On Interference Cancellation with a Focal Plane Array

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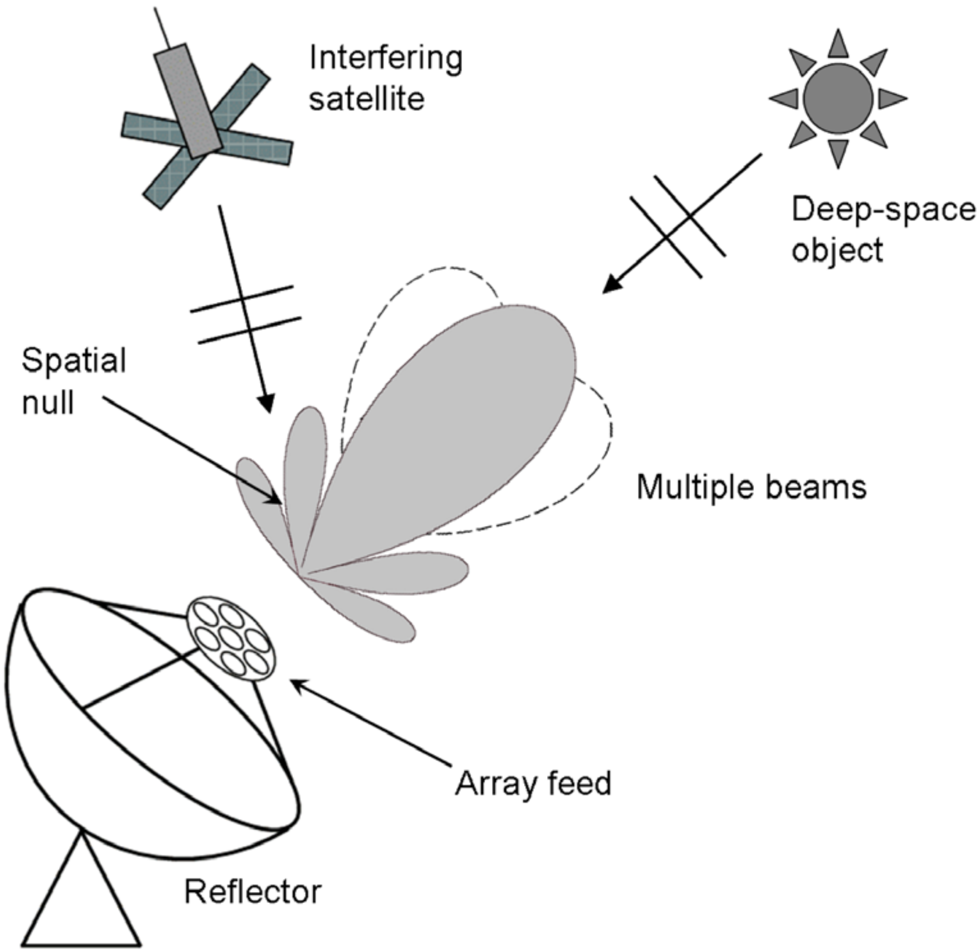
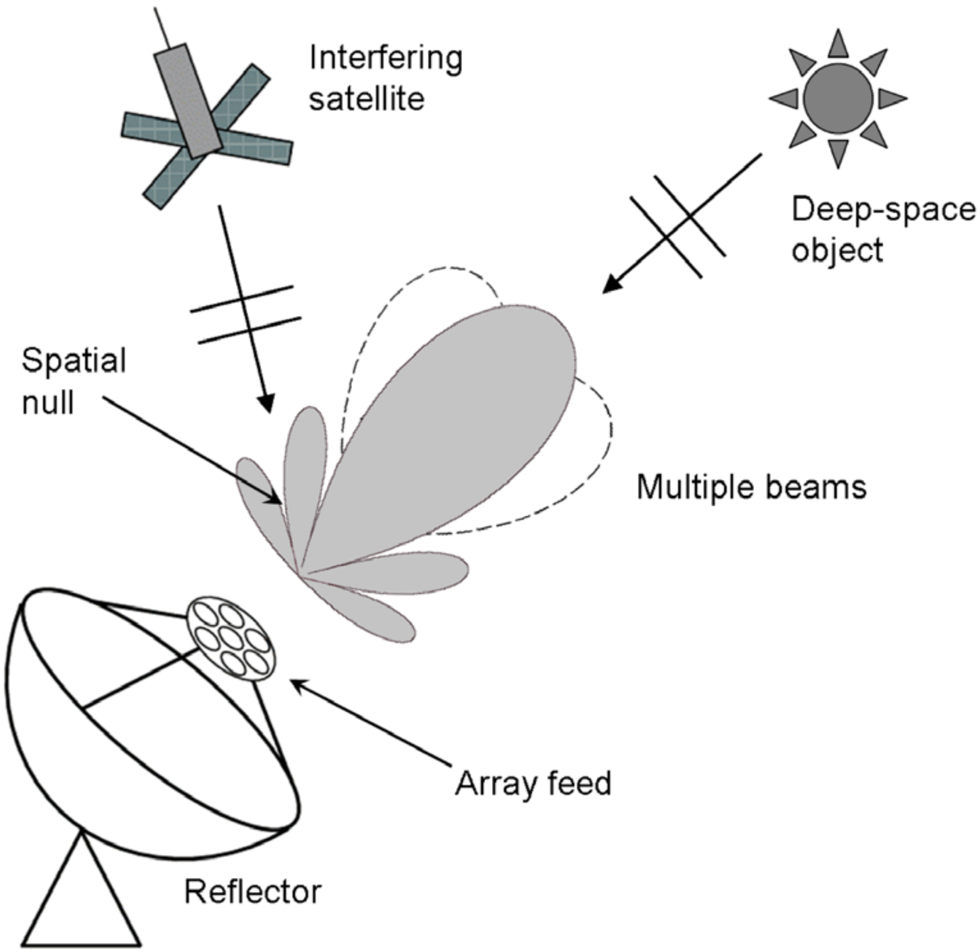
The Promise of Array Feeds for Radio Astronomy



- Multiple simultaneously formed beams in different look directions.
- Fast survey capability.
- Adaptive beamforming to cancel RFI.
- Direct and adaptive control of dish illumination pattern.
- Increased sensitivity and spillover efficiency.
- Some SKA configurations may require this to deal with RFI.

The Challenges of Array Feeds for Radio Astronomy



- Complexity.
- Beampattern rumble.
- Difficult to maintain calibrated, stable beam response with moving interference.
- High stability requirements make even variations in sidelobe patterns problematic.

Why are adaptive beamforming feeds not already in wide use?

- Much work is needed to develop wideband, very low noise, cryo-cooled, densely packed feed array systems.
  - e.g. Bradley and Fisher, NRAO.

