Blind Multiframe Point Source Image Restoration Using MAP Estimation

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Abstract

This paper introduces a Bayesian method for blind restoration of images of sparse, point-like objects. Examples of such images include astronomical star field frames and magnetoencephalogram imaging of current dipole distributions of brain neural activity. It is assumed that these images are corrupted by unknown blurring functions and noise. Both single and multiple frame observation cases are addressed. The proposed method uses maximum a posteriori *estimation techniques to recover both the unknown object and blur. Markov random field (MRF) models are used to represent prior information about both the sparse, point-like structure of the object, and the smoothed random structure of the blur. As compared with general purpose blind algorithms, incorporating a sparse point source MRF model enables much higher resolution restorations, improves point localization, and aids in overcoming the convolutional ambiguity in the blind problem*

1. Introduction

In this paper we consider the blind restoration of point-like source images that have been corrupted by noise and blurred by unknown point spread functions (psf). These problems arise in processing astronomical star field frames, magnetoencephalograms imaging of current dipole distributions of brain neural activity, and other targeting applications where high resolution localization of a few discrete sources is the primary aim. We assume one or more frames of blurred observation data are available with no knowledge of the blurring psf, and that blur psf's may be different from frame to frame if multiple observations are available

An application of particular interest to us is blind restoration of adaptive optics (AO) telescope image sequences of star fields. The AO system removes much of the atmospheric turbulence induced blurring, but a residual random, unknown blur remains that changes from frame to frame in an image sequence over a period of milliseconds. Though the gross structure of the blur is known on average [1], the specific form of each individual blur cannot be easily ascertained, and is thus best modeled with the MRF approach, proposed in this paper. Identifying individual stars in dense star clusters, forming accurate photometry estimates, and computing star-positions to sub-pixel accuracy are primary goals. For example, precise measurement of relative positions can help identify the "wobble" associated with stars orbited by massive planets. This situation lends itself well to the blind restoration technique presented in this paper.

Bayesian maximum a posterior (MAP) estimation has been shown to be effective in blind restoration. In particular, Jeffs, Hong, and Christou [3] have recently demonstrated the effectiveness of generalized Gauss Markov random fields (GGMRF) in blind restoration of extended objects. Here both the source and blur were modeled as GGMRF's which have a parametric form allowing a great variety of image representations, including hard edged fields typical of real images and smooth fields typical of blurring point spread functions. However, the GGMRF model is not well suited to point-like sparse images. A Markov random field model which favors sparse solutions is essential if high resolution restorations and accurate point localizations are to be achieved, particularly in the blind case. The ability to exploit known structure in the problem and impose a sparse form on the solution is essential in overcoming convolutional ambiguity in the blind problem. Blind restoration is a highly ill-posed inverse problem, and algorithms which incorporate known image struc-

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ture in solutions will invariably perform better.

Phillips and Leahy [6], have presented an MRF model that exploits the sparse nature of point source input images in the context of MEG-based imaging. Their model involves a dual field representation: first, a binary activity process determines which pixels have non-zero amplitudes, then a Gaussian amplitude process represents active point intensity levels. We demonstrate that this model can be effectively extended to the *blind* point source restoration case. In fact, the prior information provided by this sparse model image prior pdf is the key to overcoming inherent ambiguity when blur psf's are unknown.

2 Problem Formulation

We adopt the following image observation model for both single and multiple frame data representation

$$\begin{aligned} \bar{\mathbf{g}} &= \mathcal{H}\mathbf{f} + \bar{\eta} \qquad (1) \\ \bar{\mathbf{g}} &= [\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_M]^T, \bar{\eta} = [\eta_1, \eta_2, \cdots, \eta_M]^T \\ \mathcal{H} &= [\mathbf{H}_1^T, \cdots, \mathbf{H}_M^T]^T \end{aligned}$$

where M is the number of frames, $\mathbf{g}_i$, $\mathbf{f}$, and η_i are vectors formed by column scanning the 2-D images of the i^{th} observation frame, the true image, and the i^{th} noise frame respectively. $\mathbf{H}_i$, is the doubly block Toeplitz convolution matrix formed from the i^{th} frame psf, h_i. $\bar{\mathbf{g}}$ is the extended vector formed by concatenating all M column scanned observation frames, and $\mathcal{H}$ represents all M distinct frame blur psf's as a single system matrix. This formulation also works for the single frame case by setting $M = 1$.

