On Interference Cancellation with a Focal Plane Array

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The Promise of Array Feeds for Radio Astronomy

- Multiple simultaneously formed beams in different look directions.
- Fast survey capability.
- Adaptive beamforming to cancel RFI.
- Direct and adaptive control of dish illumination pattern.
- Increased sensitivity and spillover efficiency.
- Some SKA configurations may require this to deal with RFI.
The Challenges of Array Feeds for Radio Astronomy

- Complexity.
- Beampattern rumble.
- Difficult to maintain calibrated, stable beam response with moving interference.
- High stability requirements make even variations in sidelobe patterns problematic.
Why are adaptive beamforming feeds not already in wide use?

- Much work is needed to develop wideband, very low noise, cryo-cooled, densely packed feed array systems.
  - e.g. Bradley and Fisher, NRAO.

- Mutual coupling and impedance matching are still being studied.
  - e.g. optimal multiport lossless matching networks: Jensen and Warnick, BYU.

- Beyond these hurdles, astronomers can’t trust the time-varying beampattern needed to adaptively cancel interference.
  - Mainlobe shape constraints are possible (e.g. LCMV), but inadequate for demanding astronomical requirements.
  - We introduce a viable solution to truly correct for beam-distortion-induced PSD estimation bias.
The BYU Experimental Array Feed
BYU Prototype Array

Geometry
• Seven dipole elements
• Hexagonal grid
• 0.6λ element spacing

Configuration
• Ground-plane backing
• 3 meter parabolic reflector
• Full sampling
• Seven channel receiver
Element Design

- Balun-fed dipole
- Thickened arms for 30% bandwidth (BW = -10 dB reflection coefficient)
- Impedance matched to 50 Ω without a matching network.
- Center frequency = 1600 MHz
- λ/4 above the ground plane.
Array Receivers

- Eight receiver channels (7 array elements plus auxiliary antenna).
- Scalable modular design (Future array will require 19 elements).
- Connectorized to avoid cross-talk.
- 2 Channels per box.
Element Directivity and Coupling Measurements

- A characterization of antenna array electrical and radiation properties is required.
- Used NRAO Green Bank outdoor antenna test range.
- Patterns were measured one element at a time with others terminated or open.
- Lock-in amplifier enabled accurate phase vs bearing measurements.
RFI Mitigation Experiment

- Signal at boresight
  - Standard gain horn (14 dBi)
  - CW transmission
  - 1611.3 MHz

- Interferer at 30°
  - Dipole antenna
  - 1 kHz modulated FM
  - 30 kHz deviation
  - 1611.3 MHz
PSD Estimate with Moving RFI

Angular Velocity ≈ 0.1 °/s
Correlation time = 4.9 ms

Adaptive beamforming even works with a non-stationary interferer.
Other Array Feed Studies Under Way

- Spillover efficiency and sensitivity optimization.
- Pattern rumble minimization.
- Optimal element spacing and array plane placement.
- Aperture efficiency.
- Beam shape control for off-axis beams.
Bias Corrected Interference Canceling Array Power Spectral Density Estimation
The Need for Bias Correction

- Null placement for interference canceling distorts nominal beampattern.
- Most severe as interferer enters mainlobe.
- Even if mainlobe is well controlled, pattern rumble affects noise floor.
- Particularly problematic when sidelobe rumble affects dish illumination spillover.

Examples of beampattern distortion for two adaptive cancelation algorithms with a mainlobe interference.
Signal And Beamforming Model

\[ y[n] = \mathbf{w}^H \mathbf{x}[n] \]

- Signal source of interest: \( s[n] \)
- Interference: \( d[n] \)
- Noise: \( \eta_1[n] \), \( \eta_p[n] \)

Narrowband array operation is assumed.
Algorithm Overview

- Beamformed feed output, $y[n]$, is processed over a long term integration interval (LTI) for a PSD estimate, $S_y$.
- Interference must move relative to source of interest.
- Array covariance is assumed stationary over a short term integration (STI) window.
- Over the LTI, beam distortion effects in $S_y$ are removed.
- Does not yield a corrected time series, $y[n]$. 
Subspace Projection Beamforming

- Signal Model for \( j^{th} \) STI window of length \( L \):
  \[
x[n] = [x_1[n], \ldots x_p[n]]^T = a_s s[n] + A_d[n]D[n] + \eta[n]
  \]
  \[
  X_j = [x[jL], x[jL+1], \ldots, x[(j+1)L-1]].
  \]

- Sample covariance for \( j^{th} \) STI:
  \[
  \hat{R}_j = \frac{1}{L} X_j X_j^H
  \]

- Partitioned eigen decomposition estimates subspace \( U_d \) spanning interference array signature, \( A_d[jL] \):
  \[
  \hat{R}_j[U_d \mid U_{s+\eta}] = [U_d \mid U_{s+\eta}] \Lambda
  \]