- Mutual coupling and impedance matching are still being studied.
  - e.g. optimal multiport lossless matching networks: Jensen and Warnick, BYU.
- Beyond these hurdles, astronomers can't trust the time-varying beampattern needed to adaptively cancel interference.
  - Mainlobe shape constraints are possible (e.g. LCMV), but inadequate for demanding astronomical requirements.
  - We introduce a viable solution to truly correct for beam-distortion-induced PSD estimation bias.

The BYU Experimental Array Feed

BYU Prototype Array

Geometry

- Seven dipole elements
- Hexagonal grid
- 0.6λ element spacing

Configuration

- Ground-plane backing
- 3 meter parabolic reflector
- Full sampling
- Seven channel receiver

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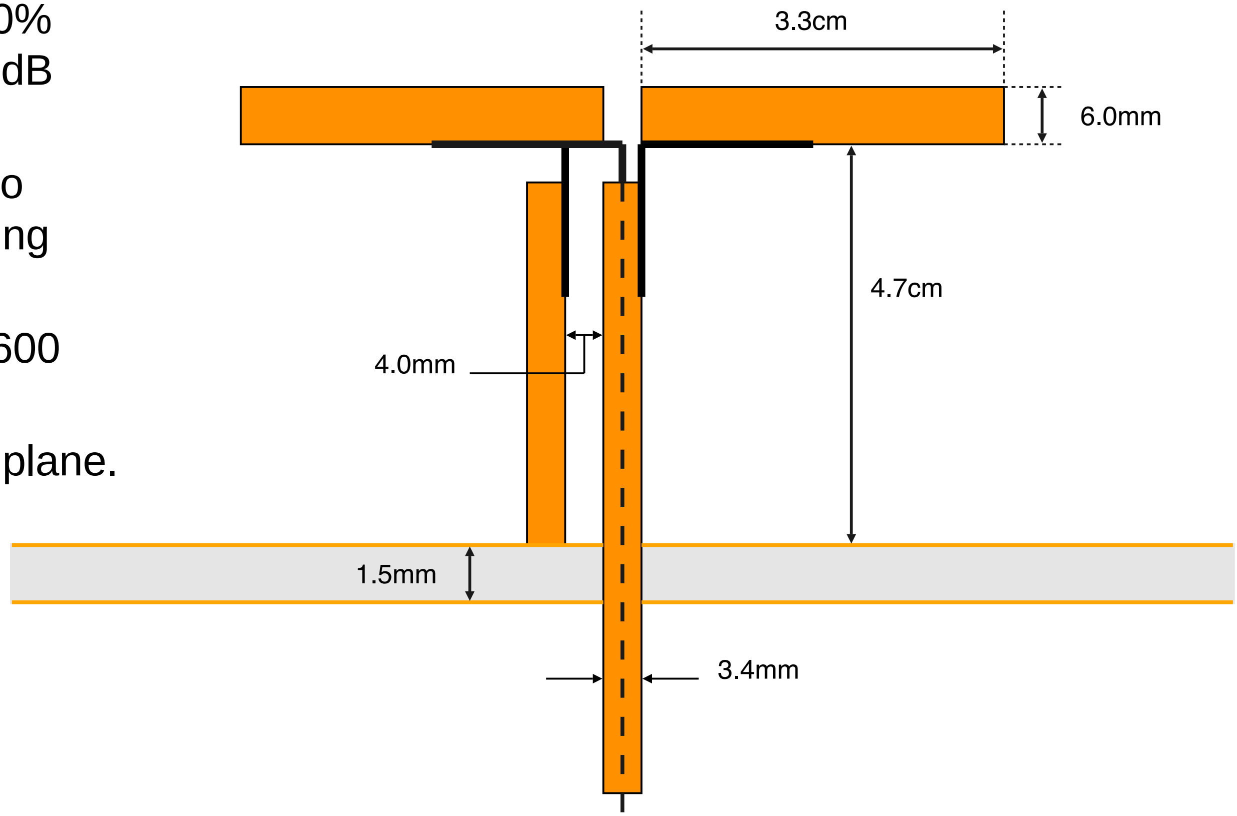
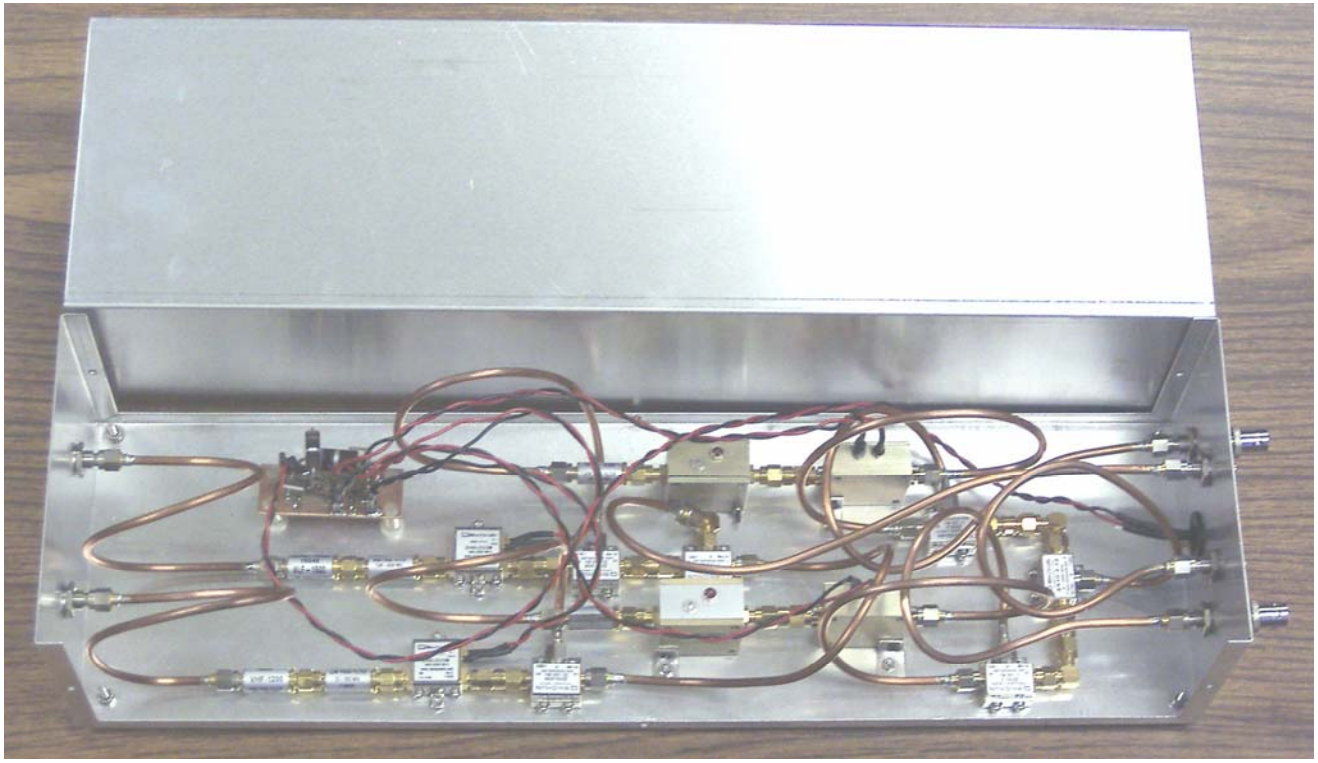
Element Design

- Balun-fed dipole
- Thickened arms for 30% bandwidth (BW = -10 dB reflection coefficient)
- Impedance matched to 50 Ω without a matching network.
- Center frequency = 1600 MHz
- λ/4 above the ground plane.



