$

where $\rho(y_{dj})$ is a function that returns the distance, from y_{dj}, of the spacecraft that is in the agents field of view and is closest to the desired position y_{dj}.

We interpret the probability mass function $p_{R_2|C_3}(\beta_2|\alpha_3)$ to be the rejectability X_2 would ascribe to location β_2 given that X_3 places its entire unit of credibility mass on α_3. The structure of this function is simple, since it is easily seen that X_2 should place its entire unit of rejectability on the location that X_3 places its unit of credibility mass, thus

$$p_{R_2|C_3}(\beta_2|\alpha_3) = \begin{cases} 1 & \beta_2 = \alpha_3 \\ 0 & \beta_2 \neq \alpha_3. \end{cases}$$

The probability mass function $p_{C_2|C_3}(\alpha_2|\alpha_3)$ corresponds to the credibility X_2 would ascribe to α_2 given that X_3 places its entire unit of credibility on α_3. Setting $p_{C_2|C_3}(\alpha_2|\alpha_3) = 0$ ensures that X_2 will not ascribe any credibility to a cell that X_3 is committed to achieving. If $\alpha_2 \neq \alpha_1$, then X_2 should apportion its credibility in accordance with the fundamental objective of minimizing the distance traveled, yielding a function of the form

$$p_{C_2|C_3}(\alpha_2|\alpha_3) = \begin{cases} 0 & \alpha_2 = \alpha_3 \\ \frac{\frac{1}{\|y_{d\alpha_3} - y_3\|}}{\sum_{\gamma \in U \setminus \{\alpha_3\}} \frac{1}{\|y_{d\gamma} - y_3\|}} & \alpha_2 \neq \alpha_3. \end{cases}$$

By similar reasoning we get that

$$p_{R_1|C_2C_3}(\beta_1|\alpha_2, \alpha_3) = \begin{cases} 0 & \beta_1 \notin \{\alpha_2, \alpha_3\} \\ 1 & \beta_1 \in \{\alpha_2, \alpha_3\}, \end{cases}$$

$$p_{C_1|C_2C_3}(\alpha_1|\alpha_2, \alpha_3) = \begin{cases} 0 & \alpha_1 \in \{\alpha_2, \alpha_3\} \\ 1 & \alpha_1 \notin \{\alpha_2, \alpha_3\}. \end{cases}$$

Simulation results for five spacecraft with several different initial conditions are shown in Figure 8. The X's represent the desired locations, the O's represent the initial location of each spacecraft, and the dashed lines represent their trajectories. With the credibility and rejectability functions defined above, the agents behave the way five humans would in similar circumstances. As can be seen from scenarios 1 and 3, each spacecraft will move to the closest desired location, unless there is another spacecraft in its field of view that is closer. In scenarios 2 and 4 we see the effect of local decision making. In scenario 2, the light blue and dark blue spacecraft are not in their respective fields of view. Both spacecraft initially decide to move toward the same location. The decision making mechanism is triggered when a collision becomes imminent, at which point the dark blue spacecraft decides to move to another location. In scenario 4, the magenta spacecraft must wander around before it finds an empty location. Of particular interest is the fact that a spacecraft will free up an occupied location for another spacecraft if it is easier for it to move to another position. Note also that chaotic behavior does not occur.

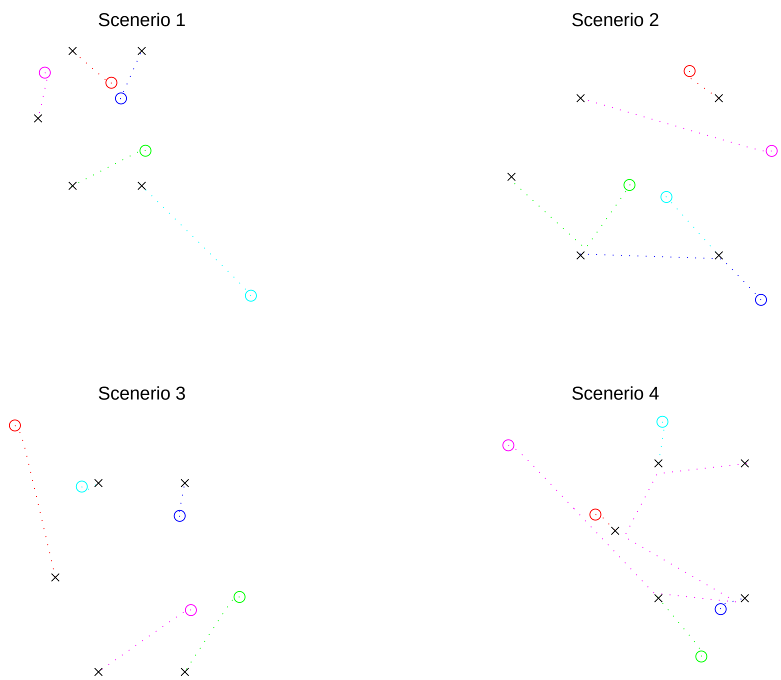

Figure 8: Simulation Results with Five Spacecraft with Limited Field of View.