Assuming $\mathbf{f}$ and $\mathcal{H}$ are statistically independent, the blind MAP restoration problem may be stated as

$$\hat{\mathbf{f}}, \hat{\mathcal{H}} = \arg\max_{\mathbf{f},\mathcal{H}} p_{g|f,h}(\bar{\mathbf{g}}|\mathbf{f},\mathcal{H}) p_f(\mathbf{f}) p_h(\mathcal{H}) \qquad (2)$$

We will assume that $p_\eta(\bar{\eta})$ is zero mean, i.i.d. Gaussian. This implies that $p_{g|f,h}(\bar{\mathbf{g}}|\mathbf{f},\mathcal{H})$ is i.i.d. Gaussian with a mean of $\mathcal{H}\mathbf{f}$.

In order to model $\mathbf{f}$ as a sparse MRF, we follow the development of Phillips and Leahy by defining

$$\mathbf{f} = \mathbf{X}\mathbf{z} \qquad (3)$$

where $\mathbf{X}$ is a diagonal matrix with elements of either 0 or 1. The vector $\mathbf{z}$ is vector of amplitudes. The vector $\mathbf{x} = diag\{X\}$, represents an indicator process that determines whether or not a particular pixel is active. In the solution of equation (2) we make the replacement

$$p_f(\mathbf{f}) = p_{x,z}(\mathbf{x}, \mathbf{z}) = p_x(\mathbf{x}) p_z(\mathbf{z}) \qquad (4)$$

where we assume the indicator process and the amplitude process are independent. The indicator function can be modeled as a binary Markov random field whose probability density function follows a Gibb's distribution,

$$p(\mathbf{x}) = \frac{1}{K} \exp\left(-V(\mathbf{x})\right),\ x_i \in \{0, 1\} \qquad (5)$$

where K is a normalizing constant and the Gibbs distribution potential function, $V(\mathbf{x})$, is given by

$$V(\mathbf{x}) = \sum_i \alpha_i x_i + \beta_i C_i\{x_i, x_j, j \in \mathcal{N}_i\} \qquad (6)$$

where α_i and β_i are weighting constants and C_i is a clustering function that operates on pixels in the neighborhood $\mathcal{N}_i$ [6]. Arguing that there is no reason to suspect any clustering *a priori* of adjacent pixels in star images (clusters may exist, but individual stars would be separated by some black space, which is different from the Phillips-Leahy clustering model), we have omitted the clustering term and focused on the first term in equation (6) which enforces sparseness in the final image.

The density function for the amplitudes is assumed to be Gaussian in the Phillips-Leahy approach. They are dealing with MEG processing in which the amplitudes can be either positive or negative and thus introduce a zero mean Gaussian. In star images, where we deal only with intensities, negative amplitudes are unacceptable. This can be enforced by adding a mean to the Gaussian, m_z. Another method is to simply draw the amplitudes from a uniform distribution over some positive range and let the noise term in Equation (2) dictate the final values.

The blur pdf is modeled as a GGMRF with density

$$p_h(\mathcal{H}) = \frac{1}{Z_h} \exp\left\{ -\alpha \sum_{s \in S_h} d_s |h_s - \mu_{h,s}|^q - \alpha \sum_{<s,t> \in C_h} c_{s,t} |(h_s - \mu_{h,s}) - (h_t - \mu_{h,t})|^q \right\} \qquad (7)$$

where C_h is the set of all cliques for the blur neighborhood system, q is the blur GGMRF shape parameter, S_h is the set of all points in the blur lattice over all frames and $c_{s,t}$ and d_s are neighborhood influence weights.

It has been shown in previous publications, for example [3], that $q > 2$ does a good job modeling smooth features such as those found in a typical blur psf. The mean, $\mu_{h,s}$, allows the restoration to maintain fidelity with prior knowledge about the blur. For example, in