Array Receivers

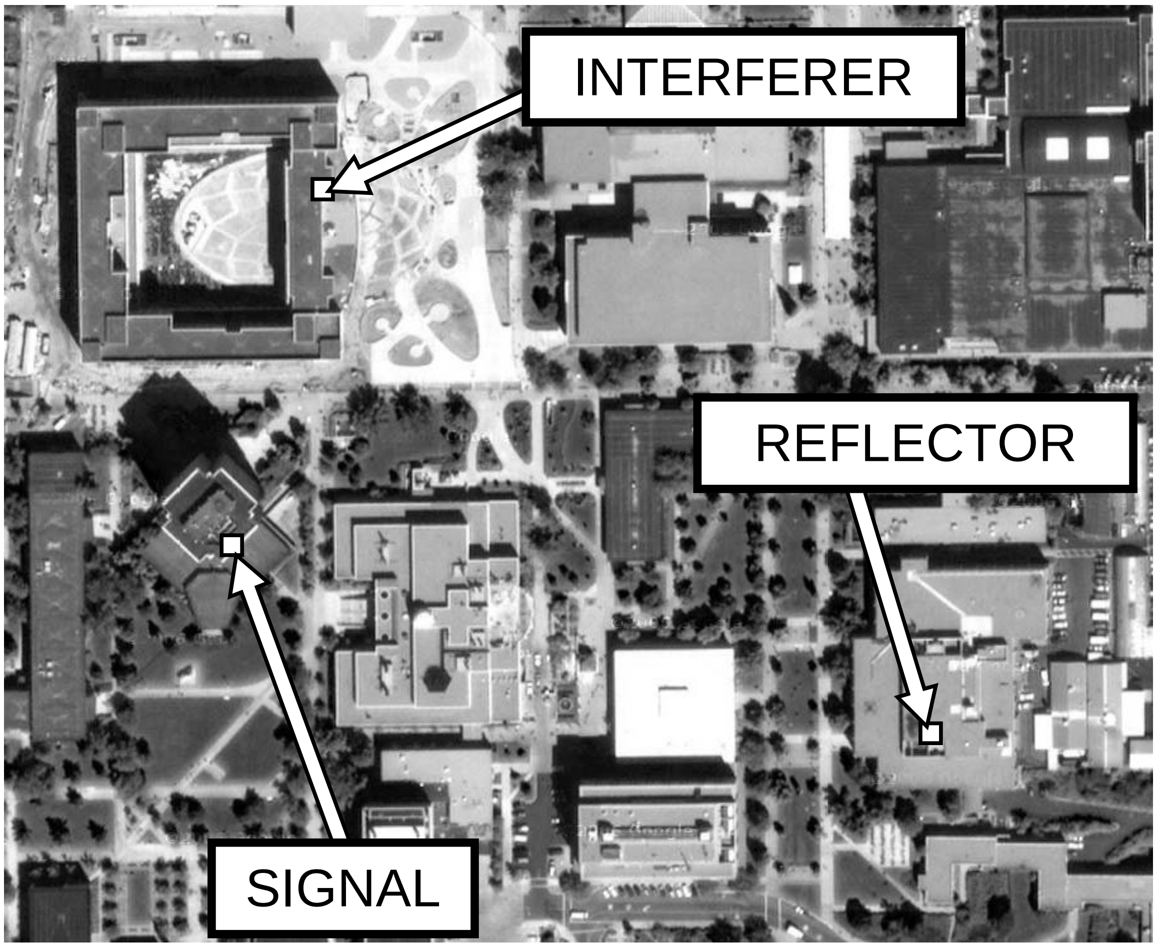


- Eight receiver channels (7 array elements plus auxiliary antenna).
- Scalable modular design (Future array will require 19 elements).
- Connectorized to avoid cross-talk.
- 2 Channels per box.

Element Directivity and Coupling Measurements

- A characterization of antenna array electrical and radiation properties is required.
- Used NRAO Green Bank outdoor antenna test range.
- Patterns were measured one element at a time with others terminated or open.
- Lock-in amplifier enabled accurate phase vs bearing measurements.

RFI Mitigation Experiment



- Signal at boresight
  - Standard gain horn (14 dBi)
  - CW transmission
  - 1611.3 MHz
- Interferer at 30°
  - Dipole antenna
  - 1 kHz modulated FM
  - 30 kHz deviation
  - 1611.3 MHz