- Projection beamformer weight for \( j^{th} \) STI:
  \[
  w_j = P_j \tilde{w} \text{ where } P_j = I - U_d U_d^H. \text{ Note } P_j A_d[jL] = 0
  \]
Uncorrected Array PSD Estimator

- Beamformed output vector for \( j^{th} \) STI:
  \[
  y_j^T = w_j^H X_j = \tilde{w}^H P_j X_j
  \]

- Welch’s PSD estimator for projection beamformer:
  \[
  \hat{S}_y^T = \frac{1}{M} \sum_{j=0}^{M-1} \left| \text{DFT}\{y_j^T G\} \right|^2 \\
  = \frac{1}{M} \sum_{j=0}^{M-1} \left| \tilde{w}^H P_j X_j GF \right|^2 \\
  = \frac{1}{M} \left( \tilde{w}^H \otimes \tilde{w}^T \right) \sum_{j=0}^{M-1} \left( P_j \otimes P_j^* \right) \left( (X_j GF') \circ (X_j GF')^* \right)
  \]

\( G \) is diagonal windowing matrix and \( F \) is the unitary DFT matrix.
The Corrected Array PSD Estimator

- Bias corrected PSD estimator:

\[
\hat{S}_{y,c}^T = \frac{1}{M} (\tilde{\mathbf{w}}^H \otimes \tilde{\mathbf{w}}^T) C^{-1} \sum_{j=0}^{M-1} (\mathbf{P}_j \otimes \mathbf{P}_j^*) (X_j \text{GF}) \circ (X_j \text{GF})^*
\]

\[
C = \frac{1}{M} \sum_{j=0}^{M-1} (\mathbf{P}_j \otimes \mathbf{P}_j^*)
\]

- Since \( \mathbf{P}_j \) removes the the nonstationary interference from \( X_j \), on average \( C^{-1} \) cancels bias caused by \( \mathbf{P}_j \).

\[
E\{\hat{S}_{y,c}^T\} = \frac{1}{M} (\tilde{\mathbf{w}}^H \otimes \tilde{\mathbf{w}}^T) C^{-1} \sum_{j=0}^{M-1} (\mathbf{P}_j \otimes \mathbf{P}_j^*) E\{(X_j \text{GF}) \circ (X_j \text{GF})^*\}
\]

\[
= (\tilde{\mathbf{w}}^H \otimes \tilde{\mathbf{w}}^T) E\{(X_{s+\eta,l} \text{GF}) \circ (X_{s+\eta,l} \text{GF})^*\}
\]

same as Welch’s PSD w/o interference!
Simulation Results

- Seven element uniform line array, no reflector.
- Half wavelength element spacing.
- $10^5$ samples in the long-term integration (LTI).
- 512 samples per STI window, no overlap.
- Hamming window shaping.
- Two moving interferers, starting at -$40^\circ$ and $33^\circ$.
- One stationary desired source at $5^\circ$.
- Array is calibrated in source direction.
- Response in all other directions is unknown.
Subspace Projection Beam Response vs Time

- **Interferers**
  - 10 dB INR for both.
  - Moving at $4.5 \times 10^{-4}$ and $3.0 \times 10^{-4}$ degrees/sample.
  - FM modulation.
  - One interferer freq. band overlaps desired source.

- **Source**
  - -30 dB SNR.
  - Narrowband.

- Adaptive null cuts into main lobe near the end of long term integration.
- Significant beamshape distortion.
Last STI Beampatterns

- Subspace projection keeps a null on the mainlobe interferer.
  - High sidelobes raise noise power in beamformer output.
- LCMV has less distortion due to minimum variance criterion.
  - Trades-off noise v.s. interference.
- All have calibrated 0 dB response to source signal at 5°.
Effective Average Beampatterns over all STIs

- Computed as follows:
  - Use beamformer weights for each STI as computed from interference data set.
  - Place narrowband unit power source at a test bearing $\theta$.
  - Calculate PSD using all STIs. Power at source frequency is the response for direction $\theta$.
  - Repeat for all $\theta$.

- Bias correction removes subspace projection distortions!

- Effective response exactly matches conventional beamformer!
PSD Estimate Comparisons

- Only subspace projection methods adequately cancel mainlobe interferer.
- Only bias corrected subspace projection keeps noise floor low enough to see signal.
- No bias in noise PSD level, but noise estimation variance is a bit higher.
When Interferer Stays Outside Mainlobe

Sidelobe rumble is evident, but mainlobe is relatively unperturbed.

All methods but non-adaptive conventional beamforming yield good PSDs.
When Interferer Stays Outside Mainlobe

Applications sensitive to small changes in sidelobe pattern shape would still benefit from bias correction.
Conclusions

- Effective bias correction has been demonstrated for array PSD estimation during RFI cancellation.
- Unbiased array PSD estimator can be applied to any array configuration (not just array feeds).
- This will be useful for beamforming clusters of dishes being proposed for some SKA configurations. Also LOFAR stations.
- Next step is accurate feed array simulation.
- Real data experiments are underway.
- Full effective beampattern is hard to determine experimentally.