6 Conclusions and Extensions

This report has developed algorithms that effect reorientation, rotation, expansion/contraction and initialization maneuvers for a constellation of N spacecraft flying in precise, pseudo-rigid body formations. The spacecraft are modeled by second order, rigid body dynamics and kinematics. We summarize and list extensions for each of these problems separately.

Constellation Reorientation. The algorithm for constellation reorientation developed in the report is based on the idea of creating a pseudo-rigid body, called a template, that models the desired constellation. The reorientation problem is accomplished by deriving a control law that reorients the template, mapping the motion of the template to desired translational and rotational trajectories for each spacecraft, and designing control laws for each spacecraft that track these trajectories. The point of rotation is derived *a priori* to tradeoff minimizing the total fuel used by the constellation, and equalizing the fuel used across the constellation. An important conclusion of our research is that equalizing the fuel used by each spacecraft may cause the overall fuel to be quickly depleted.

Possible extensions include the following list.

- The algorithm needs to be extended to on/off thrusters.
- Reorienting the constellation, to maintain a particular orientation of every spacecraft with respect to the sun, needs to be investigated. Previous work on this problem, for single spacecraft, should be applicable.
- Disturbance rejection characteristics of discrete on/off thrusters in the template architecture need to be investigated.
- Extension of these ideas to near earth orbits could also be investigated.
- The fuel equalization problem has a number of subtle challenges that will require further investigation to fully understand. In particular, we need to investigate dynamically changing the point of rotation, and how this effects the overall fuel depletion across the constellation.

- The fuel equalization problem assumed that all spacecraft were equal in the amount of fuel that they were allowed to burn. However, the combiner and collector spacecraft may be given different amounts of fuel to compensate for the relative distances that they will be required to travel. Unequal weightings of the spacecraft needs to be investigated.

Constellation Rotation. Constellation rotation algorithms have been developed for both continuous thrusters and on/off thrusters. In both cases, the general algorithm developed for reorientation is used to specify desired position and orientation trajectories for the spacecraft. For on/off thrusters a novel "tolerance maintaining" algorithm has been derived, that only fires the thrusters when a tolerance boundary is about to be crossed.

Possible extensions include the following list.

- The method outlined in the report needs to be generalized to non-impulsive thrusts.
- A similar "tolerance maintaining" algorithm needs to be developed for relative spacecraft attitude control.
- The algorithm developed in the report maintains a particular tolerance with respect to the center of the rotation. The velocity might also need to be adjusted to maintain the spacecraft within a tolerance ball. This needs to be investigated.
- The algorithm needs to be extended to include starting and stopping the rotation of the constellation.
- Disturbance rejection needs to be analyzed for the "tolerance maintaining" algorithm.

Constellation Expansion/Contraction. An algorithm for constellation expansion/contraction maneuvers has been developed using the notion of a constellation template. The template is expanded according to second order dynamics.

Possible extensions include the following list.

- The effect of the placement of the center of expansion on the total fuel usage, needs to be analyzed.

- The expansion problem using on/off thrusters needs to be investigated. An algorithm similar to the tolerance maintaining algorithm used for constellation rotations needs to be developed and analyzed for constellation expansions.
- The expansion/contraction problem outlined in Section 4 is only relevant if the constellation is required to expand uniformly. This would be the case, for example, if the metrology sensors are required to remain locked on during expansions/contractions. Otherwise, each spacecraft could move to their desired locations independent of the motion of the other spacecraft in the constellation. Our assumption is that the sensors are required to remain locked.

Constellation Initialization. The constellation initialization problem is performed using an approach that is radically different than the one used for reorientation, rotation, and expansion/contraction. The theory of satisficing games has been applied to this problem, assuming range-limited sensors and lack of communication between spacecraft. Collisions are avoided by detecting the location of a spacecraft's nearest neighbors and invoking a collision avoidance routine when a collision is eminent. The particular collision avoidance routine used in our simulations treated spacecraft as particles and created a virtual force field that repels all particles away from each other. After the spacecraft are separated by a distance of D, the decision making mechanism is invoked to decide which targets each spacecraft should seek. The lack of complete information precludes *a priori* fuel optimization. Even with limited information, however, the spacecraft do remarkably well at picking maneuvers that minimize the total fuel used by the constellation. Although the algorithm described above is sub-optimal with respect to fuel consumption, we believe that is does as well as possible in a dynamic environment with limited information.

Possible extensions include the following list.

- In the implementation, each spacecraft only knows the location of its two nearest neighbors and its three nearest desired locations. The result is that a spacecraft may get into a loop where it wanders from one filled location to another, never seeing the unfilled position. This can be corrected by allowing each spacecraft to know all desired positions.