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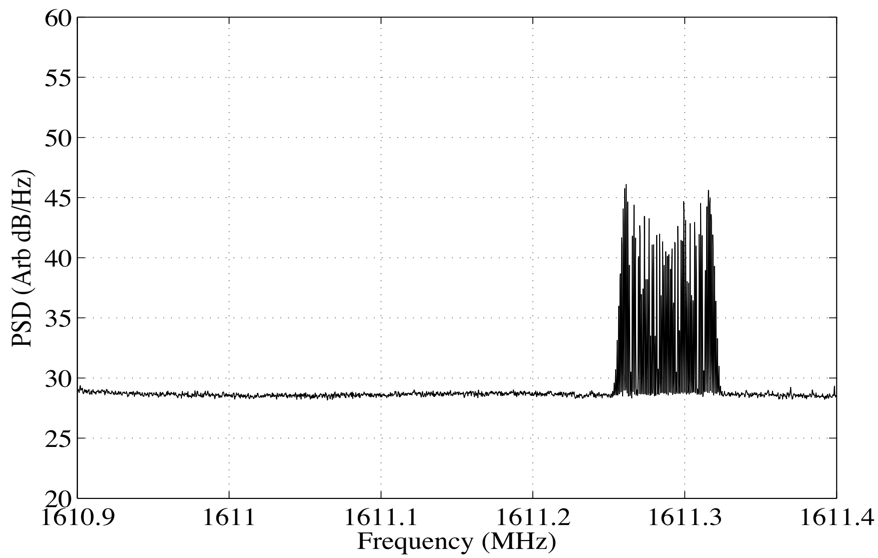
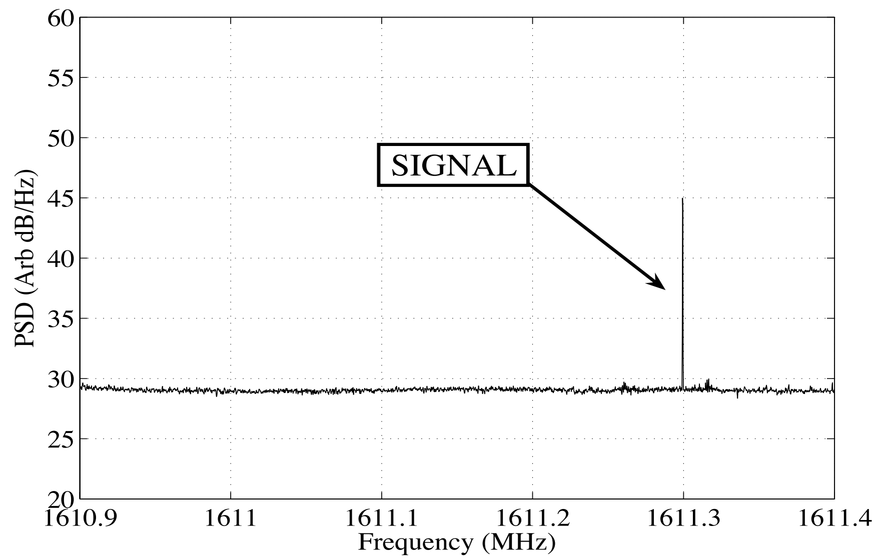
PSD Estimate with Moving RFI





Angular Velocity ≈ 0.1 °/s
Correlation time = 4.9 ms

Adaptive beamforming even works with a non-stationary interferer.

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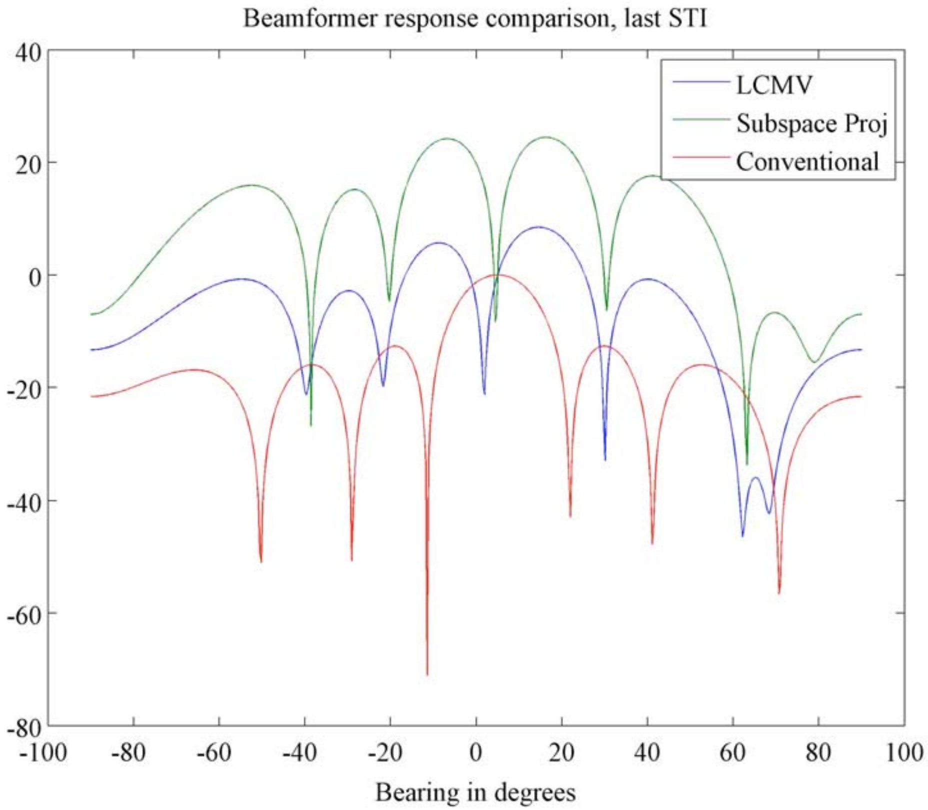
Other Array Feed Studies Under Way

- Spillover efficiency and sensitivity optimization.
- Pattern rumble minimization.
- Optimal element spacing and array plane placement.
- Aperture efficiency.
- Beam shape control for off-axis beams.

Bias Corrected Interference Canceling Array Power Spectral Density Estimation

The Need for Bias Correction



Examples of beampattern distortion for two adaptive cancelation algorithms with a mainlobe interference.

- Null placement for interference canceling distorts nominal beampattern.
- Most severe as interferer enters mainlobe.
- Even if mainlobe is well controlled, pattern rumble affects noise floor.
- Particularly problematic when sidelobe rumble affects dish illumination spillover.