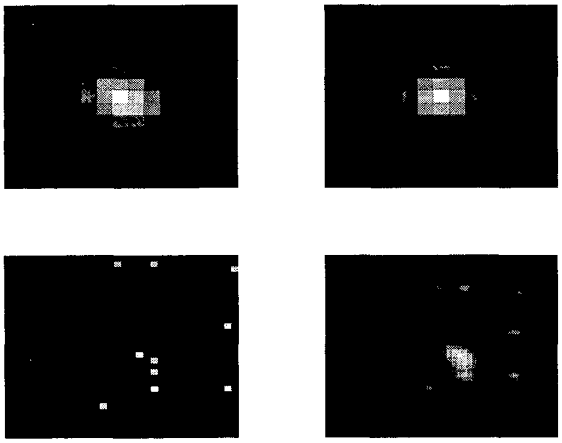

Figure 1. Image to be restored. Top left: Actual blur psf formed as an elliptical Lorentzian function on a GGMRF residual halo producing a realistic AO low-pass "mottled" halo field. Top right: Circularly symmetric Lorentzian blur model to be used as the reference mean, $\mu_{\mathbf{h}}$. Bottom left: Actual truth image. Bottom right: Observed blurred, noisy output data.

astronomical imaging isolated stars in a nearby field can be averaged to give a reference mean.

Combining the density functions we can re-express

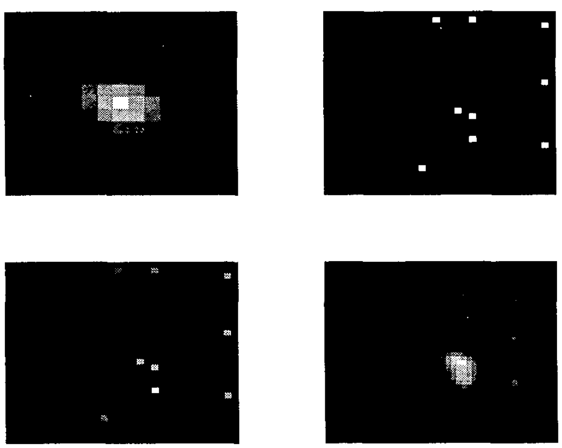

Figure 2. Restoration results. Top left: Estimated blur. Top right: Restored activity matrix. Bottom left: Restored input image. Bottom right: Restored image blurred with restored blur psf. Compare with Figure 1.

equation (2) by taking the logarithm of both sides and dropping the additive constants

$$\begin{aligned}\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\mathcal{H}} = \arg \min_{x,z,\mathcal{H}} \sum_{i=1}^{M} ||\mathbf{g}_i - \mathbf{H}_i \mathbf{X} \mathbf{z}||^2 + \\ \gamma \sum_j x_j + \lambda ||\mathbf{z} - \mu_{\mathbf{z}}||^2 \\ + \alpha \sum_{s \in S_h} d_s |h_s - \mu_{h,s}|^q + \\ \alpha \sum_{<s,t> \in C_h} c_{s,t} |(h_s - \mu_{s,t}) - (h_t - \mu_{h,t})|^q\end{aligned} \tag{8}$$

Here λ,γ,α control the relative influence as regularizing terms that the activity matrix, amplitudes, and blur have on the solution.

Equation (8) represents a very complicated nonlinear minimization problem. The approach taken to solve it is simulated annealing [2, 4], specifically the Metropolis algorithm [5]. λ,γ,α are set manually and adjusted for best restoration performance.

3 Results

In this section we present examples of the new blind algorithm using simulated adaptive optics telescope data. In Figure (1), the actual data for the first example is presented. This shows a blur with region of support that is 15×15 pixels. The blur is a rotated Lorentzian function with an elongated axis. This has been shown to be an excellent model for AO residual blur [1]. The reference mean used for the blur is a circularly symmetric Lorentzian shape, with different radius than either axis of the actual blur. The truth image is also shown consisting of ten isolated points. The bottom right image shows the resulting blurred, noise corrupted output. The noise is zero-mean white Gaussian noise at a level of 32 dB peak SNR.

Note that only one observed frame is used in this restoration example. This is actually a more difficult problem than the case of multiple frames with different blurs for each frame. In the multiframe case, the diversity provided by distinct, unknown blurs aids in the blind restoration because only the true image is common to all frames.

Figure (2) shows the results of the algorithm described above. The blur has been estimated quite accurately. The main axis is quite clearly defined in the restoration and the extent of the restored blur matches the actual blur very well. The source restoration matches the actual image to a high degree. However, two points in the L-shaped structure of the original source were blurred into a single point in the solution and some positions are inaccurate by a single pixel.