References

[1] P. K. C. Wang and F. Y. Hadaegh, "Coordination and control of multiple microspacecraft moving in formation," *The Journal of the Astronautical Sciences*, vol. 44, no. 3, pp. 315–355, 1996.

[2] J. R. Wertz, ed., *Spacecraft Attitude Determination and Control.* Kluwer Academic Publishers, 1978.

[3] P. C. Hughes, *Spacecraft Attitude Dynamics.* John Wiley & Sons, 1986.

[4] J. T.-Y. Wen and K. Kreutz-Delgado, "The attitude control problem," *IEEE Transactions on Automatic Control*, vol. 36, pp. 1148–1162, October 1991.

[5] M. Athans and P. Falb, *Optimal Control.* New York, New York: McGraw-Hill Book Company, 1966.

[6] P. K. C. Wang, F. Y. Hadaegh, and K. Lau, "Synchronized formation rotation and attitude control of multiple free-flying spacecraft," in *Proceedings of the AIAA Navigation, Guidance and Control Conference*, 1997.

[7] T. C. Schelling, *The Strategy of Conflict.* Oxford University Press, 1960.

[8] D. K. Lewis, *Convention.* Harvard University Press, 1969.

[9] G. N. Saridis, "Toward the realization of intelligent controls," *Proceedings of the IEEE*, vol. 67, no. 8, 1979.

[10] G. N. Saridis, "Analytic formulation of the IPDI for intelligent machines," *Automatica*, vol. 25, no. 3, pp. 461–467, 1989.

[11] G. N. Saridis and K. P. Valavanis, "Analytical design of intelligent machines," *Automatica*, vol. 24, pp. 123–133, 1988.

[12] J. Pearl, *Probabilistic Reasoning in Intelligent Systems.* San Mateo, Ca: Morgan Kaufmann, 1988.

[13] M. Slote, *Beyond Optimizing.* Harvard University Press, 1989.

[14] C. Bicchieri, *Rationality and Coordination.* Cambridge, 1993.

[15] G. Gigerenzer and D. G. Goldstein, "Reasoning the Fast and Frugal Way: Models of Bounded Rationality," *Psychological Review*, vol. 103, no. 4, pp. 650–669, 1996.

[16] C. E. Zsambok and G. Klein, eds., *Naturalistic Decision Making.* Lawrence Erlbaum Associates, 1997.

[17] D. Kahneman and A. Tversky, "On the reality of cognitive illusions," *Psychological Review*, vol. 193, no. 3, pp. 582–591, 1996.

[18] G. Gigerenzer, "On narrow norms and vague heuristics: A reply to Kahneman and Tversky (1996)," *Psychological Review*, vol. 103, no. 3, pp. 592–596, 1996.

[19] L. J. Ball, L. Maskill, and T. C. Ormerod, "Satisficing in Engineering Design: Causes, Consequences and Implications for Design Support," in *Proceedings of the First International Symposium on Descriptive Models of Design*, (Istanbul), pp. 317–332, July 1996.

[20] Y. C. Ho, "Heuristics, Rules of Thumb, and the 80/20 Proposition," IEEE *Transactions on Automatic Control*, vol. 39, no. 5, pp. 1025–1027, 1994.

[21] Y.-C. Ho, "On the numerical solutions of stochastic optimization problems," *IEEE Transactions on Automatic Control*, vol. 42, pp. 727–729, May 1997.

Acknowledgments

The research described in this document was carried out for the Jet Propulsion Laboratory, California Institute of Technology and sponsored by the National Aeronautics and Space Administration. The research was performed for the JPL Microspacecraft Systems Technology Office of the Technology and Applications Programs Directorate and for the JPL Guidance and Control Analysis Group of the Avionics Systems and Technology Division by Randy Beard (beard@ee.byu.edu).

The author gratefully acknowledges technical discussion and assistance from the following individuals.

Fred Hadaegh (JPL) who has provided partial funding for this work, and who introduced me to the fascinating problem of spacecraft formation flying. I am also grateful to Dr. Hadaegh for sponsoring me as a NASA/JPL summer faculty fellow and for numerous technical discussions which I have throughly enjoyed.

John West (JPL) who has provided partial funding for this work and with whom I have had many technical discussions regarding its progress.

Jonathan Lawton (BYU) my PhD student, who co-authored Section 4.

Wynn Stirling and Rick Frost (BYU) who are the principle developers of the theory of Satisficing Games used in Section 5, and who co-authored the paper:

> Beard, R., Stirling, W., Frost, R. "A Hierarchical Coordination Scheme for Satellite Formation Initialization," Submitted to the *1998 AIAA Guidance, Navigation and Control Conference.*

Most of Section 5 was taken from this paper.

Tim McLain (BYU) who was of significant help in working out the bugs in Section 2.

Paul Wang (UCLA), Mehran Mesbahi (JPL), Ken Lau (JPL), Wei-Min Lu (JPL), and Sanjay Joshi (JPL) with whom I had the privilege of working on the problem of formation flying while I was a Summer Faculty Fellow at JPL.