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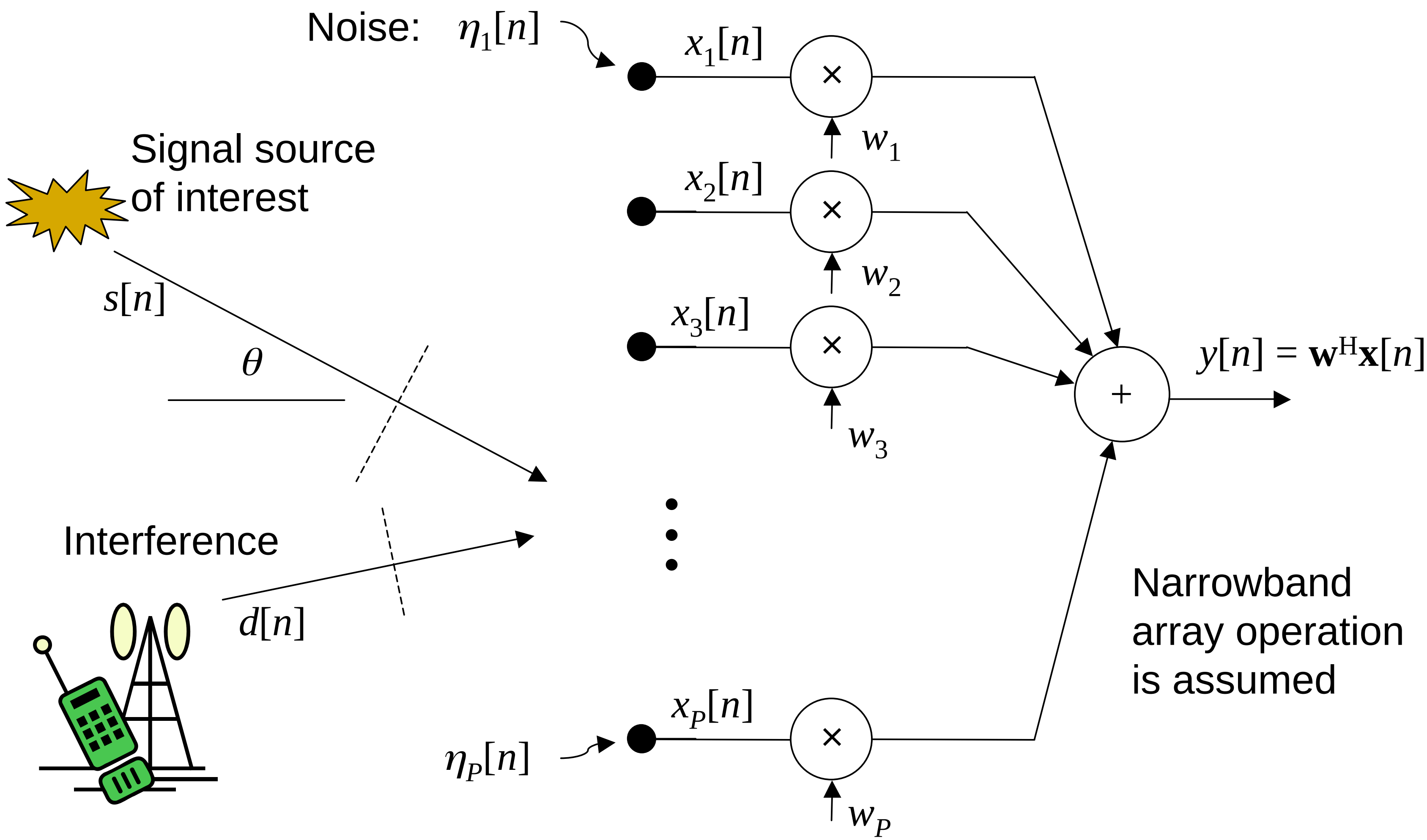
Signal And Beamforming Model



Narrowband array operation is assumed

Algorithm Overview

- Beamformed feed output, $y[n]$, is processed over a long term integration interval (LTI) for a PSD estimate, $\mathbf{S}_y$.
- Interference must move relative to source of interest.
- Array covariance is assumed stationary over a short term integration (STI) window.
- Over the LTI, beam distortion effects in $\mathbf{S}_y$ are removed.
- An extension of Leshem and Van der Veen's bias corrected covariance subspace filtering for synthesis imaging: *IEEE Trans. Inform. Theory, vol. 46, no 5.*
- Does not yield a corrected time series, $y[n]$.

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Subspace Projection Beamforming

- Signal Model for j^{th} STI window of length L:

$$\mathbf{x}[n] = \left[x_1[n], \cdots x_P[n]\right]^T = \mathbf{a}_s s[n] + \mathbf{A}_d[n]\mathbf{D}[n] + \boldsymbol{\eta}[n]$$

$$\mathbf{X}_j = \left[\mathbf{x}[jL], \mathbf{x}[jL+1], \cdots, \mathbf{x}[(j+1)L-1]\right].$$

- Sample covariance for j^{th} STI: $\hat{\mathbf{R}}_j = \frac{1}{L}\mathbf{X}_j\mathbf{X}_j^H$
- Partitioned eigen decomposition estimates subspace $\mathbf{U}_d$ spanning interference array signature, $\mathbf{A}_d[jL]$:

$$\hat{\mathbf{R}}_j[\mathbf{U}_d \mid \mathbf{U}_{s+\eta}] = [\mathbf{U}_d \mid \mathbf{U}_{s+\eta}]\Lambda$$

- Projection beamformer weight for j^{th} STI:

$$\mathbf{w}_j = \mathbf{P}_j\tilde{\mathbf{w}} \quad \text{where} \quad \mathbf{P}_j = \mathbf{I} - \mathbf{U}_d\mathbf{U}_d^H. \quad \text{Note} \quad \mathbf{P}_j\mathbf{A}_d[jL] = \mathbf{0}$$

Uncorrected Array PSD Estimator

- Beamformed output vector for j^{th} STI:

$$\mathbf{y}_j^T = \mathbf{w}_j^H \mathbf{X}_j = \tilde{\mathbf{w}}^H \mathbf{P}_j \mathbf{X}_j$$

- Welch's PSD estimator for projection beamformer:

$$\hat{\mathbf{S}}_y^T = \frac{1}{M} \sum_{j=0}^{M-1} \left| \mathrm{DFT}\{\mathbf{y}_j^T \mathbf{G}\} \right|^{\bullet 2}$$

$$= \frac{1}{M} \sum_{j=0}^{M-1} \left| \tilde{\mathbf{w}}^H \mathbf{P}_j \mathbf{X}_j \mathbf{G}\mathbf{F} \right|^{\bullet 2}$$

$$= \frac{1}{M} (\tilde{\mathbf{w}}^H \otimes \tilde{\mathbf{w}}^T) \sum_{j=0}^{M-1} (\mathbf{P}_j \otimes \mathbf{P}_j^*) \Big((\mathbf{X}_j \mathbf{G}\mathbf{F}) \circ (\mathbf{X}_j \mathbf{G}\mathbf{F})^* \Big)$$

$\mathbf{G}$ is diagonal windowing matrix and $\mathbf{F}$ is the unitary DFT matrix.

The Corrected Array PSD Estimator

- Bias corrected PSD estimator:

$$\hat{\mathbf{S}}^{T}_{y,c} = \frac{1}{M}(\tilde{\mathbf{w}}^{H} \otimes \tilde{\mathbf{w}}^{T})\mathbf{C}^{-1}\sum_{j=0}^{M-1}(\mathbf{P}_j \otimes \mathbf{P}_j^{*})\left((\mathbf{X}_j\mathbf{GF}) \circ (\mathbf{X}_j\mathbf{GF})^{*}\right)$$

$$\mathbf{C} = \frac{1}{M}\sum_{j=0}^{M-1}(\mathbf{P}_j \otimes \mathbf{P}_j^{*})$$

- Since $\mathbf{P}_j$ removes the the nonstationary interference from $\mathbf{X}_j$, on average $\mathbf{C}^{-1}$ cancels bias caused by $\mathbf{P}_j$.

$$E\{\hat{\mathbf{S}}^{T}_{y,c}\} = \frac{1}{M}(\tilde{\mathbf{w}}^{H} \otimes \tilde{\mathbf{w}}^{T})\mathbf{C}^{-1}\sum_{j=0}^{M-1}(\mathbf{P}_j \otimes \mathbf{P}_j^{*})E\left\{(\mathbf{X}_j\mathbf{GF}) \circ (\mathbf{X}_j\mathbf{GF})^{*}\right\}$$