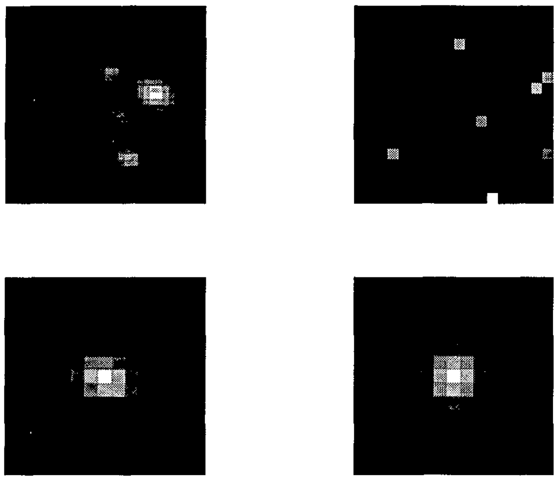

Figure 3. Second example. Top left: Observed blurred, noisy output data. Top right: Actual truth image. Bottom left: Actual blur psf. Bottom right: Circularly symmetric Lorentzian reference mean.

The actual data for the second example is found in Figure (3). Here we have a similar blur with a input image. Figures (4) and (5) show two different restorations on the data. In each of the restorations, the blur is only slightly elongated suggesting too much weight on the blur self term (i.e. on μ_h). Each of the input image restorations are fairly good. The first misses a point to the bottom right of the input image and some other positions are off by about a pixel. The second get all points but in the upper portion of the input image, one of the points is split. This is likely due to very circular restoration of the blur. These problems would likely be solved with better weights on the various priors.

4 Conclusions

The restoration example shown above demonstrates the power of the model described in this paper to emphasize the sparse character of a source image. Though GGMRFs have been shown to work well for restoring extended objects, the model does not allow the user to explicitly enforce point-like structure on the solution. The specific point-source prior of Phillips and Leahy in conjunction with the GGMRF prior for the psf leads to blind restoration solutions which are truly sparse and are an excellent estimate of the truth. We have demonstrated that the Phillips - Leahy MRF model for point sources is well suited, with minor modifications, to *blind* star field image restoration. Previously this model had been used only with known blurring or system functions.

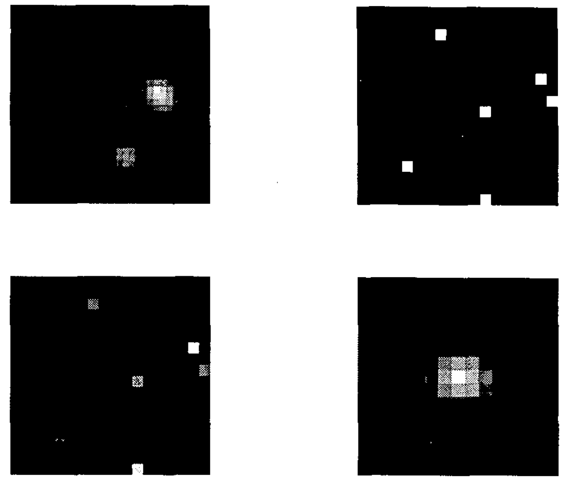

Figure 4. Second example, first restoration. Top left: Restored image blurred with restored blur psf. Top right: Restored activity matrix. Bottom left: Restored input image. Bottom right: Restored blur psf.

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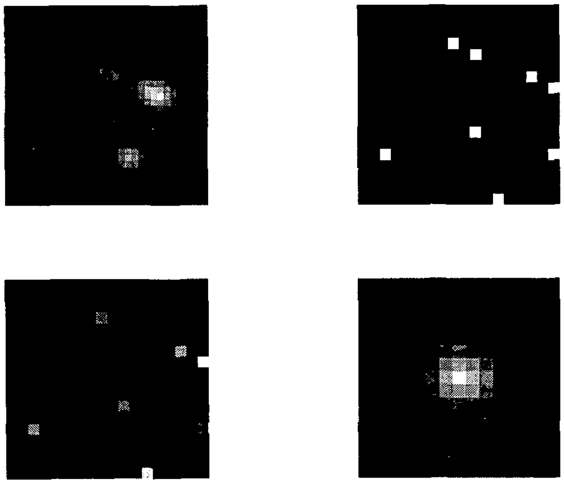

Figure 5. Second example, second restoration. Top left: Restored image blurred with restored blur psf. Top right: Restored activity matrix. Bottom left: Restored input image. Bottom right: Restored blur psf.

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