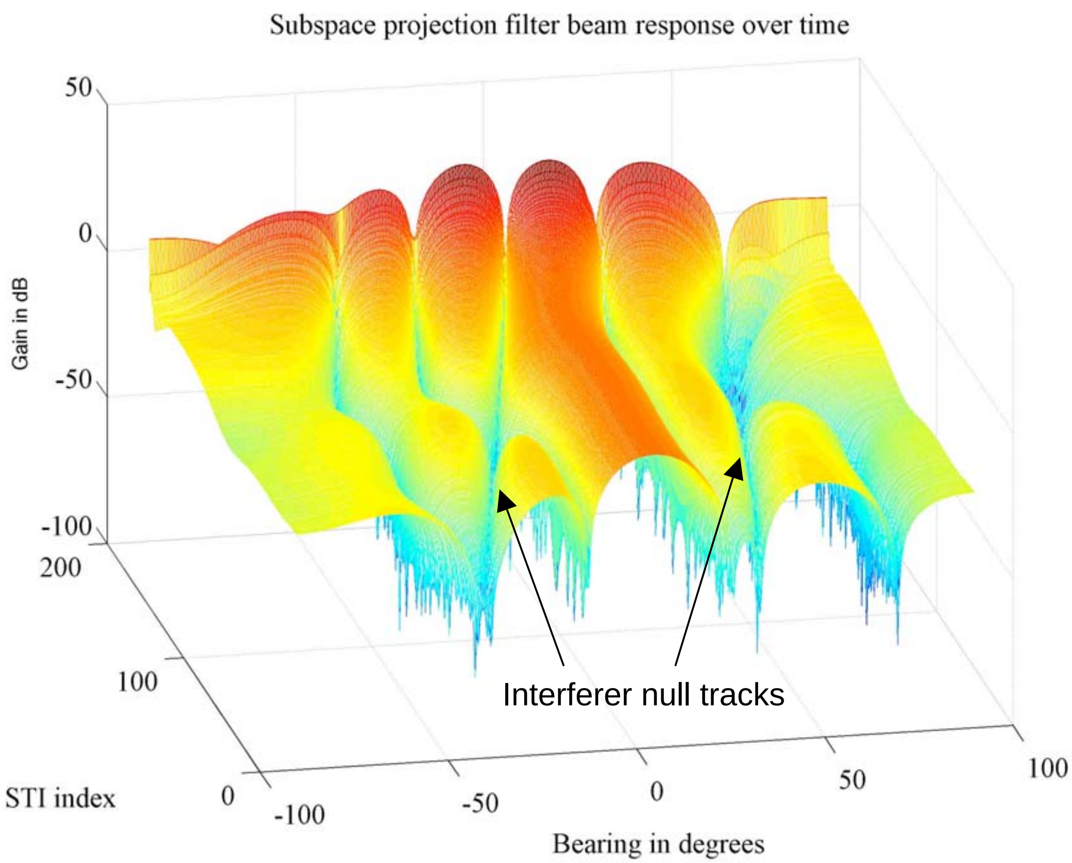
$$= (\tilde{\mathbf{w}}^{H} \otimes \tilde{\mathbf{w}}^{T})E\left\{(\mathbf{X}_{s+\eta,1}\mathbf{GF}) \circ (\mathbf{X}_{s+\eta,1}\mathbf{GF})^{*}\right\}$$

same as Welch's PSD w/o interference!

Simulation Results

- Seven element uniform line array, no reflector.
- Half wavelength element spacing.
- 10^5 samples in the long-term integration (LTI).
- 512 samples per STI window, no overlap.
- Hamming window shaping.
- Two moving interferers, starting at -40º and 33º.
- One stationary desired source at 5º.
- Array is calibrated in source direction.
- Response in all other directions is unknown.

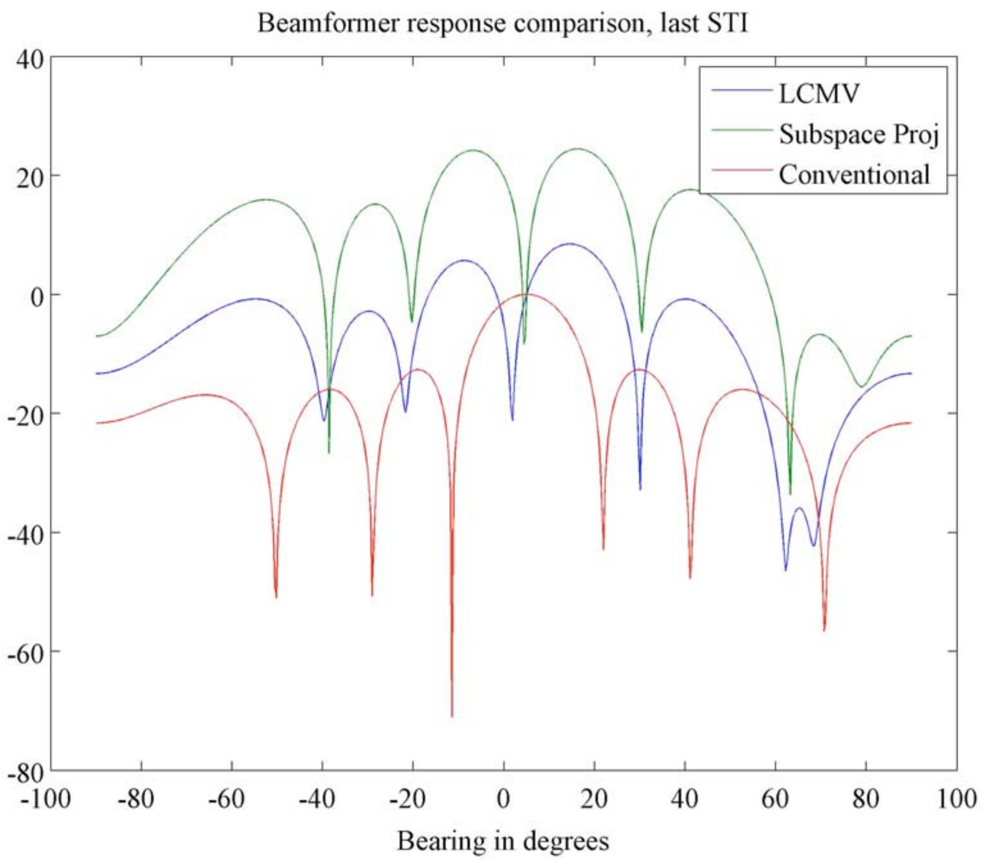
Subspace Projection Beam Response vs Time



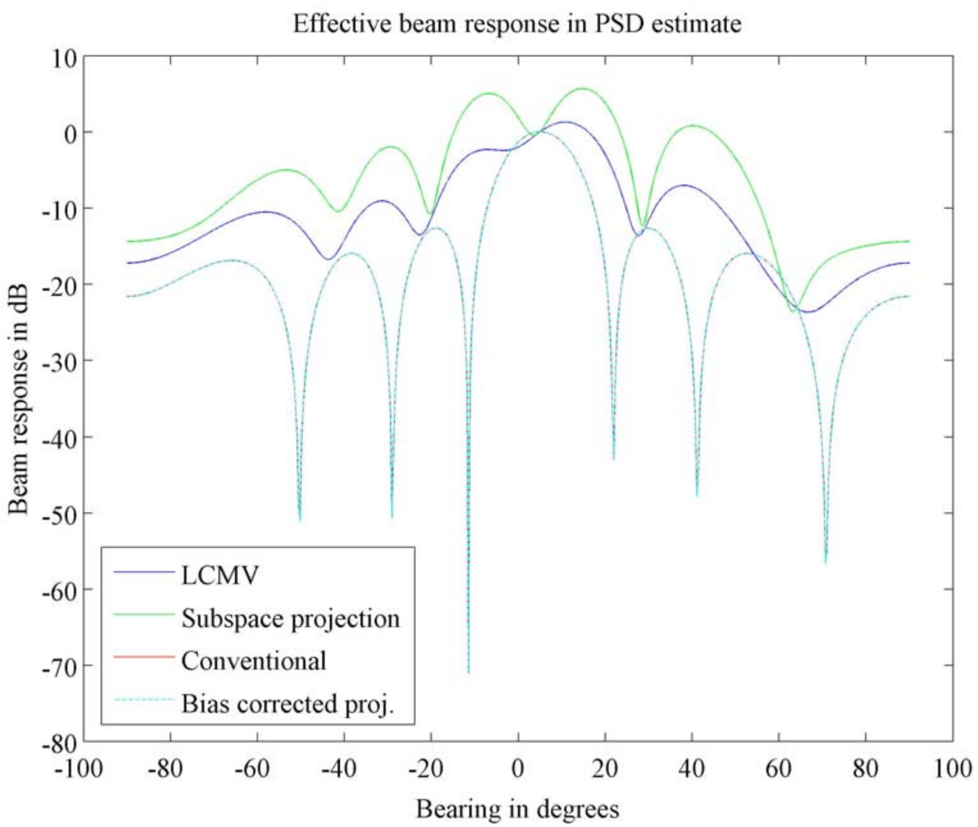
- Interferers
  - 10 dB INR for both.
  - Moving at 4.5e^{-4} and 3.0e^{-4} degrees/sample.
  - FM modulation.
  - One interferer freq. band overlaps desired source.
- Source
  - -30 dB SNR.
  - Narrowband.
- Adaptive null cuts into main lobe near the end of long term integration.
- Significant beamshape distortion.

Last STI Beampatterns

- Subspace projection keeps a null on the mainlobe interferer.
  - High sidelobes raise noise power in beamformer output.
- LCMV has less distortion due to minimum variance criterion.
  - trades-off noise v.s. interference.
- All have calibrated 0 dB response to source signal at 5º.

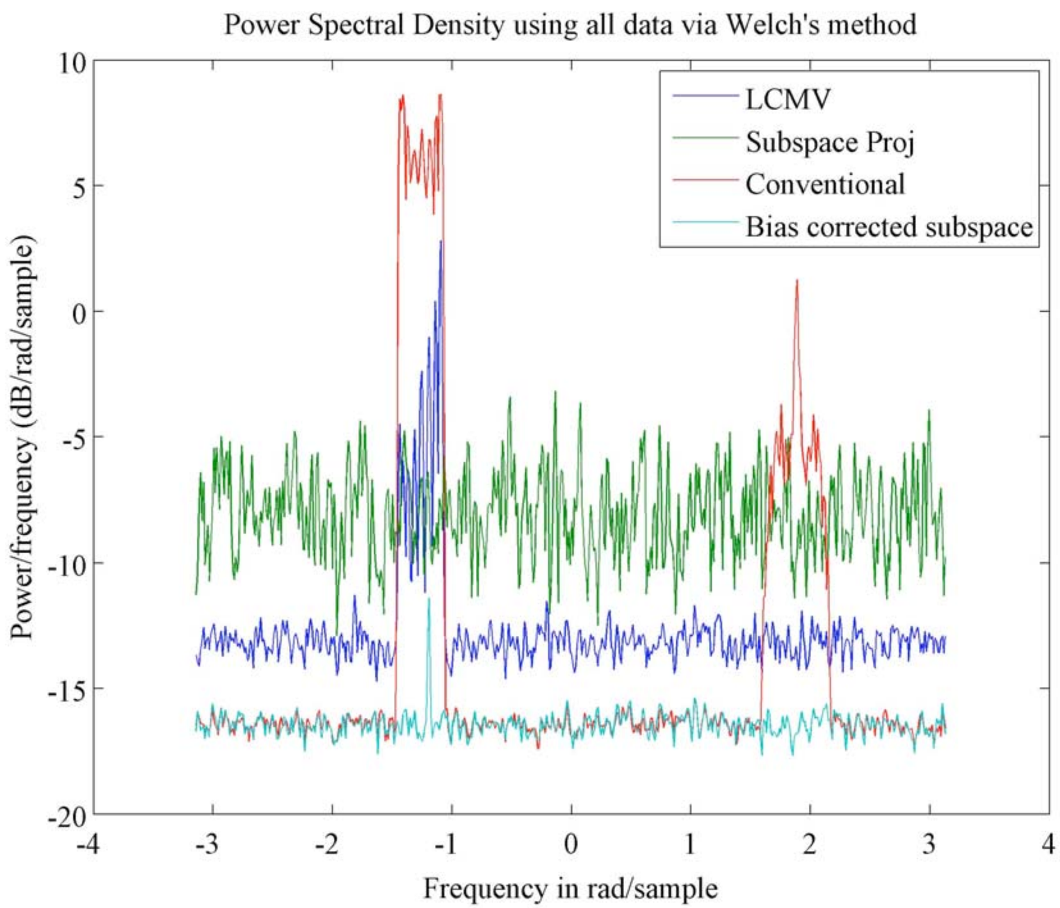


Effective Average Beampatterns over all STIs



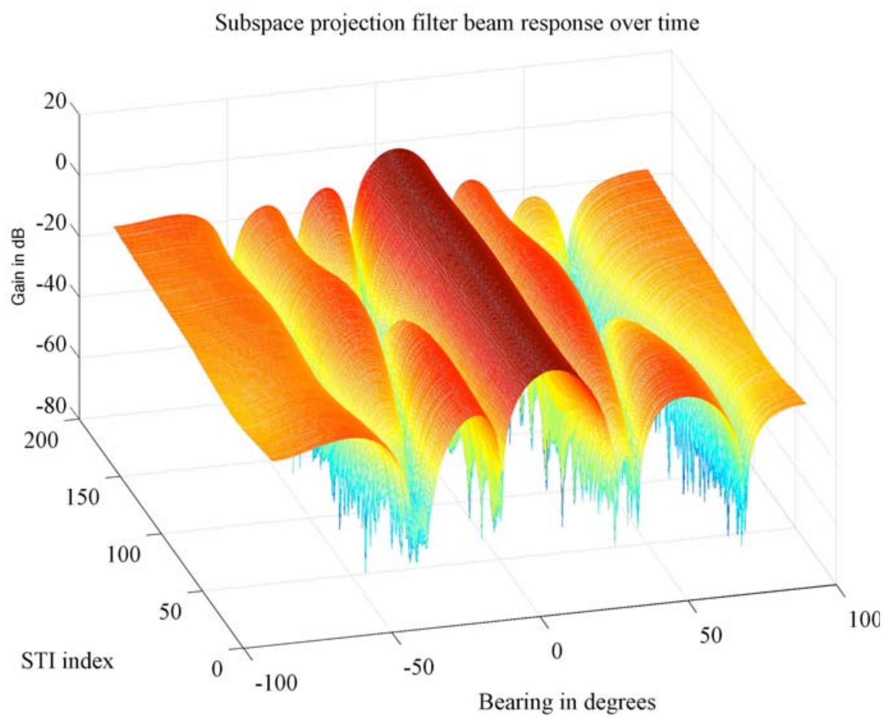
- Computed as follows:
  - Use beamformer weights for each STI as computed from interference data set.
  - Place narrowband unit power source at a test bearing θ.
  - Calculate PSD using all STIs. Power at source frequency is the response for direction θ.
  - Repeat for all θ.
- Bias correction removes subspace projection distortions!
- Effective response exactly matches conventional beamformer!

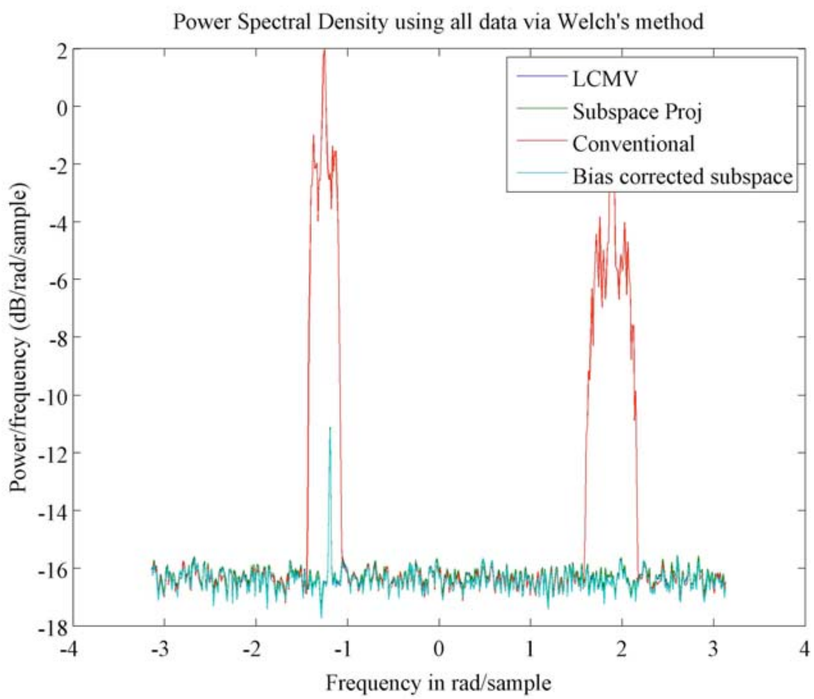
PSD Estimate Comparisons



- Only subspace projection methods adequately cancel mainlobe interferer.
- Only bias corrected subspace projection keeps noise floor low enough to see signal.
- No bias in noise PSD level, but noise estimation variance is a bit higher.

When Interferer Stays Outside Mainlobe

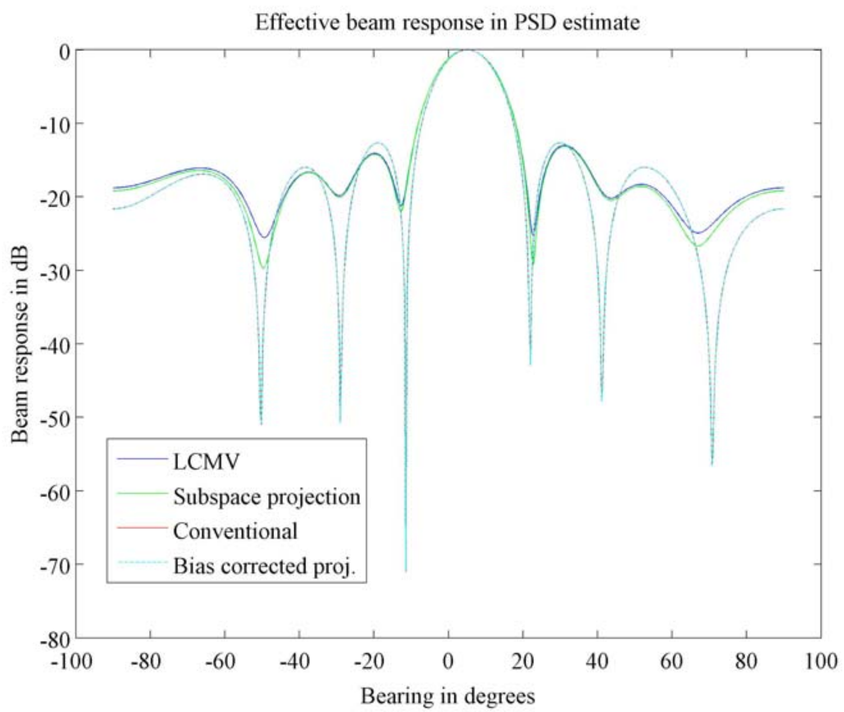




Sidelobe rumble is evident, but mainobe is relatively unperturbed

All methods but non-adaptive conventional beamforming yield good PSDs.

When Interferer Stays Outside Mainlobe



Applications sensitive to small changes in sidelobe pattern shape would still benefit from bias correction.

Conclusions

- Effective bias correction has been demonstrated for array PSD estimation during RFI cancellation.
- Unbiased array PSD estimator can be applied to any array configuration (not just array feeds).
- This will be useful for beamforming clusters of dishes being proposed for some SKA configurations. Also LOFAR stations.
- Next step is accurate feed array simulation.
- Real data experiments are underway.
- Full effective beampattern is hard to determine experimentally.